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STRATEGIES TO IMPROVE THE DIRECT SEARCH METHOD FOR NON-CONVEX WIND-THERMAL COORDINATION DISPATCH PROBLEM CONSIDERING TRANSMISSION CAPACITY

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STRATEGIES TO IMPROVE THE DIRECT SEARCH METHOD FOR NON-CONVEX WIND-THERMAL COORDINATION DISPATCH PROBLEM CONSIDERING TRANSMISSION CAPACITY

Chun-Lung Chen

Key words: wind-thermal coordination dispatch, transmission capacity limits, wind power penetration level, area spinning reserve, stochastic direct search method.

ABSTRACT

This paper presents some strategies to improve the direct search method (DSM) for solving the multi-area wind-thermal coordination dispatch (MWCD) problem considering the non-linear characteristics of a generator such as valve-point effects. Although the DSM approaches have several advantages suitable to tackle the difficult non-convex economic dispatch (NED) problems, they still have the drawbacks such as exploration problem, exploitation problem and constraint handling problem. The main problem of the conventional DSM is that it gets easily trapped in a local optimal solution due to premature convergence. This paper proposes a stochastic direct search method (SDSM) employing the parallel stochastic searching mechanism to increase both exploration and exploitation capability of the DSM. Numerical experiments are included to demonstrate that the proposed SDSM approach can obtain a higher quality solution with better performance.

I. INTRODUCTION

The rise of fuel prices and the progressive exhaustion of traditional fossil energy sources have increased the interests in economic dispatch (ED) problem, which is considered as one of the complex problems in modern energy management systems to be tackled. The objective of ED is to determine an optimal combination of power output so that the fuel cost of generation can be minimized, while simultaneously satisfying the equality and inequality constraints (Wood and Wollenberg, 1996). For simplicity, the associated incremental costs of the units are assumed to be monotonically increasing and is solved using several classical mathematical programming techniques, such as the lambda dispatch approach, the gradient method, the linear programming and the Netwon's method (Wood and Wollenberg, 1996). Unfortunately, the generating units exhibit a greater variation in the fuel cost functions due to valve-point loading, prohibited operating zones, etc. (Wood and Wollenberg, 1996). The inclusion of non-smooth cost function increases the non-linearity as well as the number of local optima in the solution space. These complex conditions make it very difficult to solve the non-convex economic dispatch (NED) problem. Besides, the problem is further complicated to the NED problem imposed by adding the large-scale integration of wind power. The traditional mathematical approaches cannot be used to solve the practical NED problem due to the inclusion of non-smooth fuel cost functions. Development of more advanced algorithms is necessary to produce more economic schedules.

Dynamic programming (DP) solution is one of the approaches to solve the difficult NED problem without restrictions on the shape of fuel cost functions. However, the DP method may suffer from the curse of dimensionality (Wood and Wollenberg, 1996) or local optimality (Liang and Glover, 1992). Over the past decades, many stochastic searching techniques have been developed to solve the highly nonlinear NED problem, including simulated annealing (SA) (Wong and Fung, 1993), genetic algorithm (GA) (Walters and Sheble, 1993), tabu search algorithm (TSA) (Lin et al., 2002; Sa-ngiamvibool et al., 2011), evolutionary programming (EP) (Yang et al., 1996), differential evolution (Noman and Iba, 2008), particle swarm optimization (PSO) (Gaing, 2003; Park et al., 2005; Chaturvedi et al., 2008; Subbaraj et al., 2010; Hosseinnezhad and Babeei, 2013), hybrid stochastic search (Victoire and Jeyakumar, 2004; Selvkumar and Thanushkodi, 2007; Alsumait et al., 2010; Kumar et al., 2011; Subbaraj et al., 2011; Subathra et al., 2015) and direct search method (DSM) (Chen and Chen,

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2001; Chen, 2006; Chen et al., 2014). Among all, the DSM approach is of particular interest because of its simple concept, easy implementation and computational efficiency. Although DSM approaches provide several advantages to tackle the difficult NED problems, they have drawbacks in exploration, exploitation, and constraint handling. Conventional DSM requires further research to improve its performance and robustness.

To increase the possibility of exploring the search space where the global optimal solution exists, the selection of calculation step S in the direct search procedure is vital to the success of DSM for preventing premature convergence problem. This study, considering valve-point effects, extends the existing work on the DSM solution to solve the multi-area wind-thermal coordination dispatch (MWCD) problem. Instead of deterministic rules, another stochastic calculation step is designed to further provide a well-balanced mechanism of global and local optimization in the direct search procedure. Using the stochastic searching mechanism, the proposed stochastic direct search method (SDSM) searches for many optimum points to increase the DSM diversity for restraining early convergence. The proposed SDSM also incorporates sequential dispatch into direct search procedure to provide coordination of energy and reserve dispatch without restrictions on the shape of cost functions. To deal with the coupling constraints of the MWCD problem, an effective constraint handling technique is also proposed to dispatch the multi-area wind and thermal generation concurrently. Appropriate setting of control parameters of the SDSM is recommended to enhance its search capacity for preventing premature convergence. Numerical experiments are included to demonstrate the merits of the proposed algorithm.

II. FORMULATION OF MWCD PROBLEM AND CONSTRAINTS

1. Notation

The following notation is used throughout the paper.

a_i, b_i, c_i, e_i, f_i:	cost coefficients of thermal unit i
ASR_U^A, ASR_U^B:	additional up-reserve requirements in each area (considering wind power generation)
ASR_D^A, ASR_D^B:	additional down-reserve requirements in each area (considering wind power generation)
$d\%$:	percentage of maximum unit capacity
DC_{iq}:	decrement cost of thermal unit i for candidate q
DC_{Wjq}:	decrement cost of wind unit j for candidate q
DS_i:	down-reserve contribution of thermal unit i
$DS_i^{\max}$:	maximum down-reserve contribution of thermal unit i
DSM:	direct search method
EDSM:	enhanced direct search method
$F_i(\bullet)$:	production cost function of thermal unit i
FT:	total operating cost
i:	index for thermal units
IC_{iq}:	incremental cost of thermal unit i for candidate q
IC_{Wjq}:	incremental cost of wind unit j for candidate q
j:	index for wind units
L_T:	total number of convergence level
MWCD:	multi-area wind-thermal coordination dispatch
NED:	non-convex economic dispatch
NP_B:	base number of saved candidates at each convergence level (NP_B = 10 in the study case)
NT:	number of thermal units in system
NW:	number of wind units in system
P_D:	total load demand
P_D^A, P_D^B:	load demand in each area
P_{AB}, $P_{AB}^{\max}$:	transfer power and flow limits from area A to area B respectively
P_i:	generation of thermal unit i
$P_i^{\max}$:	upper generation limit of thermal unit i
$P_i^{\min}$:	lower generation limit of thermal unit i
$P_{Wj}^{\max}$:	upper generation limit of wind unit j
$P_{Wj}^{\min}$:	lower generation limit of wind unit j
P_{Wj}^{*}:	available generation of wind unit j
P_{Wj}:	actual generation of wind unit j
P_{WT}^{A*}, P_{WT}^{B*}:	available area wind power generation
P_{WT}^{A}, P_{WT}^{B}:	actual area wind power generation
r%:	percentage of actual wind power generation
S_q:	random calculation step for candidate q
SDSM:	stochastic direct search method
U_{AB0}, D_{AB0}:	transfer up and down reserves from area A to area B respectively (only for satisfying the additional reserve requirements in each area)
U_{AB1}:	transfer up reserve from area A to area B
U_{AB2}:	transfer up reserve from area B to area A
USR_b^A, USR_b^B:	basic up-spinning reserve requirements in each area (not considering wind power generation)
US_i:	up-reserve contribution of thermal unit i
$US_i^{\max}$:	maximum up-reserve contribution of thermal unit i

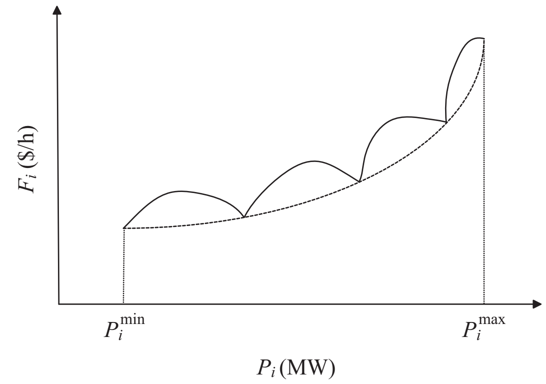

Fig. 1. Fuel cost curve of units with valve-point effects.

v :	wind speed
v_{Ij} :	cut in wind speed of wind unit j
v_{Rj} :	rated wind speed of wind unit j
v_{Oj} :	cut out wind speed of wind unit j
$z_U^A\%$, $z_U^B\%$:	percentage of local up reserve in each area
$\varphi_j(\bullet)$:	wind power curve of wind unit j
α:	scaling constant ($\alpha = 0.6$ in the study case)

2. Formulation

The main objective of solving the MWCD problem is to minimize the total fuel cost considering various constraints, such as import/export power balance, area basic up-spinning reserve requirements, area additional up/down reserve requirements, transmission capacity limits and constrained resource capacity shared between generation and reserve. Generally, the fuel cost of a generation unit will be a second-order polynomial function (Wood and Wollenberg, 1996). However, the thermal units with multi-valve steam turbines exhibit a greater variation in the fuel cost functions. A practical cost function encompasses a series of nonsmooth curves to represent the non-differentiable points due to the presence of the valve-point loading effects and the multiple fuel option. Walters and Sheble (1993) have shown the input-output performance curve for a typical thermal unit with many valve points. To include the valve-point loading effects, an additional rectified sinusoidal term is considered in the quadratic cost function. The cost curve function of units with valve point effects is depicted in Fig. 1. According to the network shown in Fig. 2, the mathematical model of the MWCD can be stated as follow.

Objective function:

$$Minimize\ FT = \sum_{i=1}^{NT} [a_i + b_i P_i + c_i P_i^2 + \left| e_i \sin(f_i (P_i^{\min} - P_i)) \right|] \quad (1)$$

subject to the following constraints.

1) System Constraints

(a) Import/export power balance constraints

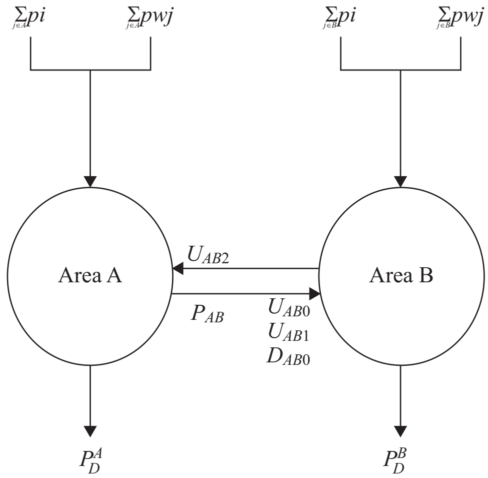

Fig. 2. A simple network model of multi-area wind-thermal system.

$$\sum_{i \in A} P_i + \sum_{j \in A} P_{Wj} - P_{AB} = P_D^A \quad (2)$$

$$\sum_{i \in B} P_i + \sum_{j \in B} P_{Wj} + P_{AB} = P_D^B \quad (3)$$

(b) Area additional up reserve requirement constraints

$$z_U^A \% \times \sum_{i \in A} US_i - U_{AB0} = ASR_U^A (\sum_{j \in A} P_{Wj}) \quad (4)$$

$$z_U^B \% \times \sum_{i \in B} US_i + U_{AB0} = ASR_U^B (\sum_{j \in B} P_{Wj}) \quad (5)$$

(c) Area basic up reserve requirement constraints

$$(1 - z_U^A)\% \times \sum_{i \in A} US_i + U_{AB2} \geq USR_b^A \quad (6)$$

$$(1 - z_U^B)\% \times \sum_{i \in B} US_i + U_{AB1} \geq USR_b^B \quad (7)$$

(d) Area additional down reserve requirement constraints

$$\sum_{i \in A} DS_i - D_{AB0} \geq ASR_D^A (\sum_{j \in A} P_{Wj}) \quad (8)$$

$$\sum_{i \in B} DS_i + D_{AB0} \geq ASR_D^B (\sum_{j \in B} P_{Wj}) \quad (9)$$

(e) Transmission capacity limits constraints

$$-P_{AB}^{\max} \leq P_{AB} + U_{AB1} + U_{AB0} - D_{AB0} \leq P_{AB}^{\max} \quad (10)$$

$$-P_{AB}^{\max} \leq -P_{AB} + U_{AB2} + D_{AB0} - U_{AB0} \leq P_{AB}^{\max} \quad (11)$$

2) Thermal Generator Constraints

(f) Unit capacity constraints

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{12}$$

(g) Unit's maximum up/down reserve contribution constraints

$$US_i^{\max} = d\% \times P_i^{\max} \tag{13}$$

$$DS_i^{\max} = d\% \times P_i^{\max} \tag{14}$$

(h) Unit's up/down spinning reserve contribution constraints

$$US_i = \min\left\{US_i^{\max}\ ,\ P_i^{\max} - P_i\right\} \tag{15}$$

$$DS_i = \min\left\{DS_i^{\max}\ ,\ P_i - P_i^{\min}\right\} \tag{16}$$

(i) Generation and reserve capacity coupling constraints

$$P_i + US_i \le P_i^{\max} \tag{17}$$

$$P_i - DS_i \ge P_i^{\min} \tag{18}$$

3) Wind Generator Constraints

(j) Wind power curve and wind speed limits

$$P_{Wj}^{*} = \begin{cases} 0 & v \le v_{Ij}\ or\ v > v_{Oj} \\ \varphi_j(v) & v_{Ij} \le v \le v_{Rj} \\ P_{Wj}^{\max} & v_{Rj} \le v \le v_{Oj} \end{cases} \tag{19}$$

(k) Available area wind power generation

$$P_{WT}^{A*} = \sum_{j \in A} P_{Wj}^{*} \tag{20}$$

$$P_{WT}^{B*} = \sum_{j \in B} P_{Wj}^{*} \tag{21}$$

(l) Actual area wind power generation

$$0 \le P_{WT}^{A} \le P_{WT}^{A*} \tag{22}$$

$$0 \le P_{WT}^{B} \le P_{WT}^{B*} \tag{23}$$

III. BRIEF REVIEW OF DSM AND ITS SOLUTION DIFFICULTY

The DSM is one of the modern heuristic algorithms suitable to solve the large-scale NED optimization problems. The DSM, first suggested by Chen and Chen (2001), has been successfully applied to ED problem considering transmission capacity constraints. Like DP algorithm, the DSM performs a direct search of solution space without restrictions on generator cost function. Several inequality and equality constraints can be handled properly in the direct search procedure without introducing any multipliers. The multi-level convergence strategy is also used to reduce the step size gradually to guarantee a possible complete examination of the solution space. The outline of the simple DSM algorithm is shown in the flow chart in Fig. 3. Experimental results reveal the proposed algorithm is an efficient approach for determining the optimal generation schedules when the generator incremental cost curves are monotonically increasing. However, the conventional DSM makes no guarantee that the solutions are optimal or even close to the optimal solution when the nonlinear characteristics of a generator are considered. The solutions obtained from the DSM largely depend on the parameter selection, such as initial random starting points and the values of initial step size S_1 and reduced factor K.

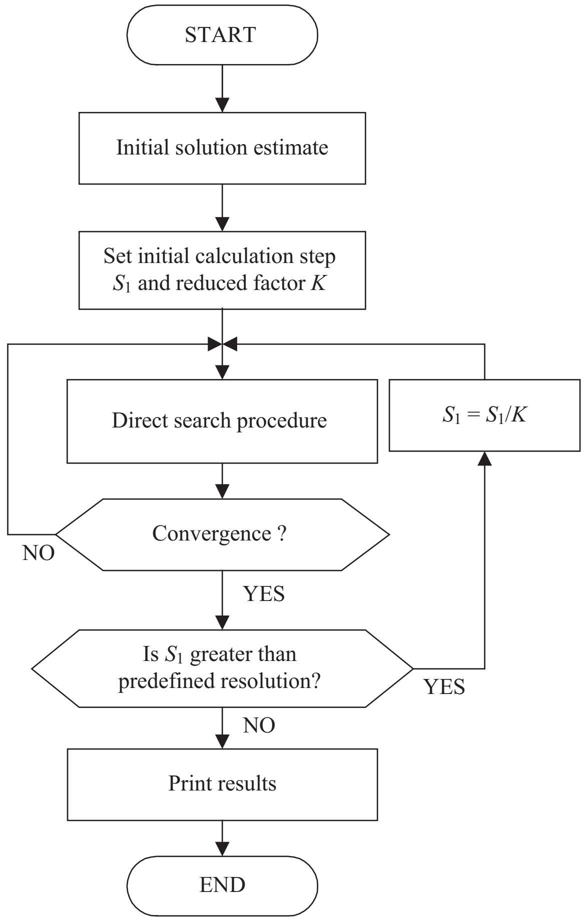

Fig. 3. Simplified flow chart of the DSM approach.

To improve the global searching capability, an enhanced DSM (EDSM) employing the parallel nature of evaluation programming is proposed to solve the NED problem (Chen, 2006).

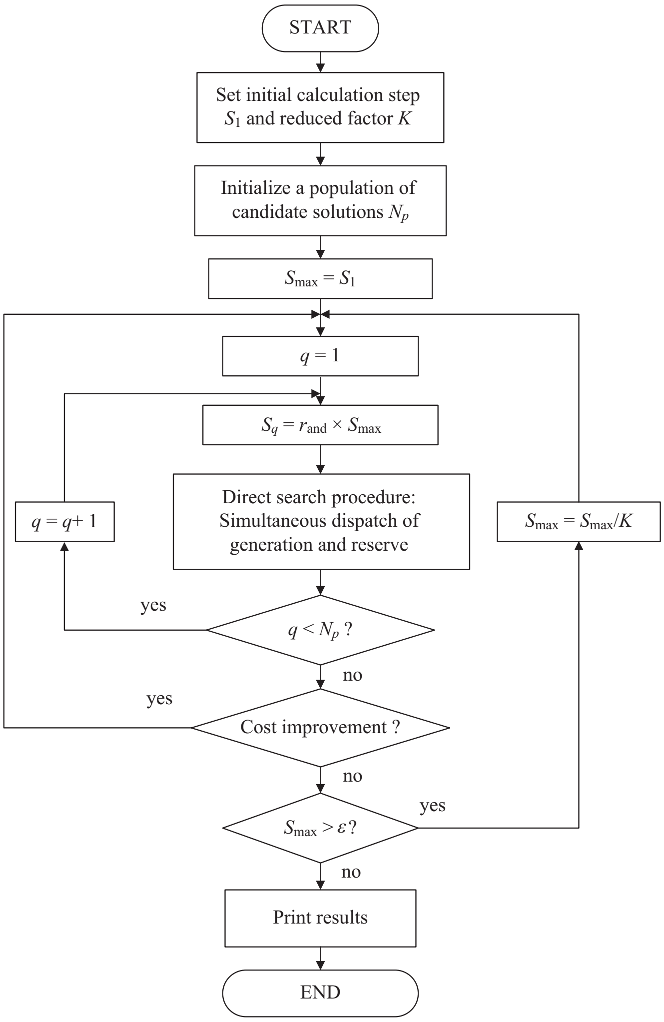

Fig. 4. Simplified flow chart of the SDSM approach.

Recently, a penalty function-direct search method (PF-DSM) employing an effective constraint handling technique is also developed to solve the problem of MWCD in a hybrid power system (Chen et al., 2014). In a previous work on DSM approaches, a larger initial step size S_1 is desired to make the search effective and the step size is then successively refined until the calculation step is less than the predetermined resolution. It is obvious that the conventional DSM with a coarse convergence step can enhance the global exploration ability but results insufficient capability to find nearby extreme points (exploitation problem). In contrast, the DSM with a refined convergence step can improve the local exploitation ability but is easily trapped in local minima (exploration problem). As a result, the standard DSM with the selection of predetermined calculation step may mislead the search and it gets easily trapped in a local optimal solution due to lack of a well-balanced mechanism between the global exploration and local exploitation abilities. To increase the possibility of exploring the search space where the global optimal solution exists, the parallel stochastic searching mechanism is employed in the study to increase the diversity of DSM and overcome trapping into local minimum problem.

IV. PROPOSED SOLUTION METHODOLOGY AND IMPLEMENTATION OF SDSM

Like many stochastic methods, the proposed SDSM is using the parallel stochastic searching mechanism to enhance its global searching capability. The outline of the proposed SDSM algorithm is shown in the flow chart in Fig. 4. Several heuristic strategies are applied to improve the SDSM solution quality and performance. The overall procedure of the proposed algorithm can be stated in detail as follows:

1. Strategy for Constraints Handling

Incorporating wind units into the existing utility NED problem adds further complexity to the solution methodology due to the constraint handling problem. It is very important to develop an effective strategy for satisfying the equality and inequality constraints. One of the most widely used techniques to handle the constraints is through the use of penalty functions. The constraints represented by (2)-(23) will be treated in different ways. The system power balance equality (2) and (3), the generation limits inequality (12) with the generation and reserve capacity coupling inequality (17) and (18) can be handled properly in the stochastic direct search procedure. The available area wind generation can be obtained from the wind speed by applying the wind power curve (constraints (19)-(21)). The area actual output of WTGs can also be controlled to any desired value through blade pitch control (constraints (22) and (23)). To account for area additional up reserve requirement violations (4) and (5), area basic up reserve requirement violations (6) and (7), area additional down reserve requirement violations (8) and (9) and transmission capacity limit violations (10) and (11), the total operating cost is augmented by nonnegative penalty terms PC_1, PC_2, PC_3 and PC_4, respectively, penalizing constraint violations. Thus, the augmented cost function is formed

$$FT_A = FT + \sum_{b=1}^{4} PC_b \tag{24}$$

where PC_1 is the penalty term for Eqs. (4) and (5); PC_2 is the penalty term for Eqs. (6) and (7); PC_3 is the penalty term for Eqs. (8) and (9); PC_4 is the penalty term for Eqs. (10) and (11). The penalty terms (PC_1-PC_4) are proportional to the corresponding violations and zero in case of no violation. There are chosen high enough as to make constraint violations prohibitive in the final solution.

2. Strategy for Initialization

In order to explore the search space where the global optimal solution exists, the second heuristic strategy is to generate a population of *NP* initial candidate solutions at random and finds solutions in parallel using a stochastic direct search procedure.

The computation steps of an initial candidate solution are shown as follows.

Step 1: Let rand be a uniform random value in the range [0, 1]. The initial power outputs of $NT-1$ thermal generating units and NW wind generating units, without violating generation limits, are generated randomly by

$$P_i = P_i^{\min} + rand \times (P_i^{\max} - P_i^{\min}) \tag{25}$$

$$P_{Wj} = rand \times P_{Wj}^{*} \tag{26}$$

Step 2: To satisfy the power balance equation, a dependent generating unit is arbitrarily selected among the committed NT units and the output of the dependent generating unit P_d is determined by

$$P_d = P_D - \sum_{\substack{i=1 \\ i \neq d}}^{NT} P_i - \sum_{j=1}^{NW} P_{Wj} \tag{27}$$

Step 3: If P_d with violating (12), a repairing strategy is applied to pick one thermal unit at random to increase (or decrease) its output by the random or predefined step (e.g., 10 MW), one by one, until it can satisfy the power balance constraints.

Step 4: Calculate the area additional reserve requirement according to the amount of actual area wind power generation.

Step 5: A simplified sequential dispatch method, described in Ref. (Chen et al., 2014), is used to solve the multi-area reserve dispatch when the generation dispatch solution is frozen. Note that the penalty terms (PC_1-PC_4) will be evaluated if there is any violation of the system constraints.

Step 6: Calculate the initial operating cost (including production cost and penalty cost).

3. Strategy for Preventing Premature Convergence

In applying the conventional DSM to solve the MWCD problem, it is quite likely that the final solution may lead to suboptimal solution owing to the inclusion of non-smooth cost function. In general, the initial candidate solutions are usually far from the global optimum, and hence, the larger calculation step S may prove to be beneficial. However, it is not reasonable for all candidate solutions to employ the same calculation step S in a convergence level. How to provide a well-balanced mechanism between the global and local exploration abilities becomes an important problem in the study to avoid earliness convergence. In order to improve the global searching capability, the third heuristic strategy is to employ the parallel stochastic searching mechanism to make full use of its exploration and exploitation capability. Thus, the selection of step size S for all candidates will be different in a convergence level and these calculation steps for all candidates will play the role of balancing the global and local exploration abilities. Large calculation step S enables the SDSM to explore globally and small calculation step S enables the SDSM to explore locally. During successive steps for the population, the global and local exploration abilities in the direct search procedure will be increased. After the first level converges, the step size is then successively refined with $S_1 = S_1/K$ during each convergence level until the S_1 is less than the predetermined resolution ε. It is obvious that the reduced factor K will also play the role of preventing premature convergence. In general, as the number of convergence levels increases, the balance of exploration and exploitation abilities can be enhanced, so that the solution quality can also be improved. Although an arbitrary choice of calculation step S may mislead the search, it can be improved by the multi-level convergence technique to increase the possibility of creating and exploring the new solution in the search space. Unfortunately, the appropriate selection of these parameters justifies the preliminary efforts required for their experimental determination. However, the SDSM with large S_1 and small K is usually commended, and this is confirmed through numerical experiments. From our experience, a proper initial calculation step S_1 is chosen to be 20~40% of the largest generation unit in the power system. The recommended value of the reduced factor K is 1.01~2.0 depending on the number of local minimum points in the cost functions.

4. Stochastic Direct Search Procedure for Candidates

Like many stochastic methods, multiple random starts are used in the direct search procedure to explore the search space where the global optimal solution exists. To find a direction that reduces the operating cost and leads to a point within the feasible region, another procedure may be needed to augment the searching technique with light computational expenses. The computation steps of the stochastic direct search procedure for candidate q are shown as follows:

Step 1: Generate a random calculation step S_q between 0 and S_1 for candidate q.

Step 2: Units, without violating the maximum or minimum generation limits, are chosen to increase or decrease their outputs by the random step S_q for calculating their incremental costs (IC) and decrement costs (DC). This is shown as follows:

$$\begin{cases} IC_{iq} = \dfrac{F_i(P_i + S_q) - F_i(P_i)}{S_q}, & i \in thermal\ generator \\ IC_{Wjq} = 0, & j \in wind\ generator \end{cases} \tag{28}$$

$$\begin{cases} DC_{iq} = \dfrac{F_i(P_i) - F_i(P_i - S_q)}{S_q}, & i \in thermal\ generator \\ DC_{Wjq} = 0, & j \in wind\ generator \end{cases} \tag{29}$$

subject to

$$\begin{cases} P_i + S_q \le P_i^{\max}, & i \in thermal\ generator \\ P_{Wj} + S_q \le P_{Wj}^{*}, & j \in wind\ generator \end{cases}$$

$$i = 1, 2, \ldots, NT; j = 1, 2, \ldots, NW \tag{30}$$

$$\begin{cases} P_i - S_q \ge P_i^{\min}, & i \in thermal\ generator \\ P_{Wj} - S_q \ge 0, & j \in wind\ generator \end{cases}$$

$$i = 1, 2, \ldots, NT; j = 1, 2, \ldots, NW \tag{31}$$

Step 3: All units are examined to check if there is any improvement. If no more improvement can be achieved, then stop; otherwise, go to step 4.

Step 4: An independent unit with minimum incremental cost *ICx* (assume unit *x*) is chosen to increase its output by the random step S_q, and then, only a dependent unit *DCy* (assume unit *y*, $y \ne x$) while gaining the most reduction in the total operating cost (including production cost and penalty cost), should be selected to reduce its output to satisfy the load balance equation. At each possible step, it should be noted that the total penalty cost is always calculated first by the simplified dispatch method before the generation dispatch solution is frozen.

Step 5: The outputs of this particular pair of units will be adjusted again by the random step S_q if they do not violate generation limits, and only the incremental cost of unit *x* and the decrement cost of unit *y* need to be recalculated.

Step 6: Go to step 3.

5. Strategy for Restricting the Search Range

To enhance the solution quality of SDSM, a larger population size NP is desired to increase the possibility of finding the global optimal solution for the MWCD problem. However, it is obvious that the major portion of the computing time is spent in performing the stochastic direct search procedure for evaluating the fuel costs of candidates. In order to improve the performance of SDSM, the last heuristic strategy is to restrict the number of candidate solutions to be examined during each convergence level. In general, the most economic candidate solution of the previous level will increase the possibility of finding the global optimal solution. Thus, most of the previously saved higher-cost candidate solutions could be eliminated immediately without searching again at current level. To preserve the solution optimization, more candidates may have to be saved during a coarse convergence level. The reason is that, although some candidates may not be economic solutions at current level, those candidates may have more potential to decrease the operating cost for future successive levels, but in a refined convergence level, most of the higher-cost candidate solutions could be discarded since the production cost is not sensitive to the calculation step. The following model was considered for relating the number of saved candidates (NP_L) to the convergence level (L). The number of saved candidate solutions at level L is determined as follows:

$$NP_L = Max\{NP_B\ ; NP_{L-1} \times (1 - \alpha \times \frac{L}{L_T})\} \tag{32}$$

V. NUMERICAL EXAMPLES

The proposed approach is applied to several test systems to verify the feasibility and effectiveness of the SDSM algorithm. All computations are performed on a PC Pentium (R) Dual CPU 2.00 GHz computer with 1.0G RAM size, and the following computer programs are developed in FORTRAN:

DSM: Direct search method with a single initial random solution (Chen and Chen, 2001)
EDSM: DSM with a deterministic calculation step for candidate solutions (Chen, 2006)
SDSM: DSM with a stochastic calculation step for candidate solutions
SDSM*: SDSM with restricting the search range

Because of the randomness of heuristic algorithms, their performance cannot be judged from a single run. Thirty trials with different initial conditions should be made to acquire a useful conclusion about the performance. These cases are stated in detail as follows:

1. Example 1: Test for a 3-Unit System

In the first example, a system with three generating units considering non-smooth fuel cost functions is studied. The test system unit data and the loss expression are described in (Liang and Glover, 1992). The load demand is set to 1400 MW. The classical mathematical programming techniques, such as the lambda-iteration dispatch method, cannot be used to solve the problem due to its non-smooth fuel cost function. Many stochastic searching techniques, such as DP algorithm and SA algorithm, have been developed to solve the nonlinear NED problem. However, only the local optimal solution can be founded by the DP approach (Liang and Glover, 1992) ($6642.26) and the SA approach (Wong and Fung, 1993) ($6639.5043). Note that only a single initial random solution is needed in the studied case by using the proposed SDSM algorithm to obtain the optimal solution ($6639.18) since the problem dimension is low. To illustrate the good convergence property of the proposed algorithm, Table 1 gives a comparison of the total number of iterations required and production costs during each convergence level. From this result, the total cost is not sensitive to the calculation step S_1. The execution of program is so fast that the CPU times can't be found out in this studied case. The efficient approach makes it an attractive method for the solution of the small-size NED dispatch problem.

Table 1. Comparison of iterations and costs under various S_1 for the load of 1400 MW in 3-unit example system.

Convergence		Iterations	Cost ($/h)	Losses (MW)
S_1 (MW)	S (MW)			
Initialization		---	6766.27	57.3219
120.00	11.17	12	6653.23	62.3115
80.00	56.48	1	6653.23	62.3115
53.33	3.400	3	6643.58	62.6522
35.55	20.81	1	6643.58	62.6522
23.70	16.39	1	6643.58	62.6522
15.80	7.665	1	6643.58	62.6522
10.53	6.369	1	6643.58	62.6522
7.023	1.476	2	6639.28	62.7593
4.682	1.455	1	6639.28	62.7593
3.121	1.422	1	6639.28	62.7593
2.080	0.848	1	6639.28	62.7593
1.387	1.257	1	6639.28	62.7593
0.924	0.361	3	6639.28	62.7590
0.616	0.350	2	6639.28	62.7487
0.411	0.164	1	6639.28	62.7487
0.274	0.042	5	6639.27	62.7533
0.182	0.092	1	6639.27	62.7533
0.121	0.027	4	6639.19	62.7555
0.081	0.075	1	6639.19	62.7555
0.054	0.043	2	6639.19	62.7542
0.036	0.028	1	6639.19	62.7542
0.024	0.005	1	6639.19	62.7542
0.016	0.014	1	6639.19	62.7542
0.010	0.008	1	6639.19	62.7542
0.007	0.002	8	6639.18	62.7549

Parameter Setting in SDSM: $NP = 1$; $S_1 = 120$ MW; $K = 1.5$; $\varepsilon = 0.01$ MW

2. Example 2: Test for a 40-Unit System

In the second example, a system with forty generating units considering the valve-point effects is studied to test the solution quality and performance of the proposed SDSM algorithm. The test system unit data is given in (Sinha et al., 2003) and the total load demand is set to 10500 MW. The same multiple minimum problem has been solved by the MTS (Sa-ngiamvibool et al., 2011), IFEEP (Sinha et al., 2003), PSO-SQP (Victoiro and Jayakumar, 2004), MPSO (Park et al., 2005), NPSO-LRS (Selvakumar and Thanushkodi, 2007), TSARGA (Subbaraj et al., 2011), GA-PS-SQP (Alsumait et al., 2010), HMAPSO (Kumar et al., 2011), SOH-PSO (Chaturvedi et al., 2008), PSO-MSAF (Subbaraj et al., 2010), θ-PSO (Hosseinnezhad and Babaei, 2013) and CE-SQP (Subathra et al., 2015). The corresponding costs of the obtained best solution from SDSM are compared with those of the previous researches in Table 2. From these results, the proposed algorithm can find a better solution ($121412.5) than many existing techniques, and has clearly shown the superiority to the previous researches in terms of minimum cost as well as average cost. Note that the results also highlight the superiority of the SDSM algorithm over the basic DSM and EDSM. Details of the best solutions obtained by the proposed SDSM algorithm is shown in the Table 3. To investigate effects of different parameters chosen on the final results, twelve cases were simulated for the proposed SDSM algorithm. Table 4 shows the best cost and average cost achieved for 30 trial runs. From the results, the

Table 2. Comparison of results of different methods for the 40-unit system.

Methods	Minimum cost ($)	Avg. cost ($)	Maximum cost ($)
MTS (Sa-ngiamvibool et al., 2011)	121532.10	121798.51	122022.15
IFEP (Sinha et al., 2008)	122624.35	123382.00	125740.63
PSO-SQP (Victoiro and Jayakumar, 2004)	122094.67	122245.25	---
MPSO (Park et al., 2005)	122252.265	---	---
NPSO-LRS (Selvakumar and Thanushkodi, 2007)	121664.4308	122209.3185	122981.5913
TSARGA (Subbaraj et al., 2011)	121463.07	122928.31	124296.54
GA-PS-SQP (Alsumait et al., 2010)	121458	122039	---
HMAPSO (Kumar et al., 2011)	121586.90	121586.90	121586.90
SOH-PSO (Chaturvedi et al., 2008)	121501.14	121853.57	122446.30
PSO-MSAF (Subbaraj et al., 2010)	121423.23	---	---
θ-PSO (Hosseinnezhad and Babaei, 2013)	121420.9027	121509.8423	121852.4249
CE-SQP (Subathra et al., 2015)	121412.88	121423.65	---
DSM	121476.8	122170.3	125104.9
	Avg. Time (s): 0.04		
EDSM	121412.7	121431.6	121461.8
	Avg. Time (s): 23.12		
SDSM	121412.5	121412.9	121414.6
	Avg. Time (s): 20.10		

Parameter Setting in DSM: $NP = 1$; $S_1 = 200$ MW; $K = 1.01$; $\varepsilon = 0.001$ MW.

Parameter Setting in EDSM (or SDSM): $NP = 600$; $S_1 = 200$ MW; $K = 1.01$; $\varepsilon = 0.001$ MW.

Table 3. Best dispatch results for the 40-unit system.

Unit No.	P_i	Unit No.	P_i	Unit No.	P_i	Unit No.	P_i
1	110.799600	11	94.000210	21	523.279900	31	189.999900
2	110.799600	12	94.000120	22	523.279800	32	189.999800
3	97.400350	13	214.759200	23	523.279100	33	189.999200
4	179.733600	14	394.279700	24	523.280000	34	164.799500
5	87.799680	15	394.278700	25	523.279000	35	199.999800
6	139.999200	16	394.279600	26	523.279100	36	194.396800
7	259.600200	17	489.278900	27	10.000210	37	109.999700
8	284.599300	18	489.278900	28	10.000630	38	110.000000
9	284.599300	19	511.279800	29	10.000220	39	109.999800
10	130.000600	20	511.278900	30	87.800590	40	511.278900

Table 4. Comparison of results with the different parameters chosen for the 40-unit system.

Case	S_1 (MW)	K	Minimum cost ($)	Avg. cost ($)	Avg. Time (s)
1	200	1.50	121412.5	121414.6	1.27
2	200	1.20	121412.5	121413.6	1.86
3	**200**	**1.01**	**121412.5**	**121412.9**	**20.10**
4	120	1.50	121412.5	121415.4	1.25
5	120	1.20	121412.5	121414.0	1.83
6	120	1.01	121412.5	121413.2	19.46
7	80	1.50	121412.5	121416.5	1.22
8	80	1.20	121412.5	121415.1	1.80
9	80	1.01	121412.5	121414.4	19.10
10	60	1.50	121420.9	121470.8	1.24
11	60	1.20	121414.6	121470.6	1.76
12	60	1.01	121412.5	121450.4	18.86

Parameter Setting in SDSM: $NP = 600$; $\varepsilon = 0.001$ MW.

Table 5. Comparison of results with 30 trial tests under various *NP* in the 40-unit system.

NP	Minimum cost ($)	Avg. cost ($)	Avg. Time (s)
10	121414.6	121428.1	0.32
50	121412.5	121415.6	1.61
100	121412.5	121414.2	3.23
200	121412.5	121414.0	6.55
300	121412.5	121413.7	9.76
400	121412.5	121413.4	13.03
500	121412.5	121413.1	16.30
600	121412.5	121412.9	20.10
1000	121412.5	121412.7	32.70

Parameter Setting in SDSM: $S_1 = 200$ MW; $K = 1.01$; $\varepsilon = 001$ MW.

Table 6. Comparison of results of different methods for the 80-unit system.

Methods	Minimum cost ($)	Avg. cost ($)	Maximum cost ($)
CSO (Selvakumar and Thanushkodi, 2009)	243195.38	243546.63	---
PSO (Selvakumar and Thanushkodi, 2009)	244188.35	246375.87	---
SCA (Selvakumar and Thanushkodi, 2009)	250864.05	254579.79	---
CE-SQP (Subathra et al., 2015)	242883.04	242945.25	---
DSM	**243121.9**	**245941.5**	**254515.0**
	Avg. Time (s): 0.11		
EDSM	**242909.1**	**242970.1**	**243047.3**
	Avg. Time (s): 101.41		
SDSM	**242794.7**	**242812.4**	**242826.1**
	Avg. Time (s): 102.12		

Parameter Setting in DSM: $NP = 1$; $S_1 = 200$ MW; $K = 1.01$; $\varepsilon = 0.01$ MW.

Parameter Setting in EDSM (or SDSM): $NP = 1000$; $S_1 = 200$ MW; $K = 1.01$; $\varepsilon = 0.01$ MW.

SDSM with large S_1 and small K is usually commended. In the study cases, a proper initial calculation step S_1 is chosen to be 200 MW and the recommended value of reduced factor K is 1.01~1.2 depending on the number of local minimum points in the cost functions. To investigate the effects of initial trail solutions on the final results, different initial random solutions were given to the SDSM approach. Table 5 shows the dispatch results under various population sizes for 30 trial runs. From this result, the total cost is not sensitive to the population size *NP*. The proposed SDSM algorithm has reached the optimal solution ($121412.5) with a high probability for the solution of the NED problem when the value of *NP* is chosen to be 1000. The results show that the proposed SDSM provides an accurate algorithm to tackle efficiently the difficult NED problem.

3. Example 3: Test for a 80-Unit System

In the third example, the simulation includes test runs for the large-scale system, used in Selvakumar and Thanushkodi (2009); and Subathra et al. (2015) to demonstrate the validity and effectiveness of the proposed algorithm. The 80-unit system is created simply by expanding example 2. There are many local optimal solutions for the dispatch problem and the problem is well suitable for testing and validating the developed SDSM algorithm. The results obtained by the proposed SDSM are compared with those obtained by using previously published methods, such as CSO (Selvakumar and Thanushkodi, 2009), PSO (Selvakumar and Thanushkodi, 2009), CSA (Selvakumar and Thanushkodi, 2009) and CE-SQP (Subathra et al., 2015). Table 6 depicts the numerical results of various methods. Although the best solution of SDSM is not guaranteed to be

Table 7. Best dispatch results for the 80-unit system.

Unit No.	P_i	Unit No.	P_i	Unit No.	P_i	Unit No.	P_i
1	110.799820	21	523.279372	41	110.799830	61	523.279362
2	110.799825	22	523.279363	42	110.799830	62	523.279365
3	97.399915	23	523.279374	43	97.399915	63	523.279372
4	179.733102	24	523.279376	44	179.733100	64	523.279374
5	87.799903	25	523.279363	45	87.799905	65	523.279374
6	140.000000	26	523.279374	46	140.000000	66	523.279365
7	259.599659	27	10.000007	47	259.599659	67	10.000004
8	284.599647	28	10.000005	48	284.599647	68	10.000000
9	284.599647	29	10.000014	49	284.599647	69	10.000002
10	130.000000	30	87.799903	50	130.000000	70	87.799905
11	168.799817	31	189.999986	51	168.799822	71	190.000000
12	94.000002	32	189.999995	52	94.000008	72	189.999999
13	214.759788	33	189.999996	53	214.759787	73	190.000000
14	394.279369	34	164.799820	54	394.279372	74	164.799820
15	394.279370	35	199.356192	55	394.279360	75	199.999992
16	394.279369	36	164.799832	56	304.519569	76	164.799832
17	489.279372	37	109.999996	57	489.279375	77	109.999986
18	489.279373	38	109.999995	58	489.279362	78	110.000000
19	511.279365	39	109.999997	59	511.279361	79	109.999914
20	511.279370	40	511.279373	60	511.279365	80	511.279373

Table 8. Comparison of results with 30 trial tests under various *NP* in the 80-unit system.

NP	Minimum cost ($)	Avg. cost ($)	Avg. Time (s)
10	242836.5	242956.7	1.02
100	242794.7	2428534	10.36
500	242794.7	242820.1	50.96
1000	242794.7	242812.4	102.12

Parameter Setting in SDSM: S_1 = 200 MW; K = 1.01 ; ε = 0.001 MW.

Table 9. Comparison of production costs and CPU times for various predefined resolution ε using SDSM in the 80-unit system.

ε (MW)	Minimum cost ($)	Avg. cost ($)	Avg. Time (s)
0.001	242794.7	242820.1	50.96
1.0	242798.1	242821.8	20.60
10.0	242802.4	242830.4	10.73
30.0	242815.5	242848.9	6.23
50.0	242827.1	242863.4	4.43

Parameter Setting in SDSM: NP = 500; S_1 = 200 MW; K = 1.01.

the global solution, the proposed SDSM has shown the superiority to the existing methods. Regarding the minimum and average cost, the proposed SDSM has found better solution ($242794.7) than the best solution previously found by CE-SQP, $242883.04 (Subathra et al., 2015). The basic DSM (or EDSM) offers no guarantee that the solutions are optimal or even close to the optimal solution. Table 7 contains details of the best solutions obtained using the proposed SDSM algorithm. Table 8 shows the solution obtained from SDSM depends on the population size. Increasing of population size will provide a better solution but takes longer computing time. This test case study converges within 50.96 sec for each run when the value of NP is chosen to be 500. It is obvious that the major portion of computing time is spent in performing the stochastic direct search technique. Fortunately, the numerical results show the production cost is close to optimal solution even in a coarse convergence level. Hence, only the simplified dispatch with a coarse convergence step could be used to compute the fuel costs in the process of SDSM for saving execution time. Table 9 gives a comparison of production costs and CPU times for various predefined resolution ε to demonstrate the advantage of approximate economic dispatch. This test case study converges within 4.43 sec with slightly sacrificing quality of the solution ($242827.1) when the value of ε is chosen to be 50.0. The suitableness of the algorithm presented in this paper to the solution of the optimal NED dispatch is, thus, confirmed.

4. Example 4: Test for a Two Area Wind-Thermal System

In the last example, the same 40-unit thermal system with two equivalent wind generation plants is considered (Sinha et al., 2003). We randomly divided forty thermal units into two areas of which Area A includes 20 units (1-20) and 30% of the total load demand and Area B has 20 units (21-40) and

Table 10. Comparison of results of four DSM strategies in system example 4.

Methods	*NP*	Minimum cost ($)	Avg. cost ($)	Avg. Time (s)
DSM (Chen et al., 2014)	1	110401.3	---	0.12
EDSM	100	109193.8	109535.2	76.28
SDSM	100	108767.4	109108.8	95.20
SDSM*	100	108767.4	109112.2	45.24
SDSM*	200	108725.4	109092.6	80.92

Parameter Setting: S_1 = 200 MW; K = 1.05; ε = 0.001 MW.

Table 11. Comparison of results considering the wind power generation or not for the load of 10500 MW.

Case	4.1	4.2
P_{WT}^{A*} / P_{WT}^{B*} (MW)	0/0	500/500
P_{WT}^{A} / P_{WT}^{B} (MW)	0/0	500/500
Flow Limit ($(P_{AB}^{\max})$ MW)	2000	2000
Line Flow ((P_{AB}) (MW)	1644	1600
USR_b^A / USR_b^B (MW)	600/600	600/600
ASR_U^A / ASR_U^B (MW)	0/0	100/100
$U_{AB0}/U_{AB1}/U_{AB2}$ (MW)	0/356/161	0/400/181
Local Up Spinning Reserve (Area A/B) (MW)	439/244	519/300
ASR_D^A / ASR_D^B (MW)	0/0	100/100
D_{AB0} (MW)	0	0
Local Down Spinning Reserve (Area A/B) (MW)	633 / 545	377 / 545
Fuel Cost (NT $/h)	121658.4	108725.4

70% of the total load demand. For simplicity, the available area wind power generation is assumed to be 500 MW. The basic up-spinning reserve requirements in each area are also assumed to be 600 MW. To cover the unpredictable wind generator output variations, the increased area up/down spinning reserve requirements are calculated as a simple fraction of the predicted area wind generation (r% = 20%). The flow limit from area A to area B is set to be 2000 MW. The maximum up/down spinning reserve of any single unit could not exceed more than 10 percent of its rated capacity (d% = 10%). To validate the performance of the proposed algorithm, four DSM strategies were developed for comparison. Table 10 shows the best cost and average cost achieved for 30 trial runs. The results show that the SDSM performs much better than basic DSM (or EDSM) as an optimizer and the superiority of the SDSM* algorithm over SDSM can also be noticed. Table 11 gives a comparison of results considering wind power generation or not for the load of 10500 MW. It should be noticed that the area B has limited generation capacity in this case. To compensate for possible fluctuations in power of the WTGs, part of local reserve in each area need to be first dispatched for satisfying its own area additional reserve requirement (100 MW). The basic up-reserve requirements for area B (600 MW) are then satisfied through the sum of resident local (300-100 = 200 MW) and imported reserve (U_{AB1} = 400 MW) from area A. In this test case, the fuel saving value is about 12933 $/h when the wind power generation (500 MW) in each area is considered. As a result, the proposed algorithm can be used to maximize the contribution of utility wind farms for reducing the cost of thermal dispatch while maintaining an adequate level of supply reliability.

VI. CONCLUSIONS

Incorporating wind units into the existing utility non-convex economic dispatch (NED) problem adds further complexity to the solution methodology. This paper presents a reliable and efficient method for solving the non-convex multi-area wind-thermal coordination dispatch (MWCD) problem. Instead of deterministic rules, the proposed stochastic direct search algorithm (SDSM) is using a stochastic technique to enhance its search capacity, which leads to a higher probability of obtaining the global optimal solution. The possibility of occurrence of finding the global optimal solution for the algorithm can be greatly increased by using the parallel stochastic searching mechanism. Several heuristic strategies are also used to improve the solution quality and performance. Compared to many stochastic searching techniques, the advantage of SDSM is that it is easy to implement and there are only few parameters to adjust. Several test systems with non-convex unit cost functions were used in this paper. The results show that the proposed EDSA provides a fast and accurate algorithm to tackle efficiently the difficult MWCD of a practical electric power system. It is observed that obtaining the global optimal solution is possible by using the proposed algorithm for the NED problem. The developed MWCD software will also be a useful tool to assess the impact and economic benefits of the installation of wind farms for the multi-area isolated power system.

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