



SEISMIC RESPONSE OF AN ARCH-BEAM INTERACTING WITH SEQUENTIAL MOVING TRAIN LOADS

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SEISMIC RESPONSE OF AN ARCH-BEAM INTERACTING WITH SEQUENTIAL MOVING TRAIN LOADS

Jong-Dar Yau¹, Ladislav Frýba², and Shota Urushadze²

Key words: arch beam, high-speed train, resonance, earthquake.

ABSTRACT

Considering the axial-flexural coupling nature of an arch bridge subjected to seismic wave, this paper is aimed at investigation of dynamic response for a train moving over a railway arch-bridge shaken by horizontal ground motions. For analytical formulation, the arch bridge is idealized as a flat-rise parabolic arch with constant sectional properties uniformly distributed along the horizontal span and the train moving over it as a sequence of identical sprung mass units with constant intervals. To perform dynamic analysis of vehicle-bridge system shaken by horizontal earthquakes, an incremental-iterative procedure is proposed in this study. From numerical results, the multiple support motion induced by seismic wave propagation plays a key factor in affecting the dynamic response of the arch-bridge/vehicle system during earthquakes.

I. INTRODUCTION

Because of regular feature of identical intervals d of bogie-sets for a train travelling over a bridge at speed v , the bridge may experience a *periodic* action of successive moving loads with an exciting passage frequency v/d . Once the exciting frequency matches any of circular natural frequencies (ω) of the bridge, the resonant response will be developed on the bridge and the oscillating amplitude of the vibrating bridge is amplified along with the increasing number of passing vehicles along the bridge (Yang et al., 1997). Since then, a lot of researcher and engineering scientists have devoted themselves to train-induced resonant behaviors of rail bridges (Li and Su, 1999; Yang et al., 2001; Yau, 2001; Ju and Lin, 2003; Xia et al., 2005; Yau, 2007). Considering seismic effect of earthquakes on vehicle/bridge interaction dynamics, Yang and Wu (2002)

presented a three-dimensional (3D) train-rails-bridge model to deal with the dynamic stability of trains moving over bridges shaken by ground motions. In their study, the authors pointed out that the vertical component of earthquakes plays an important role in running safety and stability of moving trains, especially for near fault ground excitations. On the other hand, to the best of our knowledge, there are comparatively few papers conducting the interaction response of a train running over arch-type bridge structures (Chatterjee and Datta, 1995; Fryba, 1999; Ju and Lin, 2003; Wu and Chiang, 2004; Huang, 2005), compared with those of traditional elevated-bridges (Yang and Wu, 2002).

Arch is usually considered as a structural component in modern railway transportation infrastructure for its gentle profile and aesthetic features. In this study, a rail arch-bridge would be simulated as a two-hinged parabolic arch with flat-rise ratio (Leontovich, 1959) for analytical investigation, and the train travelling over it as a sequence of identical sprung mass units with equal interval. Meanwhile, only the horizontal component of ground motions will be considered for simulating earthquakes to shake an arch bridge under the action of a moving train. The numerical simulation of dynamic response for the arch bridge due to both moving sprung masses and ground excitations will be carried out using the Newmark's scheme in conjunction with direct integration method (Newmark, 1959). From the numerical study, horizontal seismic ground excitations may amplify primarily both responses of the arch bridge and train, especially for the train moving at resonant speeds. Moreover, the maximum peak response of acceleration for the running sprung masses is related to the seismic wave speed passing the arch bridge supports during earthquakes.

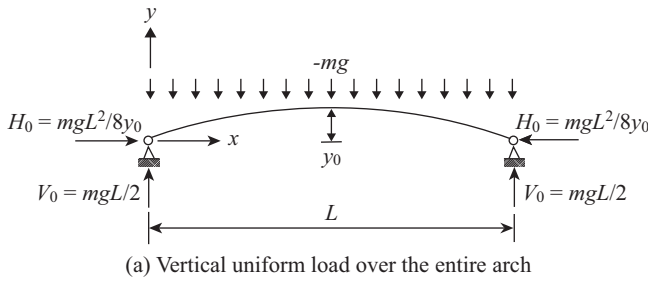
II. THEORETICAL FORMULATION

For a vertically loaded arch with hinged supports, the supports would be subjected to horizontal reactions due to load transfer of arching effect and axial-flexural coupling nature of an arch. Similarly, once the arch is subjected to horizontal seismic inputs at supports, the arch would produce vertical vibrations due to the ground excitations transmitted from the support movements. For analytical formulation, only vertical

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(a) Vertical uniform load over the entire arch
 (b) Force equilibrium of a deformed arch element

Fig. 1. Schematic diagram of a vertically loaded arch.

motions of the vehicle-bridge interaction system shaken by *in-plane* horizontal ground motions are considered in this study. The following are the assumptions adopted for the vehicle-bridge system (Leontovich, 1959):

- (1) The linear theory of a flat-rise parabolic arch is taken into account for deriving the governing equation of motion of a single-span flat-arch bridge.
- (2) The centroidal axis of cross section in the arch is a parabolic curve, and the bridge deck of the arch remains flat in a horizontal level.
- (3) The mass of the arch bridge is uniformly distributed along the horizontal axis of bridge span.
- (4) The train traveling over the arch bridge is simulated as a sequence of equidistant sprung mass units moving with constant speed.
- (5) The arch-bridge is far away from the earthquake epicenter so that the vertical components of seismic wave are negligible.

Fig. 1 depicts a theoretical model of a two-hinged parabolic arch under the action of vertical uniform load $-mg$ over the entire span. Here, m is denoted as the constant self-mass per unit length along the horizontal axis of span. The geometrical shape function $y(x)$ of the parabolic arch is given by (Leontovich, 1959).

$$y(x) = 4y_0 \left[x/L - (x/L)^2 \right], \tag{1}$$

where L = span length, and y_0 = rise of arch. Taking the equilibrium conditions of resultant forces and resultant moment at point x of the arch span, one can obtain the following cross-sectional forces:

$$\begin{aligned} H &= H_0 = \frac{-mgL^2}{8y_0}, \\ V &= V_0 \left(1 - \frac{2x}{L} \right) = \frac{wL}{2} \left(1 - \frac{2x}{L} \right), \\ Hy - Vx + \frac{mgx^2}{2} &= 0. \end{aligned} \tag{2a-c}$$

Here, (H_0, V_0) = reactions at supported ends of the parabolic arch under the self-weight of the structure, $w = mg$ self-weight of the arch bridge along the span, and g the gravity acceleration. Differentiating Eq. (2c) twice with respect to x yields $Hy'' - mg = 0$. Examination of this equality reveals that only axial force but no bending exists on the cross-sectional force of a parabolic two-hinged arch under the action of vertical uniform load over the entire span (Leontovich, 1959), and that the horizontal component of cross-sectional forces will remain constant as well, that is, $H = -mgL^2/8y_0$. Based on such a characteristic of constant horizontal component of internal forces in a parabolic arch, the equilibrium equations of shear and bending moment for the deformed arch element shown in Fig. 1(b) can be determined by

$$\begin{aligned} dV + wdx - (m\ddot{u} + c\dot{u})dx &= p(x, t)dx, \\ dM + Vdx - (H + h)d(y + u) &= 0. \end{aligned} \tag{3}$$

where V = shear force in the arch, M = bending moment in the arch, h = dynamic increment of horizontal force in the deformed arch, c = damping per unit length of the arch, $p(x, t)$ = external load function, and $u(x, t)$ = vertical displacement of the arch. The notation of over dot denotes the partial derivative with respect to time t . From the simple bending theory of a flatrise arch, one can obtain the following approximate moment curvature relationship

$$M \approx (EI_s \cos \phi)u'' = Elu'', \quad \cos \phi = \frac{1}{\sqrt{1 + (y')^2}}, \tag{4}$$

where a prime represents the derivative with respect to length x , and EI_s the flexural rigidity that varies along the centroidal axis of the arch bridge, ϕ the slope of the centroidal axis in the arch at point x with respect to the horizontal axis of span, and EI the flexural rigidity along the coordinate x -axis of the arch. Here, EI would be regarded as constant based on the consideration that $EI_s \cos \phi$ is assumed to remain constant along the coordinate x -axis of span length. By considering the relationship between bending moment and shear force and substituting Eqs. (1), (2) and (4) into Eq. (3), the differential equation of vertical motion for the flat-rise parabolic arch is written as

$$m\ddot{u} + c\dot{u} + Elu'''' - [(H + h)(u'' + y'') - mg] = p(x, t), \tag{5}$$

letting ds_0 denote the original length of the arch element and ds the deformed length, then

$$ds_0 = \sqrt{(dx)^2 + (dy)^2}, \quad ds = \sqrt{(dx)^2 + (dy + du)^2}. \quad (6)$$

According to Hooke's law, the linear stress-strain relation for a deformed flat-arch element can be determined by (Leontovich, 1959).

$$\begin{aligned} \frac{h(ds_0/dx)}{EA_s \cos \phi} &= \frac{h(ds_0/dx)}{EA} \approx \frac{ds - ds_0}{ds_0} \\ &= \frac{dy}{ds_0} \frac{du}{ds_0} = \left(\frac{dx}{ds_0} \right)^2 \times \frac{dy}{dx} \frac{du}{dx}, \end{aligned} \quad (7)$$

where h is the horizontal component of incremental force in the arch, EA_s means the axial rigidity that varies along the centroidal axis of the arch, and EA is the axial rigidity of the flat arch along the horizontal axis of the span. In this study, the axial rigidity EA is treated as constant along the coordinate x -axis of span length by regarding $EA_s \cos \phi$ as a uniform rigidity in the horizontal direction along the span of arch. The integration of Eq. (7) from 0 to L over span length yields the following horizontal component of incremental force

$$\begin{aligned} h &= \frac{EA}{L_e} \left[u_x \Big|_0^L + \int_0^L y' u' dx \right] \\ &= \frac{EA}{L_e} \left[(d_{xL} - d_{x0}) + \frac{8y_0}{L^2} \int_0^L u dx \right] \\ &= \Delta H + \alpha \int_0^L u dx \\ \Delta H &= \frac{EA}{L_e} (d_{xL} - d_{x0}), \quad \alpha = \left(\frac{8y_0}{L^2} \right)^2 \frac{EA}{L_e}, \quad L_e = \int_0^L \left(\frac{ds_0}{dx} \right)^3 dx, \end{aligned} \quad (8)$$

where L_e denotes the effective length of the arch curve. Introducing Eq. (8) into the left hand side of Eq. (5), and considering the equality $H y'' - mg = 0$ as well as the following approximation $h(u'' + y'') \approx h y''$, one can rewrite the differential equation in Eq. (5) as

$$\begin{aligned} m \ddot{u} + c \dot{u} + E I u'''' - (H + \Delta H) u'' \\ + \alpha \int_0^L u dx = p(x, t) - \kappa (d_{xL} - d_{x0}), \end{aligned} \quad (9)$$

$$\kappa = \left(\frac{8y_0}{L^2} \right) \frac{EA}{L_e}$$

where the loading function $p(x, t)$ represents sequential sprung masses moving over the arch-beam, as shown in Fig. 2.

The loading function $p(x, t)$ is given as:

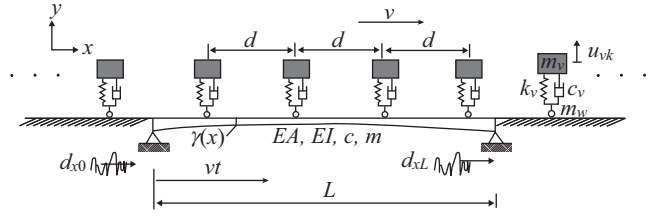


Fig. 2. Schematic diagram of successive sprung masses and flat-arch beam.

$$p(x, t) = \sum_{k=1}^N \left\{ \begin{aligned} &(P - m_v \ddot{u}_{vk} - m_w \ddot{u}) \delta(x - x_k) \\ &\left[U(t - t_g - t_k) - U(t - t_g - t_k - L/v) \right] \end{aligned} \right\}$$

$$m_v \ddot{u}_{vk} + c_v \dot{u}_{vk} + k_v u_{vk} = f_{vk}, \quad (10a-c)$$

$$f_{vk} = \begin{cases} k_v [u(x_k, t) + \gamma(x_k)] + c_v \dot{u}(x_k, t) & 0 \leq x_k \leq L \\ k_v \gamma(x_k) & x_k < 0, \quad x_k > L \end{cases}$$

in which, $P = -(m_v + m_w)g =$ lumped weight of a moving oscillator, $\delta =$ Dirac's delta function, $U(t) =$ unit step function, $k = 1, 2, 3, \dots, N$ -th moving load on the beam, $t_g =$ time lag for the first oscillator entering the arch bridge from the left hand side, $t_k = (k - 1)d/v =$ arrival time of the k -th load into the beam, $u_{vk} =$ vertical displacement of the k -th lumped mass, $f_{vk} =$ interaction force between the beam and the k -th wheel mass, and $\gamma(x_k) =$ track irregularity (vertical profile), and $x_k =$ position of the k -th load along the rail line, as defined in Eq. (9). Here, $x_k < 0$ represents the k -th load is not yet entering the bridge, $0 \leq x_k \leq L$ is running over the bridge deck, and $x_k > L$ has left the bridge. As indicated in Eq. (9), the inclusion of horizontal support movements may amplify the dynamic response of the arch-beam, which would affect the response of the vehicles running over it. On the other hand, the axial-flexural coupling nature of an arch may strengthen the flexural resistance of the flat-arch bridge.

For the arch with two-hinged ends, the boundary conditions can be expressed as follows:

$$u(0, t) = u(L, t) = 0, \quad E I u'''(0, t) = E I u'''(L, t) = 0. \quad (11)$$

Moreover, when the vibration of the arch bridge starts from rest, the initial conditions are

$$u(x, 0) = \dot{u}(x, 0) = 0. \quad (12)$$

Observation of Eq. (9) indicates that the term $(H + \Delta H)u''$ represents the second-order effect of horizontal component of cross-sectional force on the arch, and the parameter of α means the coupling effect of axial rigidity on the in-plane flexure of a flat-rise arch due to overall additional compression force h .

To account for the random nature and characteristics of rail

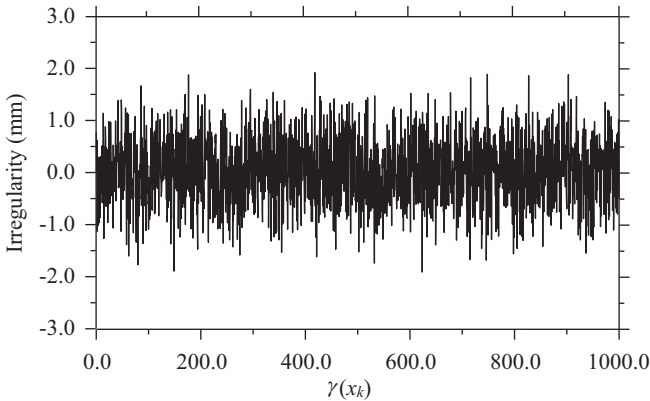


Fig. 3. Track irregularity (vertical profile).

irregularity in practice, the following *power spectrum density* (PSD) function (Yang et al., 2004) is given to simulate the vertical profile of track geometry variations

$$S(\Omega) = \frac{A_v \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (13)$$

where Ω = spatial frequency, and A_v , ($= 1.5 \times 10^{-7}$ m), Ω_r ($= 2.06 \times 10^{-6}$ rad/m), and Ω_c ($= 0.825$ rad/m) are relevant parameters. Fig. 3 plots the vertical profile of rail irregularity for the simulation of rail geometry variations in this study.

III. DYNAMIC ANALYSIS

From the homogeneous boundary conditions shown in Eq. (12), the dynamic component $u(x, t)$ of deformation of a two-hinged arch can be represented by a series of sine functions (Yang et al., 2004).

$$u(x, t) = \sum_{n=1} q_n(t) \sin \frac{n\pi x}{L}, \quad (14)$$

where $q_n(t)$ means the generalized coordinate associated with the n -th natural mode of the arch. The solution for the dynamic component $u(x, t)$ of deformation in Eq. (9) can be performed using Galerkin's method. First, substituting Eq. (14) into Eq. (9), multiplying both sides of the equation with respect to the variation of the vibration deflection $u(x, t)$, and then integrating the equation over the beam length L , one can obtain the following generalized equation of motion:

$$\begin{aligned} m\ddot{q}_n + c\dot{q}_n + k_n q_n + \Gamma_n = & \left[\sum_{k=1}^N F_k(\varpi_n, v, t) \right] + \frac{2\kappa}{n\pi} (1 - \cos(n\pi)) (d_{xL} - d_{x0}), \\ k_n = & \left(\frac{n\pi}{L} \right)^4 EI - \left(\frac{n\pi}{L} \right)^2 (H + \Delta H), \\ \Gamma_n = & \frac{2\alpha L}{n\pi^2} (1 - \cos n\pi) \left[\sum_{k=1}^1 \frac{1}{k} (1 - \cos k\pi) q_k \right] \end{aligned} \quad (15)$$

where the generalized force $F_k(\varpi_n, v, t)$ of the k -th sprung mass is expressed as

$$\begin{aligned} F_k(\varpi_n, v, t) = & \frac{2}{L} (P - m_v \ddot{u}_{v,k} - m_w \ddot{u}(x_k, t)) \psi_n(\varpi_n, t), \\ \psi_n(\varpi_n, t) = & \sin \varpi_n (t - t_g - t_k) \left[U(t - t_g - t_k) - U(t - t_g - t_k - L/v) \right], \end{aligned} \quad (16a, b)$$

and $\varpi_n = n\pi v / L$. As shown in Eqs. (15) and (16), the generalized equations for all the generalized coordinates are coupled due to the presence of the coupled terms, such as Γ_n and $\sum_{k=1}^N F_k(\varpi_n, v, t)$, in which the inertial forces in $F_k(\varpi_n, v, t)$ are time-dependent on the location of the k -th oscillator traveling over the arch beam. Obviously, the computational efforts required for solving such a set of time-dependent coupled differential equations are tremendous in CPU (Central Processing Unit) time consuming for updating the structural matrices at each time step. This is especially true for the acceleration response, rather than the displacement response, is of concern in high speed rail bridges, for which the contribution of higher modes has to be included. In this paper, an incremental-iterative procedure is used to solve the coupled generalized equations in Eq. (15) associated with the vehicle's equation in Eq. (10c), which involves three major phases: *predictor*, *corrector* and *equilibrium-checking* (Yang and Kuo, 1994).

Fig. 4 shows the flow chart to carry out the nonlinear interaction analysis of the sequential train cars running on the arch-beam shaken by horizontal ground motion.

The *predictor* is concerned with solution of the structural response for given loads including vehicular and earthquake loadings. The *corrector* phase relates to recovery of the internal resistant forces from the displacements made available in the predictor. In this phase, each of vehicle's response should be updated in an iterative way. In the *equilibrium-checking* phase, the effective internal forces computed from the *corrector* phase are compared with the external loads, the difference being regarded as the unbalanced forces. Whenever the unbalanced forces are greater than preset tolerances (say 10^{-3}), another iteration involving the three phases should be repeated.

IV. ILLUSTRATIVE EXAMPLES

As shown in Fig. 2, the properties of the flat-arch beam and moving sprung mass units are listed in Tables 1 and 2, respectively. Here, ω_1 denotes the fundamental circular frequency of the first mode and $v_{res} = \omega_1 d$ the first resonant speed of the arch beam under the moving loads. For the purpose of investigation, the damping ratio of the arch bridge is set 3%.

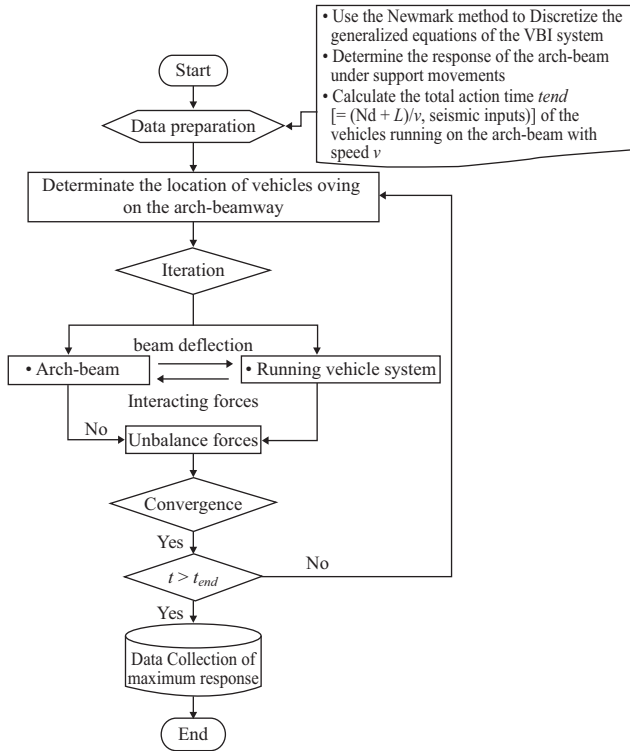


Fig. 4. Flow chart of incremental-iterative procedure.

To enhance the accuracy of dynamic response, the first 20 assumed shapes in Eq. (14) for the dynamic components of deformation are employed to formulate the coupled equations of motion for the flat-arch beam. The convergence of the assumed shape functions for a suspended beam was examined by Yau and Yang (2008). A similar convergent test can be applied to the present study.

Based on the Newmark method of direct integration with $\beta = 1/4$ and $\gamma = 1/2$ (Newmark, 1959), numerical solutions for the dynamic response of the arch bridge subjected to the action of successive moving loads and horizontal support motions have been computed using the incremental-iterative method described in Section III. Generally speaking, the resonant phenomenon of train-induced acceleration for ballasted bridges may result in drastic vibration on track structures and further bring about the problem of ballast destabilization (Museros, 2005; Yau et al., 2006). For this reason, the acceleration response of the arch bridge due to successive moving oscillators and ground excitations will be explored in the following numerical examples.

1. Phenomenon of Resonance

In order to illustrate the train-induced resonance of a railway bridge, we first compute the mid-span acceleration response of a *single-span* arch beam under the action of successive moving sprung mass units given in Table 2 with the first resonant speed of $v_{res} = \omega_1 d$ ($= 256$ km/h) using the time step of 0.005 s.

As shown in Fig. 5, the time history response of mid-span ac-

Table 1. Properties of the arch-beam.

L (m)	y_0 (m)	EA (kN)	M (t/m)	EI (kN-m ²)	H (kN)	c (kN-s/m/m)	L_e (m)	ω_1 (Hz)
45	3.2	2.0×10^6	40.0	5.33×10^8	3.1×10^4	42.7	46.9	2.84

Table 2. Properties of moving loads.

N	P (kN)	D (m)	m_v (t)	m_w (t)	c_v (kN-s/m)	k_v (kN/m)	v_{res} (km/h)
16	350	25	30.7	5	20.9	354	256

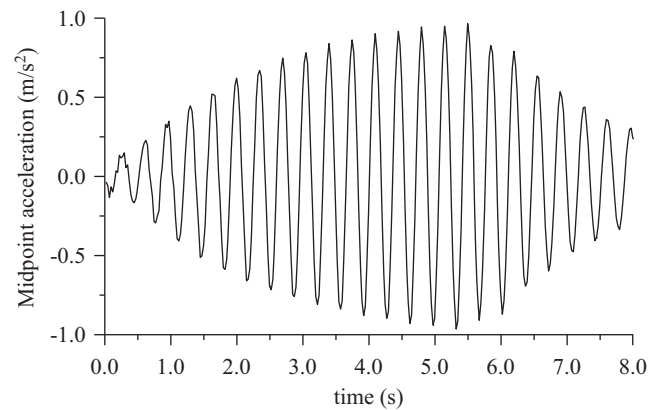


Fig. 5. Resonance of midpoint acceleration of the arch beam due to moving loads.

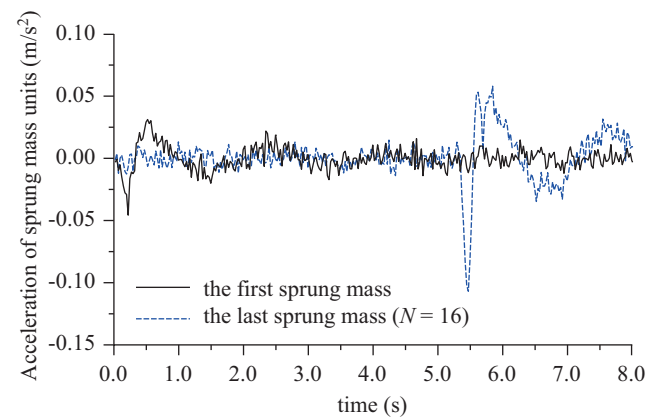


Fig. 6. Time history of vertical acceleration of sprung masses.

celeration of the arch-beam is generally built up as the increase of moving oscillators passing through the beam. In addition, the time-history responses of vertical acceleration for both the first and last ($N = 16$) sprung masses have been drawn in Fig. 6, respectively. Due to the *resonance* phenomenon occurring in the vibrating beam, the dynamic response of the last sprung mass running on the beam has been dramatically amplified in comparison with that of the first one.

To demonstrate the phenomenon of resonance for the arch

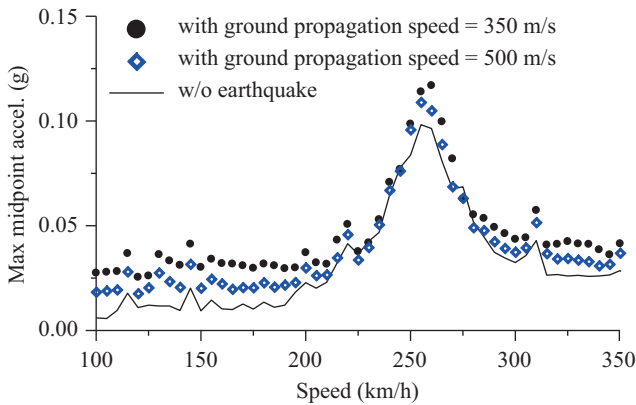


Fig. 7. a_{max} - v plot of the arch beam.

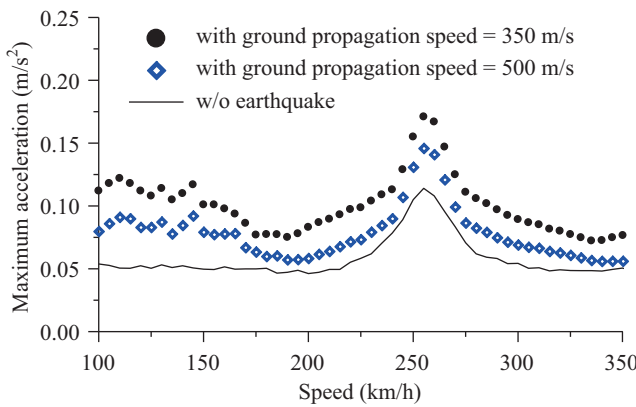


Fig. 8. Amplification of horizontal earthquakes on maximum acceleration of moving vehicles.

beam under the action of successive moving sprung mass units with various speeds, the maximum acceleration responses of midpoint of the arch beam and the moving sprung masses against speed (v) have been plotted in Figs. 7 and 8, respectively. Obviously, both response curves reveal that there exists a main peak response at the resonant speed of 256 km/h, which agrees very well with the resonant speed predicted from the formula $v_{res} = \omega_1 d$.

2. Effect of Horizontal Support Motions Due to Seismic Waves

As indicated in Eq. (9), for the case of synchronous support motions, the effect of ground motion on the response of the flat arch-beam would vanish due to the consideration of $d_{x0} = d_{xL}$. However, the travelling effect of seismic wave on the arch-beam exists in reality. For this reason, two types of soils will be considered for the construction site located of the arch-beam in this example. One is the medium soil with seismic wave velocity (v_w) of 350 m/s and the other one for the arch-beam built at bedrock with 600 m/s. Thus, the ground motion at the right bridge support exists a time lag of L/v_w behind the left one (see Fig. 2) when seismic waves travel along the arch-beam during earthquakes.

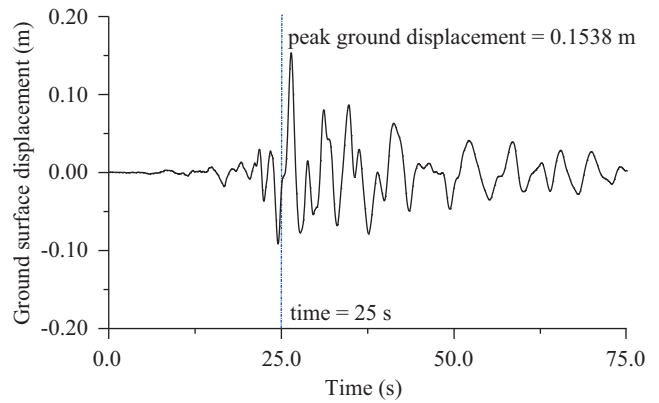


Fig. 9. Histogram of EW horizontal ground displacement of TAP003 Station.

To investigate the influence of seismic ground motion on interaction response of the vehicle/bridge coupled system, the far-field ground motion recorded at free-field station (TAP003) during the 1999 Chi-Chi Earthquake in Taiwan (Yang et al., 2004) is used to simulate the seismic support inputs acting on the arch beam. The histogram of ground displacement containing the East-West (EW) horizontal component has been plotted in Fig. 9.

As can be seen, the intensive zone of horizontal ground movements appears nearby 25 s. On the other hand, as a train travels along a bridge with resonant speed, the rear part will experience larger excitation induced by the vibrating bridge from the previous example. To let the rear part of the train model moving on the arch-beam meet the peak ground motions in the duration between 25 s and 28 s considering the TAP003 EW records, the critical time of $(25 - kd/v)_{k=13}$ is employed for the train model to enter the arch-beam. Here, k represents the k -th sprung mass entering the arch-beam during the TAP003 EW ground motions. Along with the response curves without considering earthquakes, the numerical results have been plotted in Figs. 7 and 8, respectively. Generally, the maximum acceleration amplitudes of both the sprung mass units and the mid-span of the arch-beam have been amplified significantly. However, the acceleration response curves for the sprung masses appear a noticeable amplification at lower speeds in 100~130 km/s. Since the inclusion of horizontal support movements may amplify the dynamic response of the arch-beam (see the second term on right hand side of Eq. (9)), the amplified beam oscillations would affect the response of the vehicles running over it. For this, as the train moves along the arch bridge with lower speeds during earthquakes, the car bodies may have more time to experience larger vertical excitations transmitting from the vibrating bridge deck shaken by seismic loads. Even so, the peak amplitude of maximum acceleration response still occurs at the resonant speed $v_{res} = \omega_1 d$.

V. CONCLUDING REAMRKS

With an incremental-iterative procedure in dynamic analy-

sis, the interaction response of a flat-rise parabolic arch beam under the simultaneous action of moving sprung masses and horizontal earthquake was analytically studied in this paper. From the numerical results, the following conclusions are made:

- (1) As a train travels along a rail bridge with resonant speed, the resonance phenomenon will be developed in the vibrating bridge and the dynamic response of the rear train cars will be dramatically amplified compared with the front ones;
- (2) Due to seismic travelling passage, the effect of multiple support motions play an important role in amplifying the interaction response of vehicle/bridge system;
- (3) As a train moves on a railway bridge built on a site with lower travelling wave speed (soft soils), the dynamic response of the running train is significantly amplified due to travelling effect of seismic waves.

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