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Jian-Qiang Wang
Department of Electrical Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C

Chee-Fai Yung
Department of Electrical Engineering Department, National Taiwan Ocean University, Keelung, Taiwan, R.O.C., and also with the Department of Mathematics, National Taiwan University, Taipei, Taiwan, R.O.C., yung@mail.ntou.edu.tw

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ROBUST STABILIZATION OF UNCERTAIN SWITCHED NONLINEAR SYSTEMS WITH TWO MODES

Jian-Qiang Wang[1] and Chee-Fai Yung[2]

Key words: switched nonlinear systems, control Lyapunov functions, state feedback, robust stability.

ABSTRACT

In this note, based on a control Lyapunov function approach, an integrated design of switching laws and feedback controllers for uncertain switched nonlinear control systems with two modes is discussed. A sufficient condition for the existence of globally asymptotically stabilizing state feedback laws (switching laws + controllers) is derived. Moreover, an explicit rule for constructing switching laws and an explicit formula for synthesizing feedback controllers are presented. An illustrative example is given for verifying the benefit of our approach.

I. INTRODUCTION

It is known that many physical and engineering systems can be described by switched systems (see Zefran and Burdick, 1998; Dayawansa and Martin, 1999; Liberzon, 2003). Moreover, there exist practical control systems that cannot be asymptotically stabilized by a single smooth feedback control law (Brockett, 1983). By these reasons, the study of switched systems has attracted much attention in the control community. Most of the published results on switched systems focused on stability analysis (see Ye et al., 1998; Dayawansa and Martin, 1999; Hespanha and Morse, 1999; Liberzon et al., 1999; Liberzon and Morse, 1999; Agrachev and Liberzon, 2001; Zhao and Dimirovski, 2004; Zhao and Hill, 2008; Lin and Antsaklis, 2009; Yang et al., 2011). In particular, it has been proven that a switched system is asymptotically stable under arbitrary switching if and only if a common Lyapunov function exists for all subsystems (Liberzon, 2003). By the dwell-time approach, it has been shown that a switched system is asymptotically stable if all subsystems are asymptotically stable and the switching is slow enough (see e.g., Hespanha and Morse, 1999).

For controller synthesis of switched control systems, most of the results were derived in the linear subsystems case, (see e.g., Zefran and Burdick, 1998; Daafouz et al., 2002; Xu and Antsaklis, 2004, etc). Few results have been proposed for control synthesis of switched nonlinear control systems. In (Wu, 2008) a common control Lyapunov function approach was proposed to derive necessary and sufficient conditions for the existence of stabilizing controllers for switched nonlinear control-affine systems with arbitrarily switching between two nonlinear control-affine subsystems. In addition, an explicit formula for constructing uniformly stabilizing controllers was provided. The other results were almost all derived for switched nonlinear control systems in some particular forms (e.g., strict-feedback form, lower triangular form, feed forward form, and p-normal form) and under arbitrary switching, (Wu, 2009; Ma and Zhao, 2010; Long and Zhao 2011a, 2011b ; Hou and Duan, 2013). The backstepping-based approaches were employed in these studies for constructing common control Lyapunov functions or for synthesizing stabilizing controllers. On the other hand, some of the published results dealt with the design of switching laws for stabilizing switching nonlinear systems without control inputs (Yang et al., 2009).

In the literature, few results have been presented about the integrated design of switching laws and feedback controllers for achieving stability of switched nonlinear control systems. In this paper, we try to address this problem. Based on a control Lyapunov function (CLF) approach (see e.g., Artstein, 1983; Sontag, 1983, 1989; Krstic et al., 1995), sufficient conditions for the existence of stabilizing feedback control laws (switching laws + feedback controllers) for uncertain switched nonlinear systems with two subsystems will be derived. Moreover, an explicit rule for constructing switching laws and an explicit formula for synthesizing feedback controllers will also be presented. To the best of our knowledge, the result along this direction has not been reported yet prior to this work.

Paper submitted 02/06/15; revised 05/18/15; accepted 08/18/15. Author for correspondence: Chee-Fai Yung (e-mail: yung@mail.ntou.edu.tw).
[1] *Department of Electrical Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.*
[2] *Department of Electrical Engineering Department, National Taiwan Ocean University, Keelung, Taiwan, R.O.C., and also with the Department of Mathematics, National Taiwan University, Taipei, Taiwan, R.O.C.*

II. PROBLEM FORMULATION AND PRELIMINARIES

This section formulates the problem to be solved and recalls the concept of control Lyapunov functions.

1. Problem Formulation

Consider an uncertain switched control system with two nonlinear subsystems:

$$\dot{x} = f_{\sigma(x)}(x) + \Delta f_{\sigma(x)}(x) + g_{\sigma(x)}(x)u\,,\ \sigma(x) \in \{1, 2\} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $f_i(.)$ and $g_i(.)$, $i = 1$ and 2, are known smooth functions, $\sigma(x)$: $R^n \mapsto \{1,2\}$ is a state-dependent switching signal to be designed. Suppose that the uncertain terms $\Delta f_i(x) = \rho_i F_i(x)$, $i = 1$ and 2, where $|\rho_i| < 1$ is an uncertain parameter and $F_i(x)$ is a known function with $F_i(0) = 0$. Without loss of generality, assume that $f_i(0) = 0$, $i = 1$ and 2.

The design objective is to find a switching law $\sigma(x)$ and two continuous functions $p_1(.), p_2(.): R^n \to R^m$ such that the closed-loop system

$$\dot{x} = f_{\sigma(x)}(x) + \Delta f_{\sigma(x)}(x) + g_{\sigma(x)}(x)p_{\sigma(x)}(x)\,,\ \sigma(x) \in \{1, 2\}\,, \tag{2}$$

is globally asymptotically stable for all possible uncertainties $\Delta f_i(x)$, $i = 1$ and 2.

2. Control Lyapunov Functions

To solve the problem in question, here we first recall the concept of control Lyapunov function (CLF). Consider the following (nonswitched) nonlinear system:

$$\dot{x} = f(x) + g(x)u\,. \tag{3}$$

Definition 1 (Sontag, 1989): A smooth, positive definite, and radially unbounded function $V: R^n \to R$ is a CLF of system (3) if, for each $x \in R^n \setminus \{0\}$,

$$\inf_{u \in R^m}\left\{\frac{\partial V(x)}{\partial x} f(x) + \frac{\partial V(x)}{\partial x} g(x)u\right\} < 0\,. \qquad \square$$

It is known that the existence of CLFs is a necessary and sufficient condition for the existence of globally asymptotically stabilizing controllers for (3). To ensure the existence of *continuous* stabilizing feedback laws, we still need the following *small control property* (SCP).

Definition 2 (Sontag, 1989): A CLF $V(x)$ of system (3) is said to satisfy the SCP if for each $\varepsilon > 0$ there is a $\delta > 0$ such that, if $x \neq 0$ satisfying $\|x\| < \delta$, then there is some u with $\|u\| < \varepsilon$ such that

$$\frac{\partial V(x)}{\partial x} f(x) + \frac{\partial V(x)}{\partial x} g(x)u < 0\,. \qquad \square$$

In (Sontag, 1989), it is shown that if there is a CLF $V(x)$, satisfying the SCP, for system (3), then

$$u(x) = p(x) = \begin{cases} -\dfrac{a(x) + \sqrt{a^2(x) + (b(x)b^T(x))^2}}{b(x)b^T(x)} b^T(x), & \text{if } b(x) \neq 0 \\ 0, & \text{if } b(x) = 0 \end{cases} \tag{4}$$

is a globally asymptotically stabilizing controller, where $a(x) = \dfrac{\partial V(x)}{\partial x} f(x)$ and $b(x) = \dfrac{\partial V(x)}{\partial x} g(x)$. The function $p(x)$ defined in (4) is smooth in $R^n \setminus \{0\}$ and continuous at $x = 0$ (Sontag, 1989).

To establish the main result of this note, we now review a useful lemma that will be used later.

Lemma 1 (Petersen, 1987): Given any positive definite matrix $Q(x)$ and any matrices $M(x)$ and $N(x)$ of compatible dimensions, the inequality

$$2\eta x^T M(x)N(x)x \le x^T M(x)Q(x)M^T(x)x + x^T N^T(x)Q^{-1}(x)N(x)x$$

holds for any η satisfying $|\eta| \le 1$. $\square$

III. MAIN RESULTS

This section proposes the main result of this note – an integrated design of switching laws and feedback controllers to stabilize uncertain switched nonlinear control systems.

Here we first present a sufficient condition for the existence of robustly stabilizing switching laws for an unforced switched nonlinear system.

Theorem 1: Consider the switched nonlinear system

$$\dot{x}(t) = f_{\sigma(x)}(x) + \Delta f_{\sigma(x)}(x),\ \ \sigma(x) \in \{1, 2\}\,. \tag{5}$$

with $\Delta f_i(x) = \rho_i F_i(x)$, $|\rho_i| < 1$, $i = 1,2$. If there exist a scalar $\alpha \in (0,1)$, a positive definite function $V(.): R^n \to R$, and two positive define matrix functions $Q_1(.): R^n \to R^{n\times n}$ and $Q_2(.)$: $R^n \to R^{n\times n}$ satisfying the following condition:

$$\alpha\left(\frac{\partial V(x)}{\partial x}f_1(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_1(x)\left(\frac{\partial V(x)}{\partial x}\right)^T +\frac{1}{2}F_1^T(x)Q_1^{-1}(x)F_1(x)\right) +(1-\alpha)\left(\frac{\partial V(x)}{\partial x}f_2(x) +\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_2(x)\left(\frac{\partial V(x)}{\partial x}\right)^T +\frac{1}{2}F_2^T(x)Q_2^{-1}(x)F_2(x)\right)<0,\quad \forall x\neq 0, \tag{6}$$

then there exists a switching law $\sigma(x)$ such that the system (5) is globally asymptotically stable for all possible uncertainties $\Delta f_i(x)$, $i = 1$ and 2.

Proof:

Define ($i = 1$ and 2)

$$\Phi_i \equiv \left\{ x\in R^n \left| \begin{aligned} &\frac{\partial V(x)}{\partial x}f_i(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_i(x)\left(\frac{\partial V(x)}{\partial x}\right)^T \\ &+\frac{1}{2}F_i^T(x)Q_i^{-1}(x)F_i(x)<0 \end{aligned} \right.\right\} \tag{7}$$

The condition (6) implies that $\Phi_1\cup\Phi_2 = R^n\setminus\{0\}$. Notice that

$$2\frac{\partial V(x)}{\partial x}\Delta f_i(x) \le \frac{\partial V(x)}{\partial x}Q_i(x)\left(\frac{\partial V(x)}{\partial x}\right)^T + F_i^T(x)Q_i^{-1}(x)F_i(x)$$

Choose $V(x)$ as a candidate Lyapunov function for system (5). If the switching law satisfies:

$$\sigma(x)=i \text{ only if } x\in\Phi_i,\ i=1 \text{ and } 2, \tag{8}$$

then, for each nonzero x,

$$\begin{aligned}\dot V(x) &= \frac{\partial V(x)}{\partial x}f_{\sigma(x)}(x)+\frac{\partial V(x)}{\partial x}\Delta f_{\sigma(x)}(x)\\ &=\frac{\partial V(x)}{\partial x}f_{\sigma(x)}(x)\\ &\quad+\frac{1}{2}\left(\frac{\partial V(x)}{\partial x}\Delta f_{\sigma(x)}(x)+\Delta f_{\sigma(x)}^T(x)\left(\frac{\partial V(x)}{\partial x}\right)^T\right)\\ &\le \frac{\partial V(x)}{\partial x}f_{\sigma(x)}(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_{\sigma(x)}(x)\left(\frac{\partial V(x)}{\partial x}\right)^T\\ &\quad+\frac{1}{2}F_{\sigma(x)}^T Q_{\sigma(x)}^{-1}(x)F_{\sigma(x)}\\ &<0.\end{aligned}$$

That is, the system (5) is globally asymptotically stable under any switching law satisfying (8). □

Now we consider the switched control system (1). We want to design both switching laws and feedback controllers such that the closed-loop system is robustly globally asymptotically stable.

Definition 3: A positive definite, smooth, and radially unbounded function $V(x)$ is called a *robust switched control Lyapunov function* (RSCLF) of system (1) if, for each $x\neq 0$,

$$\min_{i\in\{1,2\}}\max_{\rho_i}\inf_{u\in R^m}\left\{\frac{\partial V(x)}{\partial x}\left(f_i(x)+\Delta f_i(x)+g_i(x)u\right)\right\}<0. \quad □$$

Definition 4: A RSCLF $V(x)$ of system (1) is said to satisfy the *switched small control property* (SSCP) if for each $\varepsilon>0$ there is a $\delta>0$ such that, if $x\neq 0$ satisfies $\|x\|<\delta$, then there is some u with $\|u\|<\varepsilon$ such that

$$\min_{i\in\{1,2\}}\max_{\rho_i}\left\{\frac{\partial V(x)}{\partial x}\left(f_i(x)+\Delta f_i(x)+g_i(x)u\right)\right\}<0. \quad □$$

For a candidate RSCLF $V(x)$ of system (1), define

$$a_1(x)=\frac{\partial V(x)}{\partial x}f_1(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_1(x)\left(\frac{\partial V(x)}{\partial x}\right)^T +\frac{1}{2}F_1^T(x)Q_1^{-1}(x)F_1(x),$$

$$a_2(x)=\frac{\partial V(x)}{\partial x}f_2(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_2(x)\left(\frac{\partial V(x)}{\partial x}\right)^T +\frac{1}{2}F_2^T(x)Q_2^{-1}(x)F_2(x),$$

$$b_1(x)=\frac{\partial V(x)}{\partial x}g_1(x)$$

$$b_2(x)=\frac{\partial V(x)}{\partial x}g_2(x)$$

And let

$$D_{NP}=\{x\in R^n \mid a_1(x)<0, a_2(x)\ge 0\},$$

$$D_{PN}=\{x\in R^n \mid a_1(x)\ge 0, a_2(x)<0\},$$

$$D_{PP}=\{x\in R^n \mid a_1(x)\ge 0, a_2(x)\ge 0\},$$

$$D_Z=\{x\in R^n \mid b_1(x)=0, b_2(x)=0\}.$$

Then, we have the following main result.

Theorem 2: Consider the switched nonlinear control system (1). If there exists a smooth, proper, and positive definite function $V(x)$, satisfying the *SSCP*, such that

(1) $D_{PP} \cap D_Z = \{0\}$; and

(2) $\sup_{x \in D_{PN} \cap D_Z} \left\{ -\frac{a_1(x)}{a_2(x)} \right\} < \inf_{x \in D_{NP} \cap D_Z} \left\{ -\frac{a_1(x)}{a_2(x)} \right\}$,

then we can find a switching law $\sigma(x) \in \{1,2\}$ and feedback control laws $p_1(x)$ and $p_2(x)$ such that the closed-loop system

$$\dot{x}(t) = f_{\sigma(x)}(x) + \Delta f_{\sigma(x)}(x) + g_{\sigma(x)}(x) p_{\sigma(x)}(x) \tag{9}$$

is globally asymptotically stable for all possible uncertainties.

Proof:

By (2), let β be a positive constant satisfying

$$\sup_{x \in D_{PN} \cap D_Z} \left\{ -\frac{a_1(x)}{a_2(x)} \right\} < \beta < \inf_{x \in D_{NP} \cap D_Z} \left\{ -\frac{a_1(x)}{a_2(x)} \right\}. \tag{10}$$

We have

$$a_1(x) + \beta a_2(x) < 0, \text{ for each } x \in (D_{PN} \cup D_{NP}) \cap D_Z \tag{11}$$

Let $\alpha = \frac{1}{1+\beta}$. It is clear that $0 < \alpha < 1$. From (11) one can see that

$$\alpha a_1(x) + (1-\alpha) a_2(x) < 0, \text{ for each } x \in (D_{PN} \cup D_{NP}) \cap D_Z \tag{12}$$

Define

$$a(x) = \alpha a_1(x) + (1-\alpha) a_2(x),$$

$$b(x) = [\alpha b_1(x) \quad (1-\alpha) b_2(x)].$$

It is obvious that $b(x) = 0$ if and only if $x \in D_Z$. By condition (1) and (12) it is clear that

$$a(x) < 0, \ \forall x \in D_Z \setminus \{0\}.$$

By Sontag's formula (Sontag, 1989), let

$$p(x) = \begin{bmatrix} p_1(x) \\ p_2(x) \end{bmatrix} = \begin{cases} -\dfrac{a(x) + \sqrt{a^2(x) + (b(x) b^T(x))^2}}{b(x) b^T(x)} b^T(x), & \text{if } x \notin D_Z \\ 0, & \text{if } x \in D_Z \end{cases} \tag{13}$$

Then, we have

$$\begin{aligned} &\alpha (a_1(x) + b_1(x) p_1(x)) + (1-\alpha)(a_2(x) + b_2(x) p_2(x)) \\ &= \alpha a_1(x) + (1-\alpha) a_2(x) + \alpha b_1(x) p_1(x) \\ &\quad + (1-\alpha) b_2(x) p_2(x) = a(x) + b(x) p(x) \\ &= \begin{cases} -\sqrt{a^2(x) + (b(x) b^T(x))^2}, & \text{if } b(x) \neq 0 \\ a(x), & \text{if } b(x) = 0 \end{cases} \\ &< 0, \ \forall x \neq 0. \end{aligned} \tag{14}$$

From Theorem 1 we know that there exists a switching law such that the closed-loop system (9) is globally asymptotically stable. More precisely, by (14) it is clear that, for all $x \neq 0$,

$$a_1(x) + b_1(x) p_1(x) < 0, \tag{15}$$

or

$$a_2(x) + b_2(x) p_2(x) < 0. \tag{16}$$

Define

$$\Omega_i \equiv \{x \in R^n \mid a_i(x) + b_i(x) p_i(x) < 0\}, \ i = 1, 2 \tag{17}$$

From (16) and (17), we know that $\Omega_1 \cup \Omega_2 = R^n \setminus \{0\}$. If the switching rule satisfies

$$\sigma(x) = i \text{ only if } x \in \Omega_i, \tag{18}$$

one can see that

$$\begin{aligned} \dot{V} &= \frac{\partial V(x)}{\partial x} \left(f_{\sigma(x)}(x) + \Delta f_{\sigma(x)}(x) + g_{\sigma(x)}(x) p_{\sigma(x)}(x) \right) \\ &\leq \frac{\partial V(x)}{\partial x} f_{\sigma(x)}(x) + \frac{1}{2} \frac{\partial V(x)}{\partial x} Q_{\sigma(x)}(x) \left(\frac{\partial V(x)}{\partial x} \right)^T \\ &\quad + \frac{1}{2} F_{\sigma(x)}^T(x) Q_{\sigma(x)}^{-1}(x) F_{\sigma(x)}(x) \\ &\quad + \frac{\partial V(x)}{\partial x} g_{\sigma(x)}(x) p_{\sigma(x)}(x) \\ &= a_{\sigma(x)}(x) + b_{\sigma(x)}(x) p_{\sigma(x)}(x) \\ &< 0, \ \forall x \neq 0. \end{aligned}$$

That is, the closed-loop system (9) is globally asymptotically stable for all possible uncertainties. □

By (18), a switching law can be designed as follows: if $x(0)\in\Omega_1$, let $\sigma(0)=1$, else let $\sigma(0)=2$. For all $t>0$, define $\sigma(x)$ as:

$$\sigma(x)=\begin{cases}1, & \text{if } x(t)\in\Omega_1 \text{ and } x(t^-)\in\Omega_1, \\ & \text{or } x(t)\notin\Omega_2 \text{ and } x(t^-)\in\Omega_2 \\ 2, & \text{if } x(t)\in\Omega_2 \text{ and } x(t^-)\in\Omega_2, \\ & \text{or } x(t)\notin\Omega_1 \text{ and } x(t^-)\in\Omega_1\end{cases} \tag{19}$$

This is, switching occurs only when the state trajectory leaves region Ω_1 or leaves region Ω_2.

IV. AN ILLUSTRATIVE EXAMPLE

Consider the switched nonlinear control systems

$$\dot{x}=f_{\sigma(x)}(x)+\Delta f_{\sigma(x)}(x)+g_{\sigma(x)}(x)u\ ,\ \sigma(x)\in\{1,2\} \tag{20}$$

with

$$f_1(x)=\begin{bmatrix}x_1-x_2\\ 4x_1-x_2^3\end{bmatrix},\ f_2(x)=\begin{bmatrix}-x_2^2-x_1^2\\ -6x_1x_2\end{bmatrix},\ g_1(x)=\begin{bmatrix}1\\ -x_2^2\end{bmatrix},$$

$$g_2(x)=\begin{bmatrix}x_2\\ -2\end{bmatrix},\ F_1(x)=\begin{bmatrix}0\\ x_1x_2\end{bmatrix},\ F_2(x)=\begin{bmatrix}x_2^2\\ 0\end{bmatrix}.$$

Choose $V(x)=x_1^2+x_2^2/4$ as a candidate control Lyapunov function for the system (20). With $Q_1(x)=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$ and $Q_2(x)=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$, we have

$$\begin{aligned}a_1(x)&=\frac{\partial V(x)}{\partial x}f_1(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_1(x)\left(\frac{\partial V(x)}{\partial x}\right)^T\\ &\quad+\frac{1}{2}F_1^T(x)Q_1^{-1}(x)F_1(x)\\ &=4x_1^2-x_2^2/4+x_1^2x_2^2/4\\ a_2(x)&=\frac{\partial V(x)}{\partial x}f_2(x)+\frac{1}{2}\frac{\partial V(x)}{\partial x}Q_2(x)\left(\frac{\partial V(x)}{\partial x}\right)^T\\ &\quad+\frac{1}{2}F_2^T(x)Q_2^{-1}(x)F_2(x)\\ &=-5x_1x_2^2-2x_1^3+8x_1^2+x_2^2/4+x_2^4/8\\ b_1(x)&=\frac{\partial V(x)}{\partial x}g_1(x)=2x_1-x_2^3/2\\ b_2(x)&=\frac{\partial V(x)}{\partial x}g_2(x)=2x_1x_2-x_2\end{aligned}$$

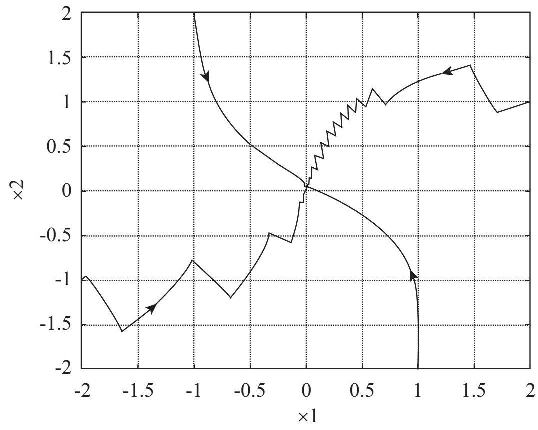

Fig. 1. State trajectories of the closed-loop systems with four different initial conditions and uncertain parameters.

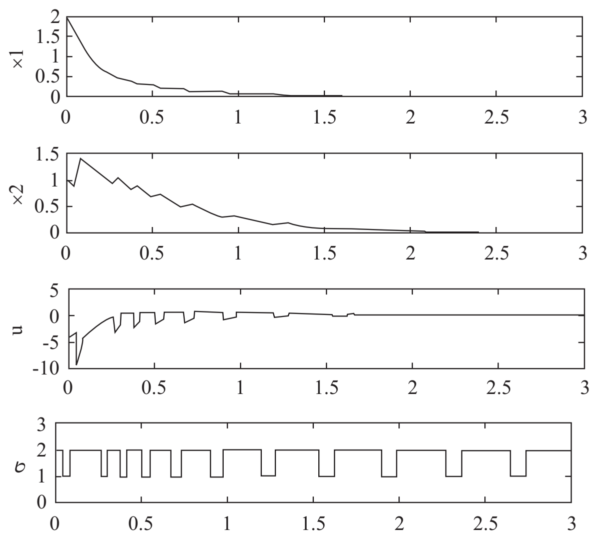

Fig. 2. System response and switching signal.

By definition, it is clear that $D_Z=\{(1/2,\sqrt[3]{2})\}$. It is easy to verify that conditions (1) and (2) in Theorem 2 hold, and therefore, we can find a switching law $\sigma(x)\in\{1,2\}$ and feedback control laws $p_1(x)$ and $p_2(x)$ such that the closed- loop system is asymptotically stable. Moreover, we can see that

$$\sup_{x\in D_{PN}\cap D_Z}\left\{-\frac{a_1(x)}{a_2(x)}\right\}<0.5<\inf_{x\in D_{NP}\cap D_Z}\left\{-\frac{a_1(x)}{a_2(x)}\right\}$$

By choosing $\beta=0.5$ and using the feedback controller constructed by (13) and the switching law implemented by (19), the state trajectories of the closed-loop system with 4 different initial conditions and different values of uncertain parameters ρ_1 and ρ_2 are shown in Fig. 1. For clarifying the switching behavior, Fig. 2 shows the state response of the closed-loop system for an arbitrary initial state and arbitrary choice of uncertain parameters. It can be seen that the state trajectory asymptotically converges to the origin.

V. CONCLUSIONS

An integrated design of switching laws and feedback controllers for asymptotically stabilizing uncertain switched nonlinear control-affine systems with two modes has been discussed. A control Lyapunov function approach has been used to derive sufficient conditions for the existence of asymptotically stabilizing feedback laws. A numerical example has also been given for illustration.

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