Journal of
Marine Science and Technology

Volume 24 | Issue 5 Article 11

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Acknowledgements

This work was supported by the Ministry of Science and Technology, Taiwan, under Contract No. MOST104-2221-E-019-015.

Journal of Marine Science and Technology, Vol. 24, No.5, pp. 1003-1010 (2016)
DOI: 10.6119/JMST-016-0622-1

ROBUST MIXED H_2/*Passivity* PERFORMANCE CONTROLLER DESIGN FOR UNCERTAIN DRUM-BOILER SYSTEM

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Key words: drum-boiler system, passivity theory, H_2 scheme, robust control, LMI.

ABSTRACT

This paper discusses a robust control problem of uncertain drum-boiler system subject to H_2 and passivity performances. The main aims of H_2 control scheme are to minimize output energy and guarantee robust asymptotical stability. Besides, passivity theory is applied to constrain effect of external disturbance on the system. Based on Lyapunov function, some sufficient conditions are derived into Linear Matrix Inequality (LMI) form that can be directly calculated by the convex optimization algorithm. Through solving the derived sufficient conditions, a controller can be established to guarantee robust asymptotical stability and mixed H_2/*Passivity* performance of uncertain drum-boiler system. Based on simulation results, validity and effectiveness of the proposed design method can be demonstrated.

I. INTRODUCTION

The boiler is a common equipment which generates high-quality steam to keep operating temperature and to supply power to marine engineering department of the ship. Thus, stability and stabilization problems of boiler systems are worth to be discussed and investigated. In order to analyze the stability of boiler, some literature (Åström and Zcklund, 1972; Abdeldjebar and Khier, 2007) proposed some mathematical equations to describe the dynamic trajectory of drum-boiler. Based on these equations, many efforts (Pellegrinetti and Bentsman, 1996; Moon and Lee, 2009) have been proposed to develop some criteria to ensure the stability of closed-loop boiler system. A robust passive controller design method was developed to guarantee robust stability and attenuation performance of uncertain boiler system by (Ku et al., 2010b). However, initial conditions and constraining system energy did not be considered in the above literature. That may cause a control problem called as "high gain effect" to make some damages during operating process. Therefore, the initial condition is needed to be considered such that stability can be guaranteed under required operation. In addition to initial condition, the constraint of system energy is also an important issue to protect components of the boiler. For the reason, robust stability, attenuation performance, and minimized energy are simultaneously considered for boiler system in this paper.

In order to achieve multiple control performances, mixed performance control scheme provides an excellent solution. Referring to literature (Nobuyama and Khargonekar, 1995; Chen et al., 2000; Karimi and Gao, 2008; Huang et al., 2011), a mixed H_2/H_∞ performance controller design method has been proposed to achieve the required performances. Based on the mixed performance schemes, H_2 control scheme (Dragan et al., 2004; Dragan, 2005; Georgiev and Tilbury, 2006; Ma and Chen, 2006) focuses on minimizing system energy under the desired initial conditions. Moreover, H_∞ control scheme (Zhang et al., 2005; Berman and Shaked, 2006; Gérard et al., 2010) is employed to deal with attenuation performance. Therefore, minimized energy and attenuation performance of disturbed systems can be simultaneously achieved by merging those two control schemes. However, the H_∞ control scheme limits the description for relationship between external disturbance and system. It is easily to find that index of attenuation performance is only established by a ratio between state and disturbance. To extend generality and flexibility of the existing mixed H_2/H_∞ performance control method, passivity theory (Lozano et al., 2000) is applied in this paper to substitute the H_∞ control scheme for discussing the energy change between system and external disturbance.

Passivity theory was developed by Willems (Lozano et al., 2000; Willems and Trentrlman, 2002a, 2002b) to analyze the stability of linear systems or nonlinear systems. The main idea of passivity theory is that if the system is passive, then energy in the system always dissipates or stores with time. Based on the idea of passivity theory, some researchers (Mahmoud and Zribi, 2002; Li et al., 2009; Ku et al., 2010a, 2010b) have investigated for the attenuation performance of disturbed

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systems. Moreover, a function describing power supply (Lozano et al., 2000) in passivity theory is a key point to decide the index of attenuation performance. Via well setting the function, the attenuation performance of the system can be transferred as several types including H_2 control scheme, H_∞ control scheme, positive real theory and different passivity schemes. Thus, passivity theory is more general and flexible than H_∞ control scheme on discussing the energy changing between external disturbance and system. Therefore, a general and flexible mixed performance design method is considered in this paper for achieving asymptotical stability, passive constraint and H_2 control performance of the boiler system.

According to the above motivation, a robust mixed H_2/*Passivity* performance control problem of uncertain drum-boiler system is discussed and investigated in this paper. Because the drum-boiler system possesses strong nonlinearity, a linearization approach (Teixeira and Żak, 1999) is applied to obtain a linear model to describe local behavior on equilibrium point. For the linear model, some sufficient conditions are derived via Lyapunov function to guarantee robust stability and the required control performances. During the derivative of this paper, a transformation technique (Kim, 2001) is employed to obtain constant terms from time-varying terms. To effectively solve the control problem of this article, the derived sufficient conditions are converted into LMI form that can be calculated by convex optimization algorithm (Boyd et al., 1994). Based on the obtained feasible solutions, one can design a mixed H_2/*Passivity* performance controller such that the uncertain drum-boiler system achieves robust stability, minimized output energy, and passivity. Finally, the effectiveness and usefulness of the proposed design method can be demonstrated by simulation results.

This paper is organized as follows. Section II describes uncertain drum-boiler system and its mixed performance control problem. Section III presents mixed performance controller design method for the system. Section IV presents some numerical simulations to demonstrate effectiveness and applicability of the proposed design method. Finally, some conclusions are stated in Section V.

II. DRUM-BOILER SYSTEM STRUCTURE AND PROBLEMS STATEMENT

In this section, a linearization technique (Teixeira and Żak, 1999) is employed to obtain a linear model to express local dynamic behavior of drum-boiler system around the chosen equilibrium point. Referring to (Nobuyama and Khargonekar, 1995; Abdeldjebar and Khier, 2007), the original nonlinear dynamic equation of drum-boiler system is provided as follows:

$$\dot{w}_1(t) = -0.00478 w_4(t) w_1^{9/8}(t) + 0.28 p_1(t) - 0.01348 p_3(t) \tag{1a}$$

$$\begin{aligned}\dot{w}_2(t) = {} & 0.1540357 w_2(t) + 0.1 v(t) + \big(103.5462 p_2(t) - 107.4835 \times p_1(t) \\ & -1.9515 p_1(t) w_2(t)\big) / \big(29.04 p_2(t) - 1.824 p_1(t)\big)\end{aligned} \tag{1b}$$

$$\begin{aligned}\dot{w}_3(t) = {} & -0.00533176 w_1(t) \\ & -0.025195 w_4(t) w_1(t) \\ & +0.7317058 p_3(t)\end{aligned} \tag{1c}$$

$$\dot{w}_4(t) = -0.04 w_4(t) + 0.029988 p_1(t) + 0.018088 \tag{1d}$$

$$y_1(t) = 14.214 w_1(t) \tag{2a}$$

$$y_2(t) = w_2(t) \tag{2b}$$

$$\begin{aligned}y_3(t) = {} & -0.1048569 w_1(t) + 0.15479 w_3(t) + 0.495 w_4(t) w_1(t) \\ & -0.2 p_3(t) + 1.272 p_1(t) - \big(324212.78 w_1(t) + 99556.25\big) \\ & \times \big(1 - 0.0012 w_3(t)\big) / \big(w_3(t)\big(w_1(t) - 1704.5\big)\big) - 103.74\end{aligned} \tag{2c}$$

$$y_4(t) = \big(0.85663 w_4(t) - 0.18128\big) w_1(t). \tag{2d}$$

where $w_1(t)$ is the drum pressure state (kgf/cm^2); $w_2(t)$ is the excess oxygen level (percent); $w_3(t)$ is the system fluid density (kg/m^3); $w_4(t)$ is the exogenous variable related to the load disturbance intensity (0-1); $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$ are the measured outputs for drum pressure (PSI), excess oxygen level (percent), drum water level (in) and steam flow rate (kg/s), respectively; $p_1(t)$, $p_2(t)$ and $p_3(t)$ are the fuel, air and feed water level rate inputs which take values between 0-1; $v(t)$ is the external disturbance input. To obtain linear system, equilibrium points of the drum-boiler system (1) are determined such as:

$$\begin{bmatrix} w_1(t) & w_2(t) & w_3(t) & w_4(t) \\ p_1(t) & p_2(t) & p_3(t) & \end{bmatrix}_{ep} = \begin{bmatrix} 22.5 & 2.5 & 621.17 & 0.8374 \\ 0.5138 & 0.5064 & 0.8127 & \end{bmatrix} \tag{3}$$

According to the equilibrium points in (3), states of (1) and (2) can be shifted such that new equilibrium points of the drum-boiler system are stated in origin. Thus, the following new states and inputs are inferred.

$$\begin{bmatrix} w_1(t) & w_2(t) & w_3(t) & w_4(t) \\ p_1(t) & p_2(t) & p_3(t) & \end{bmatrix} = \begin{bmatrix} x_1(t)+22.5 & x_2(t)+2.5 & x_3(t)+621.17 & x_4(t)+0.8374 \\ u_1(t)+0.5138 & u_2(t)+0.5064 & u_3(t)+0.8127 & \end{bmatrix} \tag{4}$$

According to (4), dynamic equations (1) can be rewritten as follows:

$$\dot{x}_1(t) = -0.00478\big(x_4(t)+0.8374\big)\big(x_1(t)+22.5\big)^{9/8} + 0.28\big(u_1(t)+0.5138\big) - 0.01348\big(u_3(t)+0.8127\big) \tag{5a}$$

$$\begin{aligned}\dot{x}_2(t) &= 0.1540357\big(x_2(t)+2.5\big)+0.1v(t) \\ &+\Big(103.5462\big(u_2(t)+0.5064\big)-107.4835\big(u_1(t)+0.5138\big) \\ &-1.9515\big(u_1(t)+0.5138\big)\big(x_2(t)+2.5\big)\Big)\Big/\Big(29.04\big(x_2(t)+2.5\big) \\ &-1.824\times\big(u_1(t)+0.5138\big)\Big)\end{aligned} \tag{5b}$$

$$\begin{aligned}\dot{x}_3(t) &= -0.00533176\big(x_1(t)+22.5\big) \\ &-0.025195\big(x_4(t)+0.8374\big)\times\big(x_1(t)+22.5\big) \\ &+0.7317058\big(u_3(t)+0.8127\big)\end{aligned} \tag{5c}$$

$$\dot{x}_4(t) = -0.04\big(x_4(t)+0.8374\big) + 0.029988\big(u_1(t)+0.5138\big)+0.018088 \tag{5d}$$

Furthermore, setting $u_1(t) = u_2(t) = u_3(t) = v(t) = 0$, Teixeira-Zak's linearization technique (Teixeira and Żak, 1999) is applied to obtain the following linear system from (5) with equilibrium point $x^{op} = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$.

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{E}v(t) \tag{6}$$

where

$$x(t) = \begin{bmatrix}x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t) & x_3^{\mathrm{T}}(t) & x_4^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},$$

$$u(t) = \begin{bmatrix}u_1^{\mathrm{T}}(t) & u_2^{\mathrm{T}}(t) & u_3^{\mathrm{T}}(t)\end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix}-0.0066 & 0 & 0 & -0.1587 \\ 0 & 0.0812 & 0 & 0 \\ -0.0264 & 0 & 0 & -0.5669 \\ 0 & 0 & 0 & -0.04\end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix}0.28 & 0 & -0.0135 \\ -8.2117 & 8.3317 & 0 \\ 0 & 0 & 0.7317 \\ 0.03 & 0 & 0\end{bmatrix} \text{ and } \mathbf{E} = \begin{bmatrix}0 \\ 0.1 \\ 0 \\ 0\end{bmatrix}.$$

Besides, uncertainty and two outputs are presented as follows.

$$\Delta\mathbf{A}(t) = \mathbf{H}\Delta(t)\mathbf{R}x(t) \tag{7a}$$

$$z_1(t) = \mathbf{C}_1x(t) + \mathbf{D}_1v(t) \tag{7b}$$

$$z_2(t) = \mathbf{C}_2x(t) + \mathbf{D}_2u(t) \tag{7c}$$

where $\Delta(t) = \sin(5t)$ is a time-varying function satisfying $\Delta(t)\Delta(t) \le \mathbf{I}$, $z_1(t) \in R^1$ is the performance output vector, and $z_2(t) \in R^2$ is the controlled output vector. Moreover, the corresponding matrices in (7) are proposed as follows:

$$\mathbf{H} = 0.1\times\mathbf{I},\ \mathbf{R} = 0.01\times\mathbf{A},\ \mathbf{C}_1 = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix},\ \mathbf{D}_1 = 1,$$

$$\mathbf{C}_2 = \begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0\end{bmatrix} \text{ and } \mathbf{D}_2 = \begin{bmatrix}1 & 0 & 0 \\ 0 & 0 & 1\end{bmatrix}.$$

In this paper, the following state feedback design technology is concerned to stabilize (6).

$$u(t) = \mathbf{F}x(t) \tag{8}$$

where $\mathbf{F}$ is a gain matrix which is needed to be found. Therefore, the following closed-loop system can be obtained by substituting (8) into (6) and (7c).

$$\dot{x}(t) = \mathbf{G}x(t) + \Delta\mathbf{A}(t)x(t) + \mathbf{E}v(t) \tag{9a}$$

$$z_1(t) = \mathbf{C}_1x(t) + \mathbf{D}_1v(t) \tag{9b}$$

$$z_2(t) = \mathbf{M}x(t) \tag{9c}$$

where

$$\mathbf{G} = \mathbf{A} + \mathbf{BF} \text{ and } \mathbf{M} = \mathbf{C}_2 + \mathbf{D}_2\mathbf{F}.$$

For the closed-loop system (9), some sufficient conditions are derived to analyze the stability for the uncertain drum-boiler system.

The following definitions are proposed to describe passivity theory and H_2 performance.

Definition 1 (Lozano et al., 2000): If there exist the known constant matrices S_1, $\mathrm{S}_2 \ge 0$ and S_3 for satisfying the following inequality, then the closed-loop system (9) is called passive with $v(t)$ and $z_1(t)$ for all $t_p > 0$.

$$2\int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_1v(t)dt > \int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_2z_1(t)dt + \int_0^{t_p} v^{\mathrm{T}}(t)\mathbf{S}_3v(t)dt \tag{10}$$

By setting constant matrices $\mathbf{S}_1$, $\mathbf{S}_2 \geq 0$ and $\mathbf{S}_3$, inequality (10) can be rewritten as several constraints. #

When disturbance input is zero $\left(v(t)=0\right)$, the following H_2 performance is considered to minimize output energy with nonzero initial condition $x(0) \neq 0$.

Definition 2 (Dragan, 2005): The H_2 performance measure of system (9) is defined as follows:

$$\int_0^\infty z_2^{\mathrm{T}}(t) z_2(t) dt < \alpha . \tag{11}$$

This definition is so-called as problem of minimizing output energy based on α. #

To convert uncertainty into the constant term, the following lemma is provided.

Lemma 1 (Xie et al., 1998): Given real compatible dimension matrices **H** and **R**, $\varepsilon > 0$ and $\Delta(t)$ with $\Delta^{\mathrm{T}}(t)\Delta(t) \leq \mathbf{I}$, one can find the following result.

$$\mathbf{H}\Delta(t)\mathbf{R} + \mathbf{R}^{\mathrm{T}}\Delta^{\mathrm{T}}(t)\mathbf{H}^{\mathrm{T}} \leq \varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} \tag{12}$$

#

According to the above definitions and lemma, some sufficient conditions are derived via Lyapunov function in the following section.

III. MIXED PERFORMANCE CONTROLLER DESIGN METHOD

In this section, a stability criterion is proposed to guarantee robust asymptotical stability and mixed *H_2/Passivity* performance of the closed-loop system (9). To apply convex optimization algorithm, the derived sufficient conditions are converted into LMI form.

Theorem 1: Given matrices $\mathbf{S}_1$, $\mathbf{S}_2 \geq 0$ and $\mathbf{S}_3$, if one can find a positive definite matrix **P**, feedback gain **F** and positive value ε to satisfy the following conditions with minimizing α, then robust asymptotical stability and mixed *H_2/Passivity* performance of the closed-loop system (9) are achieved.

$$\begin{bmatrix} \mathbf{C}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1 + \mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} & * \\ -\mathbf{S}_1^{\mathrm{T}}\mathbf{C}_1 + \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1 + \mathbf{E}^{\mathrm{T}}\mathbf{P} & \mathbf{S}_3 - \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_1 + \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{D}_1 \end{bmatrix} < 0 \tag{13a}$$

$$\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} + \mathbf{M}^{\mathrm{T}}\mathbf{M} < 0 \tag{13b}$$

$$x^{\mathrm{T}}(0)\mathbf{P}x(0) < \alpha \tag{13c}$$

Proof:

Firstly, the following Lyapunov function is chosen.

$$V\left(x(t)\right) = x^{\mathrm{T}}(t)\mathbf{P}x(t) \tag{14}$$

Calculating the difference of (14), we have

$$\dot{V}\left(x(t)\right) = \dot{x}^{\mathrm{T}}(t)\mathbf{P}x(t) + x^{\mathrm{T}}(t)\mathbf{P}\dot{x}(t). \tag{15}$$

Substituting (9a) into (15) and applying Lemma 1, one has

$$\begin{aligned} \dot{V}\left(x(t)\right) &= x^{\mathrm{T}}(t)\left(\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \mathbf{R}^{\mathrm{T}}\Delta(t)\mathbf{H}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{H}\Delta(t)\mathbf{R}\right)x(t) \\ &\quad + 2x^{\mathrm{T}}(t)\mathbf{P}\mathbf{E}v(t) \\ &\leq \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} & \mathbf{P}\mathbf{E} \\ \mathbf{E}^{\mathrm{T}}\mathbf{P} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \end{aligned} \tag{16}$$

Let us define a cost function as follows:

$$\begin{aligned} \Gamma(x,v,t) &= \int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_2 z_1(t) + v^{\mathrm{T}}(t)\mathbf{S}_3 v(t) - 2z_1^{\mathrm{T}}(t)\mathbf{S}_1 v(t) dt \\ &= \int_0^{t_p} \Big(z_1^{\mathrm{T}}(t)\mathbf{S}_2 z_1(t) + v^{\mathrm{T}}(t)\mathbf{S}_3 v(t) - 2z_1^{\mathrm{T}}(t)\mathbf{S}_1 v(t) \\ &\quad + \dot{V}\left(x(t)\right)\Big) dt - V\left(x(t_p)\right) \leq \int_0^{t_p} \Psi(x,v,t) dt \end{aligned} \tag{17}$$

where

$$\begin{aligned} \Psi(x,v,t) &= z_1^{\mathrm{T}}(t)\mathbf{S}_2 z_1(t) + v^{\mathrm{T}}(t)\mathbf{S}_3 v(t) \\ &\quad - 2z_1^{\mathrm{T}}(t)\mathbf{S}_1 v(t) + \dot{V}\left(x(t)\right). \end{aligned} \tag{18}$$

Substituting (9b) and (16) into (18), one has

$$\Psi(x,v,t) = \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^{\mathrm{T}} \mathbf{\Lambda} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \tag{19}$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{C}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1 + \mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} & * \\ -\mathbf{S}_1^{\mathrm{T}}\mathbf{C}_1 + \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1 + \mathbf{E}^{\mathrm{T}}\mathbf{P} & \mathbf{S}_3 - \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_1 + \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{D}_1 \end{bmatrix} \tag{20}$$

Obviously, if condition (13a) holds, then one can obtain $\Lambda < 0$ that implies $\Psi(x, v, t)$ from (19). From (17), $\Gamma(x, v, t) < 0$ can be found due to $\Psi(x, v, t)$. Thus, the following inequality can be obtained.

$$2\int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_1 v(t) dt > \int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_2 z_1(t) dt + \int_0^{t_p} v^{\mathrm{T}}(t)\mathbf{S}_3 v(t) dt \tag{21}$$

Because (21) is equivalent to (10), the closed-loop system

(9) is passive. Next, it is necessary to show that the closed-loop system (9) is robustly asymptotically stable. Assuming $v(t) = 0$, the following equation can be inferred from (16).

$$\dot{V}\left(x(t)\right) = x^{\mathrm{T}}(t)\left(\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R}\right)x(t) \quad (22)$$

Furthermore, the following relation can be obtained via (22) and (9c).

$$\begin{aligned}\dot{V}\left(x(t)\right) &= x^{\mathrm{T}}(t)\left(\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R}\right)x(t) \\ &\quad + z_2^{\mathrm{T}}(t)z_2(t) - z_2^{\mathrm{T}}(t)z_2(t) \\ &\le x^{\mathrm{T}}(t)\left(\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R}\right)x(t) \\ &\quad - z_2^{\mathrm{T}}(t)z_2(t) \\ &= x^{\mathrm{T}}(t)\left(\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} + \mathbf{M}^{\mathrm{T}}\mathbf{M}\right)x(t)\end{aligned} \quad (23)$$

Thus, if condition (13b) holds, then $\dot{V}\left(x(t)\right) < 0$ is easily found from (23). According to $\dot{V}\left(x(t)\right) < 0$, the closed-loop system (9) with zero external disturbance is robustly asymptotically stable. On the other hand, the following equation can be obtained by arranging the first equation in (23).

$$\begin{aligned}\dot{V}\left(x(t)\right) + z_2^{\mathrm{T}}(t)z_2(t) &= x^{\mathrm{T}}(t)\left(\mathbf{G}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{G} + \varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P}\right. \\ &\quad \left. + \varepsilon^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R}\right)x(t) + z_2^{\mathrm{T}}(t)z_2(t)\end{aligned} \quad (24)$$

Because condition (13b) holds, one has the following inequality from (24).

$$\dot{V}\left(x(t)\right) + z_2^{\mathrm{T}}(t)z_2(t) < 0 \quad (25)$$

or

$$\dot{V}\left(x(t)\right) < -z_2^{\mathrm{T}}(t)z_2(t) \quad (26)$$

Integrating both sides of (26) from 0 to T_f, we have

$$x^{\mathrm{T}}\left(T_f\right)\mathbf{P}x\left(T_f\right) - x^{\mathrm{T}}(0)\mathbf{P}x(0) < -\int_0^{T_f} z_2^{\mathrm{T}}(t)z_2(t)dt\,. \quad (27)$$

Since the closed-loop system (9) is asymptotically stable, i.e., $x\left(T_f\right) \to 0$ as $T_f \to \infty$, one has

$$\int_0^{T_f} z_2^{\mathrm{T}}(t)z_2(t)dt < x^{\mathrm{T}}(0)\mathbf{P}x(0)\,. \quad (28)$$

From (28), $x^{\mathrm{T}}(0)\mathbf{P}x(0)$ is an upper bound of H_2 performance defined in Definition 2. If (13c) holds, the following inequality can be obtained to determine the upper bound of H_2 performance.

$$x^{\mathrm{T}}(0)\mathbf{P}x(0) \le \alpha \quad (29)$$

Due to (28) and (29), one has.

$$\int_0^{T_f} z_2^{\mathrm{T}}(t)z_2(t)dt < \alpha \quad (30)$$

Through minimizing scalar α, the output energy can be constrained via initial condition and matrix. The proof of this theorem is completed. #

Referring to Theorem 1, the derived sufficient conditions are bilinear matrix inequality problems that cannot be directly calculated by the convex optimization algorithm. For this reason, these sufficient conditions are converted into LMI form in the following theorem.

Theorem 2

Given matrices $\mathbf{S}_1$, $\mathbf{S}_2 \ge 0$, and $\mathbf{S}_3$, if there exist $\mathbf{X} = \mathbf{X}^{\mathrm{T}} > 0$, $\mathbf{K}$ and positive value ε to satisfy the following conditions with minimizing α, then asymptotical stability and mixed *H*$_2$/*Passivity* performance of the closed-loop system (9) is achieved.

$$\left[\begin{array}{c|c|c|c} \begin{array}{l}\mathbf{X}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} + \mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} \\ + \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{K} + \varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}}\end{array} & * & * & * \\ \hline \begin{array}{l}-\mathbf{S}_1^{\mathrm{T}}\mathbf{C}_1\mathbf{X} \\ + \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1\mathbf{X} + \mathbf{E}^{\mathrm{T}}\end{array} & \begin{array}{l}\mathbf{S}_3 - \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_1 \\ + \mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{D}_1\end{array} & * & * \\ \hline \mathbf{C}_1\mathbf{X} & 0 & -\mathbf{S}_2^{-1} & * \\ \hline \mathbf{R}\mathbf{X} & 0 & 0 & -\varepsilon\mathbf{I} \end{array}\right] < 0 \quad (31a)$$

$$\begin{bmatrix} \mathbf{X}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} + \mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{K} + \varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}} & * & * \\ \mathbf{C}_2\mathbf{X} + \mathbf{D}_2\mathbf{K} & -\mathbf{I} & * \\ \mathbf{R}\mathbf{X} & 0 & -\varepsilon\mathbf{I} \end{bmatrix} < 0 \quad (31b)$$

$$\begin{bmatrix} -\alpha & x(0)^{\mathrm{T}} \\ x(0) & -\mathbf{X} \end{bmatrix} < 0 \quad (31c)$$

where $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{K} = \mathbf{F}\mathbf{X}$

Proof:

Multiplying both sides of (13a) by $diag\left\{\mathbf{P}^{-1}, \mathbf{I}\right\}$, one has

$$\begin{bmatrix} \mathbf{P}^{-1}\mathbf{C}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1\mathbf{P}^{-1}+\mathbf{P}^{-1}\mathbf{G}^{\mathrm{T}}+\mathbf{G}\mathbf{P}^{-1} +\varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}}+\varepsilon^{-1}\mathbf{P}^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R}\mathbf{P}^{-1} & * \\ -\mathbf{S}_1^{\mathrm{T}}\mathbf{C}_1\mathbf{P}^{-1}+\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1\mathbf{P}^{-1}+\mathbf{E}^{\mathrm{T}} & \mathbf{S}_3-\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_1-\mathbf{S}_1\mathbf{D}_1+\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{D}_1 \end{bmatrix}<0. \tag{32}$$

Using Schur complement (Boyd et al., 1994), the following inequality (32) can be obtained.

$$\begin{bmatrix} \mathbf{P}^{-1}\mathbf{G}+\mathbf{G}\mathbf{P}^{-1}+\varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}} & * & * & * \\ -\mathbf{S}_1^{\mathrm{T}}\mathbf{C}_1\mathbf{P}^{-1}+\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1\mathbf{P}^{-1}+\mathbf{E}^{\mathrm{T}} & \mathbf{S}_3-\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_1-\mathbf{S}_1\mathbf{D}_1+\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{D}_1 & * & * \\ \mathbf{C}_1\mathbf{P}^{-1} & 0 & -\mathbf{S}_2^{-1} & * \\ \mathbf{R}\mathbf{P}^{-1} & 0 & 0 & -\varepsilon\mathbf{I} \end{bmatrix}<0 \tag{33}$$

Substituting $\mathbf{G}=\mathbf{A}+\mathbf{B}\mathbf{F}$ into (33), and setting $\mathbf{X}=\mathbf{P}^{-1}$ and $\mathbf{K}=\mathbf{F}\mathbf{X}$, one has

$$\begin{bmatrix} \mathbf{X}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}+\mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}+\mathbf{A}\mathbf{X}+\mathbf{B}\mathbf{K}+\varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}} & * & * & * \\ -\mathbf{S}_1^{\mathrm{T}}\mathbf{C}_1\mathbf{X}+\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{C}_1\mathbf{X}+\mathbf{E}^{\mathrm{T}} & \mathbf{S}_3-\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_1-\mathbf{S}_1\mathbf{D}_1+\mathbf{D}_1^{\mathrm{T}}\mathbf{S}_2\mathbf{D}_1 & * & * \\ \mathbf{C}_1\mathbf{X} & 0 & -\mathbf{S}_2^{-1} & * \\ \mathbf{R}\mathbf{X} & 0 & 0 & -\varepsilon\mathbf{I} \end{bmatrix}<0 \tag{34}$$

By using Schur complement, (13b) becomes

$$\begin{bmatrix} \mathbf{G}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{G}+\varepsilon\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} & * & * \\ \mathbf{M} & -\mathbf{I} & * \\ \mathbf{R} & 0 & -\varepsilon\mathbf{I} \end{bmatrix}<0 \tag{35}$$

where $\mathbf{M}=\mathbf{C}_2+\mathbf{D}_2\mathbf{F}$.

Multiplying both sides of (35) by $diag\{\mathbf{P}^{-1},\mathbf{I},\mathbf{I}\}$, the following inequality can be obtained via applying $\mathbf{X}=\mathbf{P}^{-1}$ and $\mathbf{K}=\mathbf{F}\mathbf{X}$.

$$\begin{bmatrix} \mathbf{X}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}+\mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}+\mathbf{A}\mathbf{X}+\mathbf{B}\mathbf{K}+\varepsilon\mathbf{H}\mathbf{H}^{\mathrm{T}} & * & * \\ \mathbf{C}_2\mathbf{X}+\mathbf{D}_2\mathbf{K} & -\mathbf{I} & * \\ \mathbf{R}\mathbf{X} & 0 & -\varepsilon\mathbf{I} \end{bmatrix}<0 \tag{36}$$

By using Schur Complement, (13c) becomes

$$\begin{bmatrix} -\alpha & x(0)^{\mathrm{T}} \\ x(0) & -\mathbf{P}^{-1} \end{bmatrix}<0. \tag{37}$$

Setting $\mathbf{X}=\mathbf{P}^{-1}$, we have

$$\begin{bmatrix} -\alpha & x(0)^{\mathrm{T}} \\ x(0) & -\mathbf{X} \end{bmatrix}<0. \tag{38}$$

Through Schur complement and setting the variables, i.e., $\mathbf{X}=\mathbf{P}^{-1}$ and $\mathbf{K}=\mathbf{F}\mathbf{X}$, inequalities (34), (36), and (38) can be obtained from conditions (13) in Theorem 1. Moreover, (34), (36), and (38) are equivalent to the conditions of Theorem 2. Thus, if feasible solutions of Theorem 2 can be obtained, then the feasible solutions can also satisfy the conditions of Theorem 1. The proof of this theorem is completed. #

Based on the LMI conditions of Theorem 2, feasible solutions can be directly obtained by using MATLAB LMI-Toolbox. Furthermore, the controller (8) can be established with the feasible solutions for guaranteeing robust asymptotical stability and mixed H_2/*Passivity* performance of the closed-loop system (9).

IV. SIMULATION RESULTS

In order to apply the proposed design method, matrices $\mathbf{S}_1 = 1$, $\mathbf{S}_2 = 0.8$ and $\mathbf{S}_3 = 0.8$, and initial condition $x(0) = [20\ 2.75\ 550\ 1]^{\mathrm{T}}$ is chosen. Using LMI Toolbox of MATLAB, one can find the following feasible solutions.

$$\mathbf{P}=\begin{bmatrix} 0.0002 & -0.0001 & 0 & 0.0001 \\ -0.0001 & 0.0171 & 0.0001 & 0 \\ 0 & 0.0001 & 0.0002 & 0.0001 \\ 0.0001 & 0 & 0.0001 & 0.0002 \end{bmatrix},$$

$$\mathbf{F}=\begin{bmatrix} -1.0027 & 0.0092 & 0.0096 & 0.005 \\ -0.9595 & -6.3477 & -0.043 & -0.009 \\ 0 & 0.4305 & -1.0022 & -0.0025 \end{bmatrix},$$

$$\alpha=56.8877 \text{ and } \varepsilon=3116.6. \tag{39}$$

Based on (39), the controller (8) can be established. Let us choose the external disturbance $v(t)$ as zero-mean white noise with variance 0.2. With the added uncertainty (7a), the responses of the drum-boiler system (1) driven by the designed controller are stated in Figs. 1-4. From Figs. 1-4, the drum pressure $y_1(t)$ is stabilized on 320 PSI. The excess oxygen level $y_2(t)$ is kept near 2.5 percent with some vibrations that are caused by added external disturbance $v(t)$. From those responses,

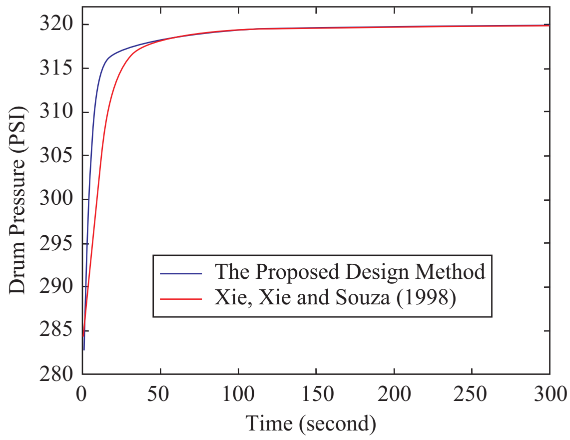

Fig. 1. Responses of measured drum pressure $y_1(t)$.

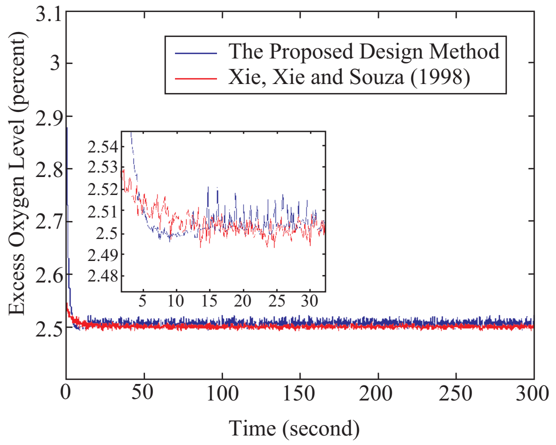

Fig. 2. Responses of excess oxygen level $y_2(t)$.

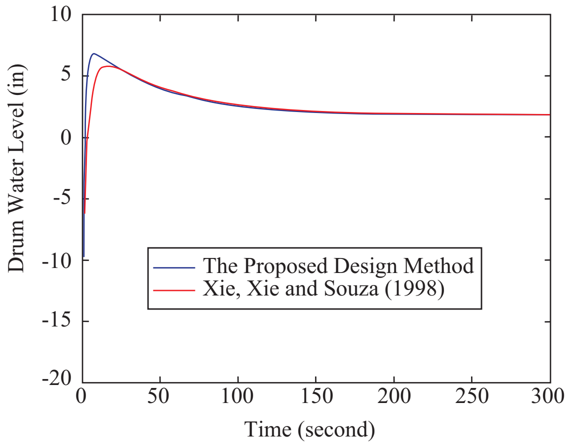

Fig. 3. Responses of drum water level $y_3(t)$.

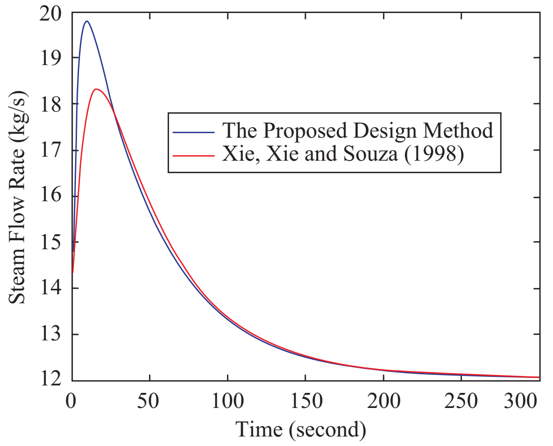

Fig. 4. Responses of steam flow rate $y_4(t)$.

one can find that the disturbance effect on the system is constrained via the designed controller. Furthermore, from Figs. 1-2, good control accuracy of drum pressure and excess oxygen level are achieved in short time. Besides, the drum water level and steam flow rate are controlled in a long time for convergence. It results in the rate of feed water limited in one. Moreover, the responses of drum water level and steam flow rate are also stabilized on the argument points during the simulation time. Besides, the following equations are used to demonstrate that the performances of the system are achieved.

$$\frac{\int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_2 z_1(t)\,dt + \int_0^{t_p} v^{\mathrm{T}}(t)\mathbf{S}_3 v(t)\,dt}{2\int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_1 v(t)\,dt} = 0.9375 \quad (40a)$$

$$\int_0^{\infty} z_2^{\mathrm{T}}(t) z_2(t)\,dt = 24.25 \quad (40b)$$

Obviously, the ratio value of (40a) is smaller than one that satisfies Definition 1. Thus, the uncertain drum-boiler system driven by the designed controller is passivity. Besides, the value of (40b) is smaller than the obtained α that means the output energy of uncertain drum-boiler system is limited. Moreover, the H_2 performance of (1) is achieved. Also, one can find that the asymptotical stability of (1) is guaranteed with the decided initial conditions. With the above illustrations, robust asymptotical stability and mixed $H_2/Passivity$ performance of the uncertain drum-boiler system is achieved via the designed controller.

To emphasize the contribution of this paper, a design method proposed by (Xie et al., 1998) is employed to control boiler system (1) with uncertainty (7a). Referring to (Xie et al., 1998), only passivity control problem was discussed. Therefore, one can apply the same matrices $\mathbf{S}_1 = 1$, $\mathbf{S}_2 = 0.8$ and $\mathbf{S}_3 = 0.8$ to find feasible solutions to satisfy the sufficient condition in (Xie, Xie and Souza 1998). And then, the following controller can be designed.

$$u(t) = \begin{bmatrix} -0.3685 & 0.92 & -0.0064 & 0.4233 \\ -0.371 & -0.9534 & -0.007 & 0.4149 \\ -0.0445 & 0.0005 & -0.4711 & 0.7069 \end{bmatrix} x(t) \quad (41)$$

Applying (41) to (1), the measured outputs responses are stated in Figs. 1-4 with the same initial condition. Based on the simulation results of (1) driven by (41), the following ratios can be obtained.

$$\frac{\int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_2 z_1(t)dt + \int_0^{t_p} v^{\mathrm{T}}(t)\mathbf{S}_3 v(t)dt}{2\int_0^{t_p} z_1^{\mathrm{T}}(t)\mathbf{S}_1 v(t)dt} = 0.8521 \quad (42a)$$

$$\int_0^{\infty} z_2^{\mathrm{T}}(t) z_2(t)dt = 1.213\times 10^5 \quad (42b)$$

It obviously shows that (42a) is smaller than one and satisfies Definition 1. Thus, the passivity of considered boiler system is achieved by (41). However, from (42b), the output energy of (1) driven by (41) is bigger than assigned value as α = 56.8877. Thus, the H_2 performance of (1) cannot be achieved by (41). Besides, from Figs. 1-2, the settling time and delay time of (1) driven by (41) is bigger than that driven by controller designed by this paper. Based on the above comparison, one can conclude that the proposed design method proposes better control performance than the method of (Xie et al., 1998) in controlling uncertain drum-boiler system (1).

V. CONCLUSIONS

A robust mixed H_2/*Passivity* performance design method for uncertain drum-boiler system has been proposed in this paper. For presenting a general and flexible controller design method, passivity theory was applied to substitute for the H_∞ scheme to achieve attenuation performance. The H_2 control scheme was employed to minimize output energy and to discuss stability issues of the system. Through the H_2 control scheme and passivity theory, some sufficient conditions were derived via Lyapunov function. Based on convex optimization algorithm, the feasible solutions can be obtained to establish a controller such that robust asymptotical stability and mixed H_2/*Passivity* performances are achieved. To increase the applicability of this paper, a relaxed mixed performance controller design method will be provided in our future works.

ACKNOWLEDGMENTS

This work was supported by the Ministry of Science and Technology, Taiwan, under Contract No. MOST104-2221-E-019-015.

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