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FUZZY CONTROL VIA IMPERFECT PREMISE MATCHING APPROACH FOR DISCRETE TAKAGI-SUGENO FUZZY SYSTEMS WITH MULTIPLICATIVE NOISES

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Wen-Jer Chang, Che-Pin Kuo, and Cheung-Chieh Ku

Key words: Takagi-Sugeno fuzzy models, multiplicative noises, Discrete Jensen Inequality, Linear Matrix Inequality.

ABSTRACT

In this paper, a fuzzy controller design problem is considered for the discrete-time stochastic Takagi-Sugeno fuzzy systems. The main purpose of this paper is to provide a less conservative fuzzy controller design method for the discrete-time nonlinear stochastic systems, which are modeled by the T-S fuzzy models. Based on the concept of imperfect premise matching, the fuzzy controller is designed without the limitation imposed from sharing the same membership functions of fuzzy model. In other words, the imperfect premise matching technique provides a generalization approach in designing fuzzy controller. Moreover, the flexibility and robustness of proposed fuzzy controller design approach can be further enhanced than the well-known parallel distributed compensation method. Moreover, based on the Lyapunov function, the stability conditions are referred to linear matrix inequality problems for applying the convex optimal algorithm. Finally, the nonlinear stochastic truck-trailer system is provided to show the utility of the proposed fuzzy controller design method.

I. INTRODUCTION

In recent years, there is an increasing interest in the fuzzy control systems that have many issues to be referred (Guerra and Vermeiren, 2001; Tanaka and Wang, 2001; Chang and Sun, 2003; Chang and Shing, 2004; Chang and Chang, 2006; Jeon and Jeong, 2006; Chang et al., 2013; Deng and Qiu, 2015; Li et al., 2015a, 2015b; Zhang et al., 2015; Tanaka et al., 2016). These works were focused on the stability and stabilization of closed-loop fuzzy systems. Especially, these approaches are developed based on the T-S fuzzy models. The Takagi-Sugeno (T-S) fuzzy models are described by a set of fuzzy "IF…THEN" rules with fuzzy sets in the antecedents and dynamic systems in the consequent, which can locally represent linear input-output relations of nonlinear systems. The Parallel Distributed Compensation (PDC) concept (Guerra and Vermeiren, 2001) provided a useful framework to design the fuzzy controller for the T-S fuzzy models. The goal of PDC concept is to design linear feedback gains for each local linear systems and let the overall control input be blended by these linear feedback gains. These local linear systems are considered as subsystems and the aggregation of subsystems represents the overall nonlinear systems.

In the literature (Tanaka et al., 1998; Kim and Lee, 2000; Teixeira et al., 2003; Er et al., 2006; Fang et al., 2006; Zhou, 2007; Li et al., 2008; Chang et al., 2009; Lam, 2009; Chang et al., 2010a, 2010b; Liu and Lam, 2015; Sun, 2016), the fuzzy controller design has been developed based on the concept of PDC for T-S fuzzy models. The PDC technique is indeed a useful tool for fuzzy controller design of T-S fuzzy model. However, under the PDC design technique, the fuzzy controller shares the same membership functions of the T-S fuzzy model. Therefore, when the premise membership functions of the T-S fuzzy model are complicated, it increases the structural complexity of the fuzzy controller leading to a higher implementation cost. Some approaches to improved fuzzy controller design via relaxed stability conditions were reported in (Tanaka et al., 1998; Teixeira et al., 2003; Fang et al., 2006; Chang et al., 2009). These stability conditions can be cast in terms of Linear Matrix Inequality (LMI) that can be solved numerically using some convex programming techniques (Boyd et al., 1994). However, the relaxed stability conditions in (Tanaka et al., 1998; Teixeira et al., 2003; Fang et al., 2006; Chang et al., 2009) are also developed based on the PDC concept. Without loss of generality, when a large number of subsystems are involved, the PDC-based fuzzy controller is conservative. In order to solve this problem, the Non-PDC fuzzy controller design (Zhou et al., 2007; Li et al., 2008; Lam, 2009; Chang et al., 2010a) has focused on developing the relaxed stability condi-

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tions based on the nonquadratic Lyapunov functions. In (Zhou et al., 2007; Li et al., 2008; Lam, 2009; Chang et al., 2010a), the application of the nonquadratic Lyapunov functions increases the complexity for the fuzzy controller derivations. Different from the approaches of (Zhou et al., 2007; Li et al., 2008; Lam, 2009; Chang et al., 2010a), this paper intends to utilize the concept of Imperfect Premise Matching (IPM) to find relaxed fuzzy controllers whose membership functions are different from that of the T-S fuzzy models.

The goal of IPM technique (Lam and Narimani, 2009) is to design different membership functions of fuzzy controllers for the T-S fuzzy models. Under the IPM concept, the T-S fuzzy model can be uncertain in value. As a result, the authors of (Lam and Narimani, 2009) proposed fuzzy controller displays the robustness property to handle parameter uncertainties. In order to keep the design flexibility and robustness property of the fuzzy controller, the fuzzy controller does not require sharing the same premises as those of the fuzzy model. Therefore, using the IPM concept, one can design a fuzzy controller with simple membership functions to handle the fuzzy model with complex membership functions. However, the different premises of influence can be unstable. In such a case, arbitrary matrices are joining to relax the stability conditions. Consequently, the less conservative stability conditions for deterministic T-S fuzzy systems were proposed and verified in (Lam and Narimani, 2009). However, the stochastic systems often appear in practical industries. Moreover, the stability and stabilization problems of stochastic systems are more difficult than that of deterministic systems. Thus, it is worthy to derive the relaxed stability conditions via the concept of IPM to find fuzzy controllers for stochastic T-S fuzzy systems. In our previous work (Chang et al., 2015) the stability analysis and controller synthesis of continuous-time stochastic nonlinear systems were discussed and investigated via applying IPM concept. Based on our experiences in (Chang et al., 2015), a relaxed fuzzy controller design method is developed for discrete-time stochastic nonlinear systems in this paper.

Recently, the fuzzy control problem for the nonlinear stochastic systems has been investigated via the T-S fuzzy models in (Chang et al., 2009; Chang et al., 2010a; Ku et al., 2010; Chang et al., 2011). In these papers, the consequent part of T-S fuzzy model is described as stochastic differential equations that are structured by deterministic differential equations with multiplicative noise terms (Eli et al., 2005). The PDC concept was employed to design the fuzzy controllers in (Chang et al., 2009; Chang et al., 2010a; Ku et al., 2010; Chang et al., 2011). The purpose of this paper is to study the fuzzy controller design for the discrete-time stochastic nonlinear systems. Based on the stochastic differential equations (Eli et al., 2005), the T-S fuzzy model with multiplicative noise (Chang et al., 2009; Chang et al., 2010a; Ku et al., 2010; Chang et al., 2011) is built to represent the discrete-time stochastic nonlinear systems. Besides, the proposed fuzzy controller design may not share the same premise membership function with the T-S fuzzy model by using the IPM concept. For the fuzzy controller design via IPM concept, arbitrary slack matrices are introduced to compensate unstable elements. Hence, the IMP-based fuzzy controller can be enhanced to increase the flexibility and robustness property. Finally, the IMP-based fuzzy controller design problem can be efficiently solved by using convex programming techniques to solve the relaxed LMI stability conditions. To validate the effectiveness of the proposed fuzzy controller design method, a numerical example for the control of discrete-time nonlinear truck-trailer system is included in this paper.

II. SYSTEM DESCRIPTIONS AND PROBLEM STATEMENTS

In this paper, a discrete-time stochastic T-S fuzzy model is used to represent a nonlinear stochastic system. The stochastic T-S fuzzy model is described by fuzzy IF-THEN rules, which represent local linear input-output relations of the nonlinear systems. Let r be the number of fuzzy rules describing the discrete-time stochastic T-S fuzzy model. The i-th rule of the stochastic T-S fuzzy model has the following form.

***Rule i*:** IF $z_1(k)$ is M_{i1} and $z_2(k)$ is M_{i2} and… and $z_\phi(k)$ is $M_{i\phi}$ THEN

$$x(k+1) = \left(\mathbf{A}_i x(k) + \mathbf{B}_i u(k)\right) + \left(\bar{\mathbf{A}}_i x(k) + \bar{\mathbf{B}}_i u(k)\right) w(k) \quad \text{for } i = 1, 2, \ldots, r \tag{1}$$

where $M_{i\phi}$ is the fuzzy set, $z_1(k)\ldots z_\phi(k)$ are the premise variables, ϕ is the number of premise variables, $\mathbf{A}_i \in \Re^{n\times n}$, $\mathbf{B}_i \in \Re^{n\times m}$, $\bar{\mathbf{A}}_i \in \Re^{n\times n}$ and $\bar{\mathbf{B}}_i \in \Re^{n\times m}$ are known constant matrices, $x(k) \in \Re^n$ is the state vector, $u(k) \in \Re^m$ is the control input vector and $w(k)$ is a scalar Brownian motion (Karatzas and Shreve, 1991) which satisfies $E\{w(k)\} = 0$ and $E\{w^2(k)\} = 1$, where $E\{\bullet\}$ denote the standard expected operator for $\{\bullet\}$. The final plant of the fuzzy model (1) can be referred as follows.

$$x(k+1) = \sum_{i=1}^{r} h_i(z(k))\left\{\left(\mathbf{A}_i x(k) + \mathbf{B}_i u(k)\right) + \left(\bar{\mathbf{A}}_i x(k) + \bar{\mathbf{B}}_i u(k)\right) w(k)\right\} \tag{2}$$

where $z(k) = \left[z_1(k)\ldots z_\phi(k)\right]$ and

$$h_i(z(k)) = \frac{f_i(z(k))}{\sum_{i=1}^{r} f_i(z(k))},\ f_i(z(k)) = \prod_{\alpha=1}^{\phi} M_{i\alpha}(z(k)),\ \sum_{i=1}^{r} h_i(z(k)) = 1 \tag{3}$$

where $M_{i\alpha}(z_\alpha(k))$ is the grade of membership function of the $z_\alpha(k)$ in $M_{i\alpha}$. It is assumed that $\sum_{i=1}^{r} f_i\left(z(k)\right) > 0$, $i = 1, 2, \ldots, r$ and $f_i\left(z(k)\right) \geq 0$ for all $z(k)$. Therefore, $h_i\left(z(k)\right) \geq 0$ and $\sum_{i=1}^{r} h_i\left(z(k)\right) = 1$ for all $z(k)$.

Based on the IPM concept, one can design a fuzzy controller that the membership function is different from that of the fuzzy plant. Thus, the fuzzy controller can be described as follows.

Rule j **:** IF $z_1(k)$ is N_{j1} and $z_2(k)$ is N_{j2} and … and $z_\phi(k)$ is $N_{j\phi}$ THEN

$$u(k) = \mathbf{G}_j x(k) \text{ for } j = 1,\ 2,\ \ldots,\ r \tag{4}$$

where $N_{j\phi}$ is the fuzzy set, $z_1(k)$ … $z_\phi(k)$ are the premise variables, ϕ is the number of premise variables, and $G_j \in \Re^{m\times n}$ is the feedback gains of rule j. The inferred output of the fuzzy controller (4) is given by.

$$u(k) = \sum_{j=1}^{r} m_j\left(z(k)\right)\mathbf{G}_j x(k) \tag{5}$$

where $z(k) = \left[z_1(k)\ldots z_\phi(k)\right]$ and

$$m_j\left(z(k)\right) = \frac{g_j\left(z(k)\right)}{\sum_{j=1}^{r} g_j\left(z(k)\right)},\ g_j\left(z(k)\right) = \prod_{\beta=1}^{\phi} N_{j\beta}\left(z(k)\right),\ \sum_{j=1}^{r} m_j\left(z(k)\right) = 1 \tag{6}$$

where $N_{j\beta}(z_\beta(k))$ is the grade of membership function of the $z_\beta(k)$ in $N_{j\beta}$. It is assumed that $\sum_{j=1}^{r} g_j\left(z(k)\right) > 0$, $j = 1, 2, \ldots, r$ and $g_j\left(z(k)\right) \geq 0$ for all $z(k)$. Therefore, $m_j\left(z(k)\right) \geq 0$ and $\sum_{j=1}^{r} m_j\left(z(k)\right) = 1$ for all $z(k)$. Substituting (5) into (2), one can obtain corresponding closed-loop system as follows.

$$\begin{aligned} x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(k)\right) m_j\left(z(k)\right)\Big\{\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)x(k) \\ +\left(\left(\bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i\mathbf{G}_j\right)x(k)\right)w(k)\Big\} \end{aligned} \tag{7}$$

In the following statements, $h_i\left(z(k)\right)$ and $m_j\left(z(k)\right)$ are denoted as h_i and m_j for brevity. In (7), the fuzzy model and fuzzy controller do not share the same membership functions that lead to IPM. The IPM concept can be referred to (Lam and Narimani, 2009). Based on the IPM concept, the purpose of this paper is to derive relaxed stability conditions for finding the fuzzy controller (5) that can stabilize the closed-loop system (7). In next section, the Lyapunov theory is used to derive the relaxed stability conditions. Besides, the LMI technique is employed to find the solutions of these relaxed stability conditions such that the fuzzy controller (5) can be obtained.

III. IPM-BASED FUZZY CONTROLLER DESIGN FOR DISCRETE STOCHASTIC T-S FUZZY SYSTEMS

According to the closed-loop fuzzy system (7), the IPM-based fuzzy controller is developed in this section by deriving the relaxed stability conditions. One can find that these stability conditions can be solved by using the convex programming technique because they can be transferred into the LMI problems. In the following theorem, the stability conditions for the existence of IPM-based fuzzy controller are first derived. By transferring the stability conditions of Theorem 1, the LMI stability conditions will be introduced in the Theorem 2.

Theorem 1

The closed-loop system (7) is asymptotically stable if the membership functions of the fuzzy model and fuzzy controller satisfy $m_j - \rho_j h_j \geq 0$ for all j and $z(k)$, where $0 < \rho_j < 1$, and there exist negative definite matrices $\mathbf{R}_{ij} = \mathbf{R}_{ji}^{\mathrm{T}}$, $\mathbf{S}_{ij} = \mathbf{S}_{ji}^{\mathrm{T}}$, positive definite matrix $\mathbf{P} > 0$, arbitrary matrices $\mathbf{\Lambda}_i$ and $\mathbf{V}_{ij}$, and feedback gains $\mathbf{G}_j$ such that the following conditions are satisfied.

$$\mathbf{\Omega}_{ij} < 0 \tag{8}$$

$$\mathbf{\Psi}_{ii} < \mathbf{R}_{ii} \tag{9}$$

$$\mathbf{K}_{ij} \leq \mathbf{R}_{ij} + \mathbf{R}_{ji}^{\mathrm{T}} \tag{10}$$

$$\mathbf{\Theta}_{ii} < \mathbf{S}_{ii} \tag{11}$$

$$\mathbf{Y}_{ij} \leq \mathbf{S}_{ij} + \mathbf{S}_{ji} \tag{12}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \cdots & \mathbf{R}_{1r} \\ \mathbf{R}_{21} & \mathbf{R}_{22} & \cdots & \mathbf{R}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{r1} & \mathbf{R}_{r2} & \cdots & \mathbf{R}_{rr} \end{bmatrix} < 0 \text{ and}$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} & \cdots & \mathbf{S}_{1r} \\ \mathbf{S}_{21} & \mathbf{S}_{22} & \cdots & \mathbf{S}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{r1} & \mathbf{S}_{r2} & \cdots & \mathbf{S}_{rr} \end{bmatrix} < 0 \tag{13}$$

where $\mathbf{\Lambda}_i = \mathbf{\Lambda}_i^{\mathrm{T}}$ and $\mathbf{V}_{ij} = \mathbf{V}_{ij}^{\mathrm{T}}$, $i, j = 1, 2, \ldots, r$ are arbitrary matrices, $\mathbf{\Omega}_{ij} = \mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i$, $\mathbf{\Psi}_{ii} = \rho_i\left(\mathbf{\Phi}_{ii} - \mathbf{\Lambda}_i - \mathbf{V}_{ii}\right)$, $\mathbf{\Theta}_{ii} = \mathbf{\Lambda}_i + \rho_i \mathbf{V}_{ii}$, $\mathbf{K}_{ij} = \left(\rho_j\left(\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i - \mathbf{V}_{ij}\right) + \rho_i\left(\mathbf{\Phi}_{ji} - \mathbf{\Lambda}_j - \mathbf{V}_{ji}\right)\right)$, $\mathbf{Y}_{ij} = \rho_j \mathbf{V}_{ij} + \mathbf{\Lambda}_j + \rho_i \mathbf{V}_{ji}$, $\mathbf{\Phi}_{ij} = \left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)^{\mathrm{T}} \mathbf{P}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right) + \left(\bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i\mathbf{G}_j\right)^{\mathrm{T}} \mathbf{P}\left(\bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i\mathbf{G}_j\right)^{\mathrm{T}} - \mathbf{P}$.

Proof :

Choose the Lyapunov function as $V\left(x(k)\right) = x^{\mathrm{T}}(k)\mathbf{P}x(k)$. From (Karatzas and Shreve, 1991), one has the properties as $E\left\{w(k)\right\} = 0$ and $E\left\{w^2(k)\right\} = 1$. Therefore, one can obtain the difference of chosen Lyapunov function with standard expectation operator such as

$$
\begin{aligned}
E\left\{\Delta V\left(x(k)\right)\right\} &= E\left\{x^{\mathrm{T}}(k+1)\mathbf{P}x(k+1) - x^{\mathrm{T}}(k)\mathbf{P}x(k)\right\} \\
&\le E\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i m_j x^{\mathrm{T}}(k)\left(\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)^{\mathrm{T}}\mathbf{P}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\right.\right. \\
&\quad \left.\left. + \left(\bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i\mathbf{G}_j\right)^{\mathrm{T}}\mathbf{P}\left(\bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i\mathbf{G}_j\right)^{\mathrm{T}} - \mathbf{P}\right)x(k)\right\} \\
&= E\left\{x^{\mathrm{T}}(k)\mathbf{\Gamma}_{ij}x(k)\right\}
\end{aligned} \tag{14}
$$

where

$$
\mathbf{\Gamma}_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i m_j \mathbf{\Phi}_{ij} \tag{15}
$$

Note that (15) implies the fuzzy model and fuzzy controller have different membership functions that lead the concept of IPM. Here, let us consider the following equality

$$
\sum_{i=1}^{r} h_i = \sum_{j=1}^{r} m_j = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j = 1
$$

and

$$
\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(h_j - m_j\right)\mathbf{\Lambda}_i + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\rho_j m_j\left(\mathbf{V}_{ij} - \mathbf{V}_{ij}\right) \\
&= \sum_{i=1}^{r} h_i\left(\sum_{j=1}^{r} h_j - \sum_{j=1}^{r} m_j\right)\mathbf{\Lambda}_i + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\rho_j m_j\left(\mathbf{V}_{ij} - \mathbf{V}_{ij}\right) = 0
\end{aligned}
$$

where $\mathbf{\Lambda}_i = \mathbf{\Lambda}_i^{\mathrm{T}} \in \Re^{n\times n}$, $\mathbf{V}_{ij} = \mathbf{V}_{ij}^{\mathrm{T}} \in \Re^{n\times n}$, and $i, j = 1, 2, \ldots, r$ are arbitrary matrices. The arbitrary matrices $\mathbf{\Lambda}_i$ and $\mathbf{V}_{ij}$ are introduced to compensate unstable elements. These terms are introduced to (15) to alleviate the conservativeness, and $0 < \rho_j < 1$, $j = 1, 2, \ldots, r$ are designed such that $m_j - \rho_j h_j \ge 0$ for all j and $z(k)$. To obtain $m_j - \rho_j h_j \ge 0$, it is necessary to join arbitrary matrices $\mathbf{\Lambda}_i$ and $\mathbf{V}_{ij}$. Therefore, the Eq. (15) can be represented as

$$
\begin{aligned}
\mathbf{\Gamma}_{ij} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(m_j + \rho_j h_j - \rho_j h_j\right)\mathbf{\Phi}_{ij} \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(h_j - m_j + \rho_j h_j - \rho_j h_j\right)\mathbf{\Lambda}_i \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\rho_j h_j\left(\mathbf{V}_{ij} - \mathbf{V}_{ij}\right) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(m_j - \rho_j h_j\right)\left(\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i\right) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \rho_j\left(\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i - \mathbf{V}_{ij}\right) + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\left(\mathbf{\Lambda}_i + \rho_j\mathbf{V}_{ij}\right) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(m_j - \rho_j h_j\right)\left(\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i\right) + \sum_{i=1}^{r} h_i h_i \rho_i\left(\mathbf{\Phi}_{ii} - \mathbf{\Lambda}_i - \mathbf{V}_{ii}\right) \\
&\quad + \sum_{i=1}^{r}\sum_{i<j} h_i h_j\left(\rho_j\left(\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i - \mathbf{V}_{ij}\right) + \rho_i\left(\mathbf{\Phi}_{ji} - \mathbf{\Lambda}_j - \mathbf{V}_{ji}\right)\right) \\
&\quad + \sum_{i=1}^{r} h_i h_i\left(\mathbf{\Lambda}_i + \rho_i\mathbf{V}_{ii}\right) + \sum_{i=1}^{r}\sum_{i<j} h_i h_j\left(\mathbf{\Lambda}_i + \rho_j\mathbf{V}_{ij} + \mathbf{\Lambda}_j + \rho_i\mathbf{V}_{ji}\right) \\
&= \mathbf{\Pi}_1 + \mathbf{\Pi}_2 + \mathbf{\Pi}_3
\end{aligned} \tag{16}
$$

where

$$
\mathbf{\Pi}_1 = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(m_j - \rho_j h_j\right)\mathbf{\Omega}_{ij},\ \mathbf{\Pi}_2 = \sum_{i=1}^{r} h_i^2\mathbf{\Psi}_{ii} + \sum_{i=1}^{r}\sum_{i<j} h_i h_j\mathbf{K}_{ij}
$$

$$
\text{and } \mathbf{\Pi}_3 = \sum_{i=1}^{r} h_i^2\mathbf{\Theta}_{ii} + \sum_{i=1}^{r}\sum_{i<j} h_i h_j\mathbf{Y}_{ij}.
$$

From (16), if the condition $m_j - \rho_j h_j \ge 0$ and the condition (8) are satisfied, then one has

$$
\mathbf{\Pi}_1 < 0 \tag{17}
$$

It is evident that if the conditions (9), (10), (11) and (12) are held, one can obtain

$$
\mathbf{\Pi}_2 < \bar{\mathbf{h}}\mathbf{R}\bar{\mathbf{h}}^{\mathrm{T}} \tag{18}
$$

$$
\mathbf{\Pi}_3 < \bar{\mathbf{h}}\mathbf{S}\bar{\mathbf{h}}^{\mathrm{T}} \tag{19}
$$

where $\bar{\mathbf{h}} = \left[h_1\ h_2 \ldots h_n\right]$. According to the inequalities (17), (18) and (19), one can obtain the following inequality from (16).

$$
\mathbf{\Gamma}_{ij} \le \bar{\mathbf{h}}\mathbf{R}\bar{\mathbf{h}}^{\mathrm{T}} + \bar{\mathbf{h}}\mathbf{S}\bar{\mathbf{h}}^{\mathrm{T}} \tag{20}
$$

Since $\mathbf{R}<0$ and $\mathbf{S}<0$, one can obtain $\mathbf{\Gamma}_{ij}<0$ from (20). Due to $\mathbf{\Gamma}_{ij}<0$, the inequality $E\left\{\Delta\mathbf{V}\left(x\left(k\right)\right)\right\}<0$ can be obtained from (14). Therefore, if the conditions of Theorem 1 hold, then one has $E\left\{\Delta\mathbf{V}\left(x\left(k\right)\right)\right\}<0$ and the closed-loop system (7) is asymptotically stable

#

Theorem 1 presents the sufficient conditions that are of bilinear matrix inequalities. In the next theorem, applying the mathematic transform technique, the bilinear matrix inequalities can be transformed into the LMI problems. Through using the MATLAB LMI-toolbox to solve the LMI problems, one can obtain the stable fuzzy controllers for the closed-loop fuzzy systems.

Theorem 2

The closed-loop system (7) is asymptotically stable if the membership functions of the fuzzy model and fuzzy controller satisfy $m_j-\rho_j h_j\geq 0$ for all j and $z\left(k\right)$, where $0<\rho_j<1$, and there exist negative definite matrices $\mathbf{R}_{ij}=\mathbf{R}_{ji}^{\mathrm{T}}$, $\mathbf{S}_{ij}=\mathbf{S}_{ji}^{\mathrm{T}}$, positive definite matrix $\mathbf{P}^{-1}=\mathbf{Q}$, arbitrary matrices $\tilde{\mathbf{\Lambda}}_i$ and $\tilde{\mathbf{V}}_{ij}$, and feedback gains $\mathbf{G}_j=\mathbf{Q}^{-1}\mathbf{N}_j$ such that the following conditions are satisfied.

$$\mathbf{\Xi}_{ij}<0 \tag{21}$$

$$\mathbf{H}_{ii}<\mathbf{D}_{ii} \tag{22}$$

$$\tilde{\mathbf{K}}_{ij}\leq\mathbf{U}_{ij} \tag{23}$$

$$\tilde{\mathbf{\Theta}}_{ii}<\mathbf{S}_{ii} \tag{24}$$

$$\tilde{\mathbf{Y}}_{ij}\leq\mathbf{S}_{ij}+\mathbf{S}_{ji} \tag{25}$$

where

$$\mathbf{D}_{ii}=\left[\begin{array}{c|c}\mathbf{R}_{ii} & 0\\ \hline 0 & 0\end{array}\right]\in\Re^{3n\times 3n},\quad \mathbf{U}_{ij}=\left[\begin{array}{c|c}\mathbf{R}_{ij}+\mathbf{R}_{ji}^{\mathrm{T}} & 0\\ \hline 0 & 0\end{array}\right]\in\Re^{5n\times 5n},$$

$$\mathbf{\Xi}_{ij}=\begin{bmatrix}-\mathbf{Q}-\tilde{\mathbf{\Lambda}}_i & \left(\mathbf{A}_i\mathbf{Q}+\mathbf{B}_i\mathbf{N}_j\right)^{\mathrm{T}} & \left(\bar{\mathbf{A}}_i\mathbf{Q}+\bar{\mathbf{B}}_i\mathbf{N}_j\right)^{\mathrm{T}}\\ * & -\mathbf{Q} & 0\\ * & * & -\mathbf{Q}\end{bmatrix},$$

$$\mathbf{H}_{ii}=\begin{bmatrix}-\rho_i\left(\mathbf{Q}+\tilde{\mathbf{\Lambda}}_i+\tilde{\mathbf{V}}_{ii}\right) & \left(\mathbf{A}_i\mathbf{Q}+\mathbf{B}_i\mathbf{N}_i\right)^{\mathrm{T}} & \left(\bar{\mathbf{A}}_i\mathbf{Q}+\bar{\mathbf{B}}_i\mathbf{N}_i\right)^{\mathrm{T}}\\ * & -\rho_i^{-1}\mathbf{Q} & 0\\ * & * & -\rho_i^{-1}\mathbf{Q}\end{bmatrix},$$

$$\tilde{\mathbf{K}}_{ij}=\begin{bmatrix}\mathbf{X} & \left(\mathbf{A}_i\mathbf{Q}+\mathbf{B}_i\mathbf{N}_j\right)^{\mathrm{T}} & \left(\bar{\mathbf{A}}_i\mathbf{Q}+\bar{\mathbf{B}}_i\mathbf{N}_j\right)^{\mathrm{T}} & \left(\mathbf{A}_j\mathbf{Q}+\mathbf{B}_j\mathbf{N}_i\right)^{\mathrm{T}} & \left(\bar{\mathbf{A}}_j\mathbf{Q}+\bar{\mathbf{B}}_j\mathbf{N}_i\right)^{\mathrm{T}}\\ * & -\rho_j^{-1}\mathbf{Q} & 0 & 0 & 0\\ * & * & -\rho_j^{-1}\mathbf{Q} & 0 & 0\\ * & * & * & -\rho_i^{-1}\mathbf{Q} & 0\\ * & * & * & * & -\rho_i^{-1}\mathbf{Q}\end{bmatrix}.$$

and "*" denotes the transposed elements or matrices for symmetric position. Besides, $\mathbf{R}_{ij}=\mathbf{R}_{ji}^{\mathrm{T}}$ and $\mathbf{S}_{ij}=\mathbf{S}_{ji}^{\mathrm{T}}$ are defined in (13), $\mathbf{Q}=\mathbf{P}^{-1}$, $\mathbf{N}_j=\mathbf{G}_j\mathbf{P}^{-1}$, $\mathbf{X}=-\rho_j\left(\mathbf{Q}+\tilde{\mathbf{\Lambda}}_i+\tilde{\mathbf{V}}_{ij}\right)-\rho_i\left(\mathbf{Q}+\tilde{\mathbf{\Lambda}}_j+\tilde{\mathbf{V}}_{ji}\right)$, $\tilde{\mathbf{\Theta}}_{ii}=\tilde{\mathbf{\Lambda}}_i+\rho_i\tilde{\mathbf{V}}_{ii}$ and $\tilde{\mathbf{Y}}_{ij}=\tilde{\mathbf{\Lambda}}_i+\rho_j\tilde{\mathbf{V}}_{ij}+\tilde{\mathbf{\Lambda}}_j+\rho_i\tilde{\mathbf{V}}_{ji}$. Moreover, $\tilde{\mathbf{\Lambda}}_i=\mathbf{P}^{-1}\tilde{\mathbf{\Lambda}}_i\mathbf{P}^{-1}$, and $\tilde{\mathbf{V}}_{ij}=\mathbf{P}^{-1}\tilde{\mathbf{V}}_{ij}\mathbf{P}^{-1}$, $i,j=1,\ 2,\ \ldots,\ r$ are arbitrary matrices.

Proof :

By using the Schur complement (Boyd et al., 1994), one can get that $\mathbf{\Omega}_{ij}=\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{P}$, $\mathbf{\Psi}_{ii}=\mathbf{P}\mathbf{H}_{ii}\mathbf{P}$, $\mathbf{K}_{ij}=\mathbf{P}\tilde{\mathbf{K}}_{ij}\mathbf{P}$, $\mathbf{\Theta}_{ii}=\mathbf{P}\tilde{\mathbf{\Theta}}_{ii}\mathbf{P}$ and $\mathbf{Y}_{ij}=\mathbf{P}\tilde{\mathbf{Y}}_{ij}\mathbf{P}$, respectively. Obviously, if (21)-(25) are held, then (8)-(12) can also be satisfied. Therefore, if the conditions of Theorem 2 are held, then the conditions of Theorem 1 can also be satisfied. According to Theorem 1, if the conditions (21)-(25) are satisfied then the closed-loop system (7) is asymptotically stable

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Theorem 2 transforms the conditions of Theorem 1 into LMI forms. Based on the stability conditions of Theorem 2, one can solve these conditions via MATLAB LMI-toolbox for obtaining the feasible solutions of fuzzy controllers (5). However, the guaranteeing of condition as $m_j-\rho_j h_j\geq 0$ is an interesting problem. To clarify the problem, a remark is added as follows:

Remark 1

Before discussing stability and stabilization problem of the closed-loop model (7), the membership functions of fuzzy controller and fuzzy model are known. According to the value of ρ_j satisfying $0\leq\rho_j\leq 1$, a small enough value ρ_j can be thus chosen such that $m_j-\rho_j h_j\geq 0$ is held.

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In the following section, a numerical example is provided to demonstrate the application and effectiveness of the proposed fuzzy control method.

IV. A NUMERICAL EXAMPLE

In this section, using a numerical example, the usefulness of the approach proposed in the previous sections is illustrated. Here, a discrete-time nonlinear truck-trailer system is considered. The dynamic equation of the discrete-time nonlinear truck-trailer system has been described in (Tanaka and Sano, 1994). In this example, the multiplicative noise $w(k)$ is considered for describing the practical stochastic behaviors. Without loss of generality, it is assumed that $x_1(k)$ and $u(k)$ are always small and the horizontal position motion $x_4(k)$ can be neglected. Therefore, one can represent and simplify the original model as follows.

$$x_1(k+1)=\left(1-\frac{\upsilon\cdot\Delta t}{L_2}\right)x_1(k)+\frac{\upsilon\cdot\Delta t}{L_1}u(k) \tag{26a}$$

$$x_2(k+1)=\frac{\upsilon\cdot\Delta t}{L_2}x_1(k)+x_2(k) \tag{26b}$$

$$x_3(k+1)=\upsilon\cdot\Delta t\cdot\sin\left(\frac{\upsilon\cdot\Delta t}{2L_2}x_1(k)+x_2(k)\right)+x_3(k) \tag{26c}$$

In this example, the following parameter values are used for simulations

$$L_1=2.8\text{ m},\ L_2=5.5\text{ m},\ \Delta t=2\text{ m},\ \upsilon=\text{-1.0 m/sec}$$

where L_1 is the length of truck, L_2 is the length of trailer, Δt is the sampling time, υ is the constant speed of backing up. For $x_1(k)$, 90° and –90° correspond to two "jackknife" positions. The jackknife phenomenon cannot be avoided if the steering is not controlled during the backward movement. To succeed in the backing control, we need to avoid the jackknife phenomenon. The controlling purpose of this paper is to back up a truck trailer along a straight line $(x_3=0)$ without forward movements as shown in (Tanaka and Sano, 1994), that is $x_1(k)\to 0$, $x_2(k)\to 0$ and $x_3(k)\to 0$.

Assume that the premise variable $\frac{\upsilon\cdot\Delta t}{2L_2}x_1(k)+x_2(k)$ is operated between $[-\pi,\ \pi]$. Then a T-S fuzzy model, which approximately represents the dynamic of the truck-trailer stochastic system (26), can be obtained as follows

T-S Fuzzy Model:

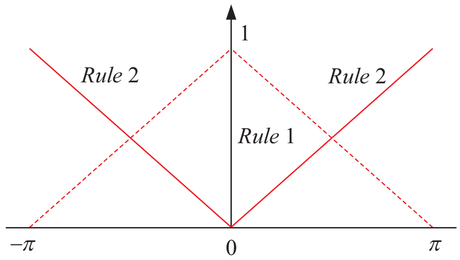

Fig. 1. Membership functions of fuzzy model.

***Rule* 1:** IF $\frac{\upsilon\cdot\Delta t}{2L_2}x_1(k)+x_2(k)$ is about 0, THEN

$$x(k+1)=\left(\mathbf{A}_1x(k)+\mathbf{B}_1u(k)\right)+\left(\bar{\mathbf{A}}_1x(k)+\bar{\mathbf{B}}_1u(k)\right)w(k) \tag{27a}$$

***Rule* 2:** IF $\frac{\upsilon\cdot\Delta t}{2L_2}x_1(k)+x_2(k)$ is about $-\pi$ or π THEN

$$x(k+1)=\left(\mathbf{A}_2x(k)+\mathbf{B}_2u(k)\right)+\left(\bar{\mathbf{A}}_2x(k)+\bar{\mathbf{B}}_2u(k)\right)w(k) \tag{27b}$$

where

$$\mathbf{A}_1=\begin{bmatrix}1-\frac{\upsilon\cdot\Delta t}{L_2} & 0 & 0\\ \frac{\upsilon\cdot\Delta t}{L_2} & 1 & 0\\ \frac{\upsilon^2\cdot\Delta t^2}{2L_2} & \upsilon\cdot\Delta t & 1\end{bmatrix},\ \mathbf{A}_2=\begin{bmatrix}1-\frac{\upsilon\cdot\Delta t}{L_2} & 0 & 0\\ \frac{\upsilon\cdot\Delta t}{L_2} & 1 & 0\\ \frac{\psi\cdot\upsilon^2\cdot\Delta t^2}{2L_2} & \psi\cdot\upsilon\cdot\Delta t & 1\end{bmatrix},$$

$$\mathbf{B}_1=\mathbf{B}_2=\begin{bmatrix}\frac{\upsilon\cdot\Delta t}{L_1}\\ 0\\ 0\end{bmatrix},\ \bar{\mathbf{A}}_a=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.1\end{bmatrix},\ \bar{\mathbf{B}}_a=\begin{bmatrix}0.1\\ 0\\ 0\end{bmatrix}$$

for $a = 1, 2$, and $\psi=10^{-2}/\pi$.

Fig. 1 shows the membership functions of the above T-S fuzzy model.

Based on the IPM technique, the proposed fuzzy controller is designed to possess different membership functions from the fuzzy model (27). In this example, it is assumed that the premise variable $\frac{\upsilon\cdot\Delta t}{2L_2}x_1(k)+x_2(k)$ of the fuzzy controller is constrained within $[-\pi,\ \pi]$ and the corresponding mem-

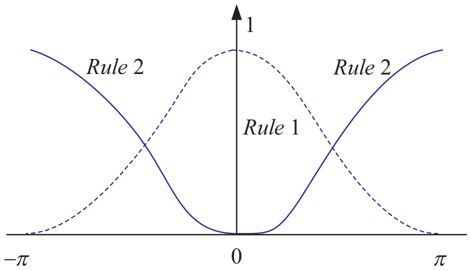

Fig. 2. Membership functions of fuzzy controller.

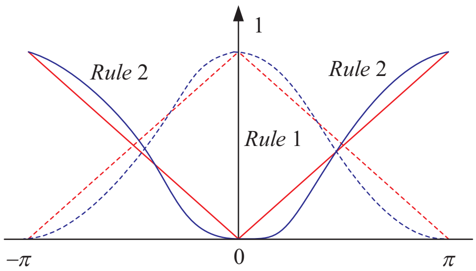

Fig. 3. Membership functions.

bership functions are given in Fig. 2. Then, the IPM-based fuzzy controller has the following form.

Fuzzy Controller:

***Rule* 1:** IF $\dfrac{\upsilon \cdot \Delta t}{2L_2} x_1(k) + x_2(k)$ is about 0, THEN

$$u(k) = \mathbf{G}_1 x(k) \tag{28a}$$

***Rule* 2:** IF $\dfrac{\upsilon \cdot \Delta t}{2L_2} x_1(k) + x_2(k)$ is about $-\pi$ or π THEN

$$u(k) = \mathbf{G}_2 x(k) \tag{28b}$$

For determining the values of ρ_1 and ρ_2 to satisfy the condition $m_j - \rho_j h_j \geq 0$, the membership functions of the fuzzy model and fuzzy controller are stated in Fig. 3. From Fig. 3, it is easy to determine $\rho_1 = \rho_2 = 0.82$ to satisfy $m_j - \rho_j h_j \geq 0$. By applying the proposed fuzzy controller design technique, one can find the feasible solutions by solving the stability conditions (21)-(25) with the setting $\rho_1 = \rho_2 = 0.82$. Then, the common positive definite matrix $\mathbf{P} = \mathbf{Q}^{-1}$ and the negative definite matrices $\mathbf{R}$ and $\mathbf{S}$ are obtained as follows.

$$\mathbf{P} = \begin{bmatrix} 0.0056 & -0.0092 & 0.0019 \\ -0.0092 & 0.0282 & -0.0046 \\ 0.0019 & -0.0046 & 0.0031 \end{bmatrix},$$

$$\mathbf{R}_{11} = \begin{bmatrix} -29.5755 & -8.109 & -2.4622 \\ -8.109 & -4.1181 & -5.3777 \\ -2.4622 & -5.3777 & -16.4058 \end{bmatrix},$$

$$\mathbf{R}_{12} = \begin{bmatrix} -0.27526 & -0.11988 & -0.12254 \\ -0.11988 & -4.5793 & -3.8367 \\ -0.12254 & -3.8367 & -2.194 \end{bmatrix} \times 10^{-15},$$

$$\mathbf{R}_{22} = \begin{bmatrix} -32.4528 & -10.1311 & 1.9026 \\ -10.1311 & -4.8137 & 0.0022 \\ 1.9026 & 0.0022 & -8.1082 \end{bmatrix},$$

$$\mathbf{S}_{11} = \begin{bmatrix} -29.5755 & -8.109 & -2.4622 \\ -8.109 & -4.1181 & -5.3777 \\ -2.4622 & -5.3777 & -16.4058 \end{bmatrix},$$

$$\mathbf{S}_{12} = \begin{bmatrix} 0.13532 & 6.0375 & 2.3898 \\ 6.0375 & 2.8693 & 2.8723 \\ 2.3898 & 2.8723 & 9.5381 \end{bmatrix} \times 10^{-16},$$

$$\mathbf{S}_{22} = \begin{bmatrix} -32.4528 & -10.1311 & 1.9026 \\ -10.1311 & -4.8137 & 0.0022 \\ 1.9026 & 0.0022 & -8.1082 \end{bmatrix}.$$

In the same time, the control feedback gains for the fuzzy controller are obtained as follows

$$\mathbf{G}_1 = \begin{bmatrix} 3.1201 & -3.4112 & 0.5284 \end{bmatrix} \tag{29a}$$

$$\mathbf{G}_2 = \begin{bmatrix} 2.8782 & -2.5396 & 0.5228 \end{bmatrix} \tag{29b}$$

Applying the IPM-based fuzzy controller (28) with the feedback gains (29) to control the discrete-time nonlinear truck-trailer system (26), the simulation results of states are shown in Figs. 4-6. The initial condition of the simulation in this example is given as $x(0) = [88° \; -135° \; -10]^{\mathrm{T}}$. From these simulated responses, one can find that the controlled discrete-time nonlinear truck-trailer system (26) is asymptotically stable. The control goal of this paper can be achieved by designed the IPM-based fuzzy controller (28). Obviously, the proposed design method is useful and effective for designing IPM-based fuzzy controller for the discrete-time nonlinear stochastic systems.

To emphasize the importance of considering stochastic behavior, a comparison between this paper and (Lam and Narimani, 2009) is proposed. Referring to (Lam and Narimani, 2009), an IPM-based fuzzy controller design method was developed for deterministic nonlinear systems. Applying method

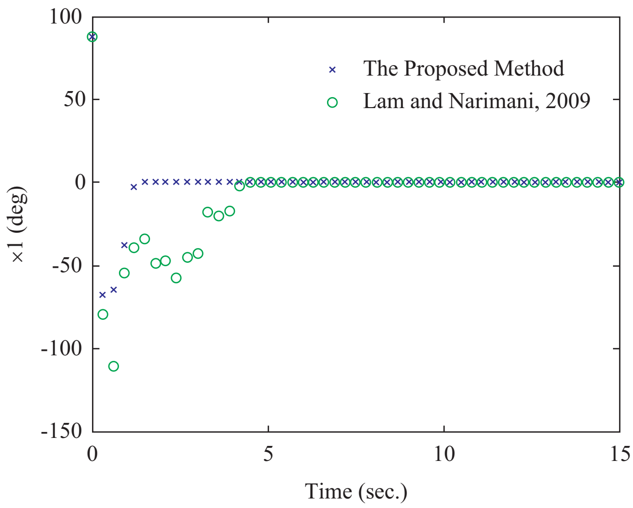

Fig. 4. Responses of state $x_1(k)$.

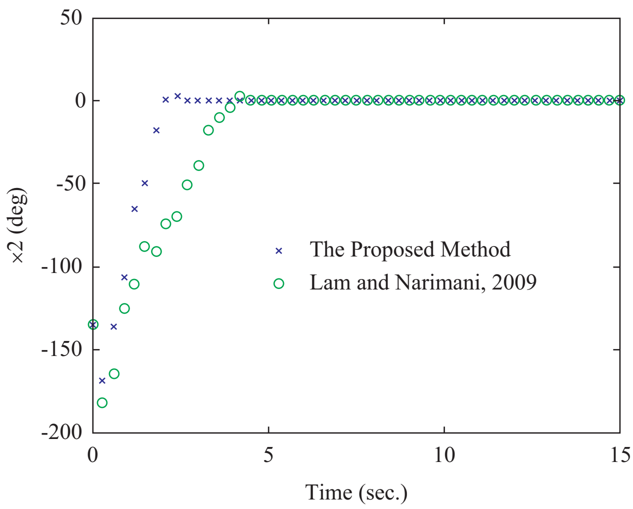

Fig. 5. Responses of state $x_1(k)$.

of (Lam and Narimani, 2009) to the T-S fuzzy model (27), the following gains can be obtained with the same $\rho_1 = \rho_2 = 0.82$.

$$\mathbf{G}_1 = \begin{bmatrix} 2.8986 & -3.2275 & 0.4752 \end{bmatrix} \text{ and}$$

$$\mathbf{G}_2 = \begin{bmatrix} 2.8148 & -2.7662 & 0.4935 \end{bmatrix} \tag{30}$$

Applying IPM-based fuzzy controller (28) with gains (30), the responses of (27) are also stated in Figs 4-6. From the figures, one can find that rise time and settling time of (27) driven by controller designed by this paper are smaller than that driven by controller designed by (Lam and Narimani, 2009). Through the simulation results, the proposed design method provides some improvements for the method of (Lam and Narimani, 2009) in controlling nonlinear stochastic systems.

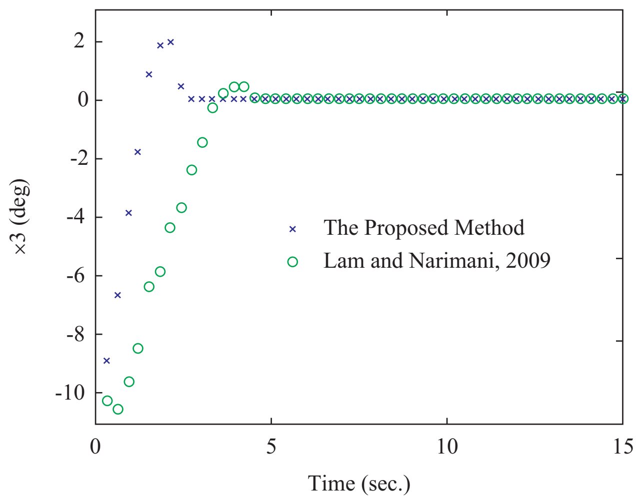

Fig. 6. Responses of state $x_3(k)$.

V. CONCLUSIONS

In this paper, a fuzzy controller design approach has been developed for a class of discrete-time stochastic nonlinear systems. The T-S fuzzy model with multiplicative noise was utilized to represent this class of nonlinear systems. Using the IPM concept, one can directly design a fuzzy controller that the membership function is different from that of the fuzzy plant. The relaxed stability conditions derived based on the IPM concept can be categorized as the LMI problems. By solving the LMI stability conditions via convex programming technique, the fuzzy controller can be obtained to stabilize the discrete-time T-S fuzzy model with multiplicative noise. Finally, a numerical example was provided to demonstrate the effectiveness and application of the proposed fuzzy controller design method.

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