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A MODIFIED SUMNER METHOD FOR OBTAINING THE ASTRONOMICAL VESSEL POSITION

Tien-Pen Hsu[1], Guo-Yu Weng[1], and Chih-Li Chen[2]

Key words: celestial navigation, Sumner line, running fix, iteration method.

ABSTRACT

An adaptive boundaries technique (ABT) yielded by the geometrical properties of celestial triangles is proposed to deal with problems resulting from ex-meridian or meridian sights when the Sumner method is used. Due to the trial-and-error characteristic of the Sumner method, an iteration method is introduced to improve numerical accuracy. Combining the ABT and the iteration method into the Sumner method, this modified Sumner method (MSM) is developed so that it successfully determines the astronomical vessel position (AVP). Especially when the non-simultaneous sights condition is encountered, based on the running fix concept, middle-latitude sailing is adopted to translate Sumner points to the fix time for determining the AVP. A program developed using the proposed approach is implemented to solve the AVP problem. Three benchmark examples are conducted to validate the accuracy and versatility of the proposed approach.

I. INTRODUCTION

In the open sea, there are two major ways to obtain the vessel position. One is to obtain it using a global navigation satellite system (GNSS); while the other is to determine it using celestial navigation. Although a GNSS provides navigators with continuous vessel position conveniently, it may suffer deliberate jamming, hostile spoofing or accidental interference, leading to inaccurate positioning (NTSB, 1997; John A. Volpe National Transportation Systems Center, 2001; Carroll, 2003; Williams et al., 2008; Grant et al., 2009). Accordingly, using independent position-fixing sources to cross check the vessel position for navigational safety is necessary (Bowditch, 1984, 2002; ICS, 1998; OCIMF, 2008; Royal Navy, 2008). As modern society grows ever more dependent on computing technology, computers are an essential tool of our working environments and academic disciplines. Not surprisingly, therefore, section B-II/1 of the 2010 Manila Amendments of International Convention on Standards of Training, Certification and Watchkeeping for Seafarers (STCW) recommends developing celestial navigation calculation software for determining astronomical vessel position (AVP) (IMO, 2010), as we set out to do here.

In the history of celestial navigation development, the American sea captain Thomas H. Sumner was the first to replace circle of position (COP) by line of position (LOP) (also called the Sumner line) to simplify calculation of the AVP. Based on this concept, the Sumner method was developed. Because the geometry of the Sumner method is so simple and obvious, it gradually gained popularity and eventually came to be used on every ship in the United States Navy (Richardson, 1946). Knowledge about the Sumner method soon spreads to the European maritime countries, opening a new era in practical navigation (Oestmann, 2011). However, the Sumner method cannot be used when the sights are taken near or at the time of the meridian passage (Gradsztajn, 1979). In addition, sizable errors in the calculation resulting from replacing COP by LOP, especially in the case of high altitude observations, commonly arise (Bowditch, 1984, 2002; Culter, 2003; Chen et al., 2003; Hsu et al., 2005; Chen et al., 2014). Finally, under conditions involving non-simultaneous sights (Gibson, 1994), determining how to use the Sumner method to determine the AVP still needs to be resolved. Elimination of these obstacles to using the Sumner method thus becomes the main challenge to our research goal of developing software that uses celestial navigation to accurately calculate AVP, which we solve via the development and implementation of a modified Sumner method (MSM) that incorporates an adaptive boundaries technique (ABT) proposed to deal with problems resulting from ex-meridian or meridian sights and an iteration method that we introduce to remedy the problem of inaccuracy arising from the trial-and-error characteristic of the Sumner method.

This paper is organized as follows: Section 2 describes theoretical background of the proposed MSM and the process of running fix. Computational procedures and the developed program are presented in Section 3. In Section 4, three benchmark examples illustrate the accuracy and versatility of the proposed method. Finally, conclusions are drawn in Section 5.

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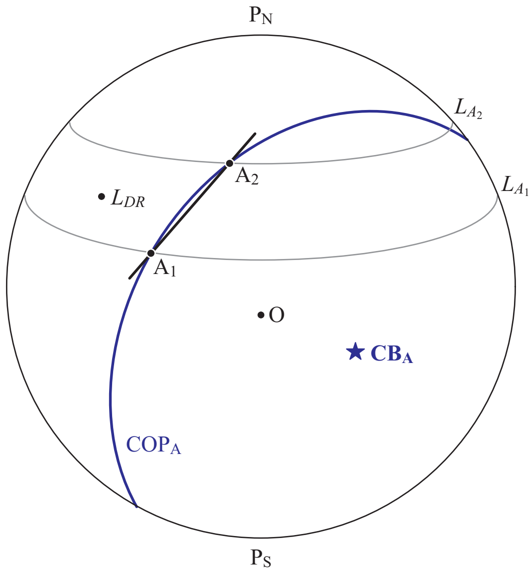


Fig. 1. Illustrations of the concept of establishing the Sumner line by the Sumner method.

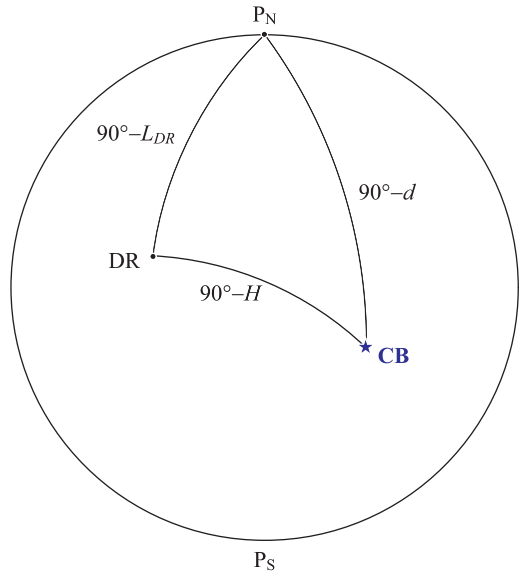


Fig. 3. Three sides of the celestial (spherical) triangle.

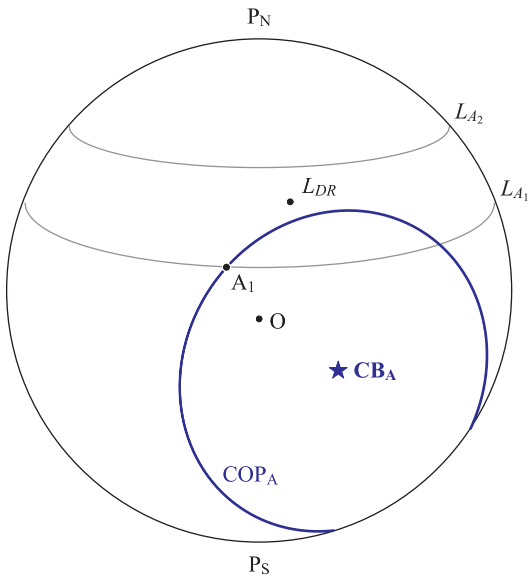


Fig. 2. The Sumner line cannot be established for ex-meridian or meridian sights.

II. THEORETICAL BACKGROUND

The Sumner method consists of establishing an LOP (also called a Sumner line) from the observed altitude of a celestial body by assuming two latitudes and then calculating the longitudes through which the Sumner line passes (Bowditch, 2002). The intersections of the two assumed latitudes and the COP, in which the Sumner line is connected as shown in Fig. 1, are hereafter called Sumner points. When another Sumner line is established, the AVP is determined by the intersection of the two Sumner lines. This method has been widely used due to its simplicity, which derives from the concept of LOP (Oestmann, 2011); however, as shown in Fig. 2, there are situations in which the Sumner points do not exists (i.e., the COP only intersects one of the assumed latitudes), because the Sumner method only considers the position of the observer, without regard to the geographical position and the observed altitude of the celestial body when locating the assumed latitudes. This makes the Sumner method unusable for ex-meridian or meridian sights. Besides, when the Sumner line is used to approximate the COP, the AVP is usually inaccurate (Bowditch, 1984, 2002; Culter, 2003; Chen et al., 2003; Hsu et al., 2005; Chen et al., 2014). Aimed at overcoming these shortcomings of the Sumner method, our modified Sumner method (MSM) first proposes an adaptive boundaries technique (ABT) to ensure the existence of two Sumner points and then, makes use of an iteration method to eliminate the approximation error induced by the Sumner line. Note that it is impossible to yield two Sumner lines at the same time in practice. Consequently, middle-latitude sailing is adopted to translate two pairs of Sumner points taken at two different observed times to a single fix time. Finally, the AVP is yielded by the intersection of the two resulting Sumner lines at the single fix time.

1. Adaptive Boundaries Technique

For the Sumner method, the normal practice is to take the assumed latitudes at 10′ north and south of the latitude of the dead reckoning (DR) position (Cotter, 1969; Chen et al., 2014), hereafter called the initial bounds. When ex-meridian or meridian sights are encountered, the Sumner points might not be determinable as explained above. To tackle this problem, we need to go back to consider the three sides of a celestial (spherical) triangle, which describes the relation of the latitude of the DR position (L_{DR}), declination (d) and observed altitude (H) as shown in Fig. 3. Based on the geometrical property, that any two sides

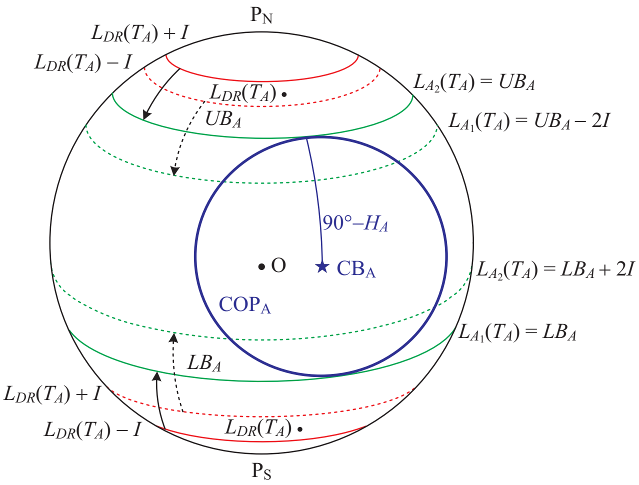


Fig. 4. Illustrations of locating the assumed latitudes by using the adaptive boundaries technique for CB_A.

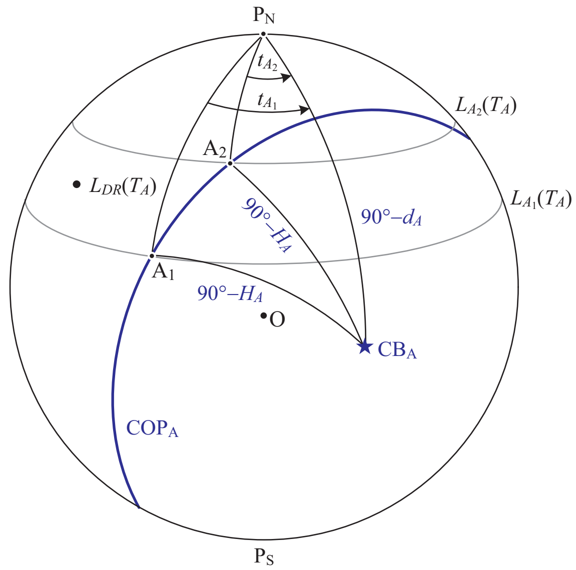


Fig. 5. The celestial spherical triangle for determining the longitudes of Sumner points.

of the spherical triangle are together greater than the third (Clough-Smith, 1966), we have

$$d - (90° - H) \le L_{DR} \le d + (90° - H), \tag{1}$$

in which the left-hand side of L_{DR} is the lower bound (LB) and the right hand side of L_{DR} is the upper bound (UB). Thus, adaptive boundaries can be located to limit the interval of assumed latitudes, where the LB and UB are decided by the declination and the observed altitude of a celestial body.

As shown in Fig. 4, if any one of the initial bounds is lower than the LB, we set the lower assumed latitude as the LB; while if any one of the initial bounds is higher than the UB, we set the higher assumed latitude as the UB. Consequently, the lower bound (LB_A), upper bound (UB_A), and two assumed latitudes for the celestial body A (CB_A) at observed time T_A, $L_{A_1}(T_A)$ and $L_{A_2}(T_A)$, are respectively expressed as

$$LB_A = d_A - (90° - H_A), \tag{2a}$$

$$UB_A = d_A + (90° - H_A), \tag{2b}$$

and

$$\left[L_{A_1}(T_A), L_{A_2}(T_A)\right] = \begin{cases} \left[L_{DR}(T_A) - I,\ L_{DR}(T_A) + I\right], \\ \text{if } LB_A \le L_{DR}(T_A) \mp I \le UB_A \\ \left[LB_A,\ LB_A + 2I\right], \\ \text{if } L_{DR}(T_A) - I < LB_A \\ \left[UB_A - 2I,\ UB_A\right], \\ \text{if } L_{DR}(T_A) + I > UB_A \end{cases}, \tag{2c}$$

where d_A is the declination; H_A is the observed altitude; $L_{DR}(T_A)$ is the latitude of the DR position at T_A; I is the increment of the assumed latitude and $I = 10'$ for the initial step.

As shown in Fig. 5, once the assumed latitudes ($L_{A_1}(T_A)$ and $L_{A_2}(T_A)$) are available, the meridian angles (t_{A_1} and t_{A_2}) and longitudes ($\lambda_{A_1}(T_A)$ and $\lambda_{A_2}(T_A)$) of Sumner points A_1 and A_2 at observed time T_A can be obtained by

$$\cos(t_{A_1}) = \frac{\sin H_A - \sin[L_{A_1}(T_A)]\sin d_A}{\cos[L_{A_1}(T_A)]\cos d_A}, \tag{3a}$$

$$\cos(t_{A_2}) = \frac{\sin H_A - \sin[L_{A_2}(T_A)]\sin d_A}{\cos[L_{A_2}(T_A)]\cos d_A}, \tag{3b}$$

$$\lambda_{A_1}(T_A) = G_A \mp (t_{A_1}), \tag{3c}$$

and

$$\lambda_{A_2}(T_A) = G_A \mp (t_{A_2}), \tag{3d}$$

where G_A is the Greenwich hour angle (GHA) of CB_A.

Similarly, the lower bound (LB_B), upper bound (UB_B), and two assumed latitudes for the celestial body B (CB_B) at observed time T_B, $L_{B_1}(T_B)$ and $L_{B_2}(T_B)$, can respectively be written as

$$LB_B = d_B - (90° - H_B), \tag{4a}$$

$$UB_B = d_B + (90° - H_B), \tag{4b}$$

and

$$\left[L_{B_1}(T_B),\ L_{B_2}(T_B)\right] = \begin{cases} \left[L_{DR}(T_B) - I,\ L_{DR}(T_B) + I\right], \\ \quad if\ LB_B \le L_{DR}(T_B) \mp I \le UB_B \\ \left[LB_B,\ LB_B + 2I\right], \\ \quad if\ L_{DR}(T_B) - I < LB_B \\ \left[UB_B - 2I,\ UB_B\right], \\ \quad if\ L_{DR}(T_B) + I > UB_B \end{cases}, \tag{4c}$$

where d_B is the declination; H_B is the observed altitude and $L_{DR}(T_B)$ is the latitude of the DR position at T_B.

Then, once the assumed latitudes ($L_{B_1}(T_B)$ and $L_{B_2}(T_B)$) are available, the meridian angles (t_{B_1} and t_{B_2}) and longitudes ($\lambda_{B_1}(T_B)$ and $\lambda_{B_2}(T_B)$) of Sumner points B_1 and B_2 at observed time T_B can be calculated by

$$\cos(t_{B_1}) = \frac{\sin H_B - \sin[L_{B_1}(T_B)]\sin d_B}{\cos[L_{B_1}(T_B)]\cos d_B}, \tag{5a}$$

$$\cos(t_{B_2}) = \frac{\sin H_B - \sin[L_{B_2}(T_B)]\sin d_B}{\cos[L_{B_2}(T_B)]\cos d_B}, \tag{5b}$$

$$\lambda_{B_1}(T_B) = G_B \mp (t_{B_1}), \tag{5c}$$

and

$$\lambda_{B_2}(T_B) = G_B \mp (t_{B_2}), \tag{5d}$$

where G_B is the Greenwich hour angle (GHA) of CB_B.

Note that north latitudes and east longitudes are positive, while south latitudes and west longitudes are negative. As for the minus-or-plus option in Eqs. (3c), (3d), (5c) and (5d), − is used for bodies east of the meridian and + is used for bodies west of the meridian.

2. Astronomical Vessel Position

By using the equation of line joining two points, simultaneous Sumner line equations at fix time T_F for CB_A and CB_B can be established as

$$(d\lambda_A)L_F - (dL_A)\lambda_F = (L_{A_1})(d\lambda_A) - (dL_A)(\lambda_{A_1}), \tag{6a}$$

and

$$(d\lambda_B)L_F - (dL_B)\lambda_F = (L_{B_1})(d\lambda_B) - (dL_B)(\lambda_{B_1}), \tag{6b}$$

where L_F and λ_F are the latitude and longitude of AVP at the fix time; dL_A and $d\lambda_A$ are the difference of latitude and longitude between Sumner points A_1 and A_2, respectively; L_{A_1} and λ_{A_1} are the latitude and longitude of Sumner point A_1, respectively; dL_B and $d\lambda_B$ are the difference of latitude and longitude between Sumner points B_1 and B_2, respectively; and L_{B_1} and λ_{B_1} are the latitude and longitude of Sumner point B_1, respectively.

The AVP (L_F, λ_F) can be obtained by Cramer's Rule as

$$\begin{aligned} L_F = &\frac{(dL_A)[(L_{B_1})(d\lambda_B) - (dL_B)(\lambda_{B_1})]}{(dL_A)(d\lambda_B) - (dL_B)(d\lambda_A)} \\ &- \frac{(dL_B)[(L_{A_1})(d\lambda_A) - (dL_A)(\lambda_{A_1})]}{(dL_A)(d\lambda_B) - (dL_B)(d\lambda_A)}, \end{aligned} \tag{7a}$$

and

$$\begin{aligned} \lambda_F = &\frac{(d\lambda_A)[(L_{B_1})(d\lambda_B) - (dL_B)(\lambda_{B_1})]}{(dL_A)(d\lambda_B) - (dL_B)(d\lambda_A)} \\ &- \frac{(d\lambda_B)[(L_{A_1})(d\lambda_A) - (dL_A)(\lambda_{A_1})]}{(dL_A)(d\lambda_B) - (dL_B)(d\lambda_A)}. \end{aligned} \tag{7b}$$

3. Iteration Method

Since the Sumner method is a trial-and-error method, an iteration method is introduced to eliminate the approximation error for obtaining the real AVP. Consequently, by decreasing the increment of assumed latitudes (I), we have

$$I = 10' \times 2^{-n}, \tag{8}$$

where n is the number of iteration.

4. Process of Running Fix

When the position (L_d, λ_d) at departure time, the speed (S) and the course angle (C) of the vessel are known, the position (L_a, λ_a) at arrival time can be obtained by using middle-latitude sailing (Bowditch, 1984, 2002; Culter, 2003). Therefore, we introduce the equations for middle-latitude sailing:

$$dL = (S \times dT)\cos C, \tag{9a}$$

$$L_m = L_d + \frac{1}{2}dL, \tag{9b}$$

and

$$d\lambda = (S \times dT)\sin C \sec L_m, \tag{9c}$$

where dL is the difference of latitude, L_m is the middle-latitude (also called the mean latitude) and $d\lambda$ is the difference in longitude between positions at departure time and arrival time, respectively; dT is the time interval (in hours) between the departure time and arrival time. Thus, the position (L_a, λ_a) at

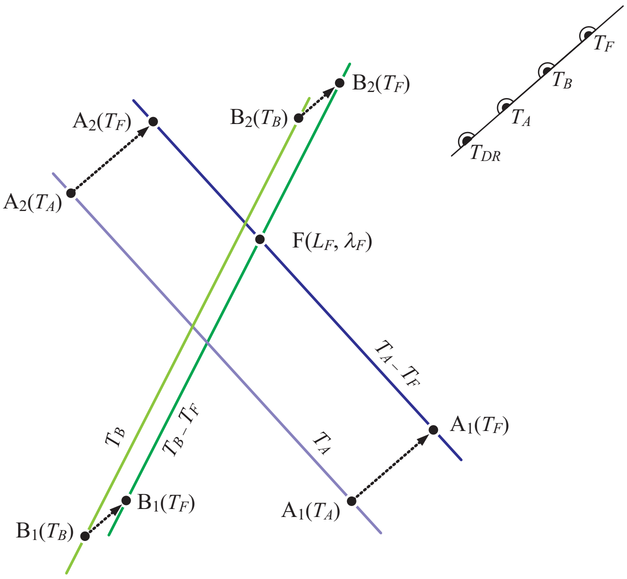


Fig. 6. Illustrations of translating the Sumner points.

arrival time can be yielded by

$$L_a = L_d + dL, \tag{10a}$$

and

$$\lambda_a = \lambda_d + d\lambda. \tag{10b}$$

III. COMPUTATION PROCEDURES AND NUMERICAL PROGRAM

1. Construction of the Computational Procedures for MSM

Step 1. Calculate DR Positions at Observed Times T_A and T_B

The DR positions at T_A and T_B with known DR position at time T_{DR} can be calculated by using Eqs (9a), (9b), (9c), (10a) and (10b), as shown in Fig. 6.

Step 2. Determine the Two Sumner Points at T_A and T_B

(1) At T_A: The lower bound and upper bound for CB_A are located by using Eqs. (2a) and (2b). Then, the assumed latitudes of the Sumner points can be determined by Eqs. (2c) and the longitudes of Sumner points A_1 and A_2 at T_A can be obtained by Eqs. (3a), (3b), (3c) and (3d).

(2) At T_B: The lower bound and upper bound for CB_B are located by using Eqs. (4a) and (4b). Then, the assumed latitudes of the Sumner points can be determined by Eqs. (4c) and the longitudes of the Sumner points B_1 and B_2 at T_B can be obtained by Eqs. (5a), (5b), (5c) and (5d).

Step 3. Determine the Two Sumner Points at Fix Time T_F

(1) At T_F for CB_A: Once the Sumner points at T_A are yielded, the two Sumner points at T_F can be determined by using Eqs. (9a), (9b), (9c), (10a) and (10b), as shown in Fig. 6.

(2) At T_F for CB_B: Similarly, once the Sumner points at T_B are yielded, the two Sumner points at T_F can be determined by using Eqs. (9a), (9b), (9c), (10a) and (10b), as shown in Fig. 6.

Step 4. Determine the AVP at Fix Time T_F

The AVP (L_F, λ_F) at T_F can be obtained by using Eqs. (7a) and (7b).

Step 5. Determine the Real AVP

The real AVP is yielded by using the iteration method. Consequently, decreasing the increment of assumed latitudes (I) of Eq. (8) and repeating the iteration steps 1 to 5 can reach the real AVP while two successive positions obtained by Step 4 does not change to the precision required.

2. Numerical Program

Based on the proposed MSM, we developed a numerical program with graphical user interface (GUI) using Visual Basic.Net 2012, which we named the AVP-MSM Prog. For the convenience of navigators, the AVP-MSM Prog calculates the real AVP and draws the Sumner lines on the built-in small area plotting sheet.

IV. VALIDATION AND DISCUSSION

Three examples are provided to validate the proposed MSM. In Example 1, we illustrate the application of ABT. Example 2 illustrates application of the running fix concept for dealing with the non-simultaneous sights at high altitude. Example 3 describes how the MSM and the running fix concept are used in combination to solve the problem of the overdetermined AVP.

Example 1

On May 16, 1995 the ZT 20-11-26 the DR position of a vessel is L25°10.0′N, λ157°10.0′W. At 20-07-43, the ***Kochab*** *is spotted with a sextant. Later, at 20-11-26 the star* ***Spica*** *is observed. The navigator records the needed information and further reduces it from the Nautical Almanac for sight reduction as shown in Table 1.* (Reediting from pp. 301-303 of Bowditch (2002)).

Table 1. Needed information for solving the AVP in Example 1.

Celestial body	Kochab	Spica
DR	1995/05/16, ZT 20-11-26,	25°10.0′ N 157°10.0′ W
ZT	20-07-43	20-11-26
H	47°13.6′	32°28.7′
d	74°10.6′ N	11°08.4′ S
G	103°43.0′	126°05.7′

Source: Reediting from pp. 301-303 of Bowditch (2002)

Table 2. Solving procedures in details by the MSM for Example 1.

Step	CB_A	Equations	Kochab	Spica	Equations	CB_B
1	$DR(T_A)$	(9a)-(9c) (10a), (10b)	25°10.0′ N 157°10.0′ W	25°10.0′ N 157°10.0′ W	(9a)-(9c) (10a), (10b)	$DR(T_B)$
2	LB_A	(2a)	31°24.2′ N	68°39.7′ S	(4a)	LB_B
	UB_A	(2b)	63°03.0′ N	46°22.9′ N	(4b)	UB_B
lower than LB_A	$L_{DR}(T_A)-I$	(2c)	25°00.0′ N	25°00.0′ N	(4c)	$L_{DR}(T_B)-I$
	$L_{DR}(T_A)+I$	(2c)	25°20.0′ N	25°20.0′ N	(4c)	$L_{DR}(T_B)+I$
	$L_{A_1}(T_A)$	(2c)	31°24.2′ N	25°00.0′ N	(4c)	$L_{B_1}(T_B)$
	$L_{A_2}(T_A)$	(2c)	31°44.2′ N	25°20.0′ N	(4c)	$L_{B_2}(T_B)$
	t_{A_1}	(3a)	000°00.0′	045°55.0′ E	(5a)	t_{B_1}
	t_{A_2}	(3b)	010°34.5′ E	045°40.4′ E	(5b)	t_{B_2}
	$A_1(T_A)$	(3c)	31°24.2′ N 103°43.0′ W	25°00.0′ N 172°00.7′ W	(5c)	$B_1(T_B)$
	$A_2(T_A)$	(3d)	31°44.2′ N 114°17.5′ W	25°20.0′ N 171°46.1′ W	(5d)	$B_2(T_B)$
3	$A_1(T_F)$	(9a)-(9c) (10a), (10b)	31°24.2′ N 103°43.0′ W	25°00.0′ N 172°00.7′ W	(9a)-(9c) (10a), (10b)	$B_1(T_F)$
	$A_2(T_F)$	(9a)-(9c) (10a), (10b)	31°44.2′ N 114°17.5′ W	25°20.0′ N 171°46.1′ W	(9a)-(9c) (10a), (10b)	$B_2(T_F)$
4	F(0)	(7a), (7b)	(33°21.8′ N, 165°53.6′ W)		(7a), (7b)	F(0)
5	F(1)	(8), (7a), (7b)	(37°46.7′ N, 159°17.6′ W)		(8), (7a), (7b)	F(1)
	F(2)	(8), (7a), (7b)	(38°59.5′ N, 156°29.0′ W)		(8), (7a), (7b)	F(2)
	F(3)	(8), (7a), (7b)	(39°00.0′ N, 156°21.7′ W)		(8), (7a), (7b)	F(3)
	F(4)	(8), (7a), (7b)	(39°00.0′ N, 156°21.7′ W)		(8), (7a), (7b)	F(4)
Fix			(39°00.0′ N, 156°21.7′ W)			

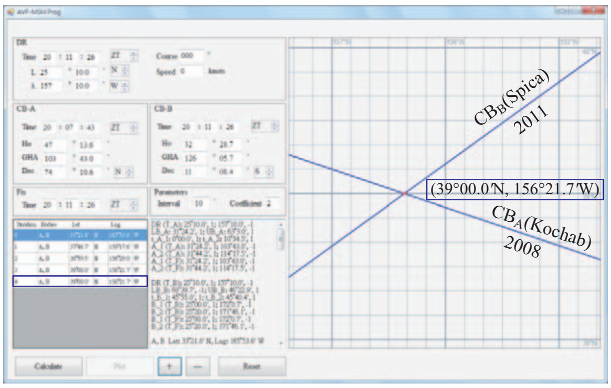


Fig. 7. Results of running the AVP-MSM Prog in Example 1.

Required

Determine the AVP at ZT 20-11-26.

Solution

The AVP (39°00.0′N, 156°21.7′W) is determined by using the MSM and the AVP-MSM Prog, respectively. Results and detailed information are shown in Table 2 and Fig. 7.

Remark

(1) We purposely moved the DR position from (L39°00.0′ N, λ157°10.0′ W) to (L25°10.0′ N, λ157°10.0′ W) for validation of the ABT. As shown in Table 2, the initial bounds ($L_{DR}(T_A) \mp I$) of the star Kochab, (25°00.0′ N, 25°20.0′ N), are lower than the lower bound (LB_A), 31°24.2′ N, and the Sumner method fails to determine the AVP. Consequently, the proposed ABT enforces the assumed latitudes ($L_{A_1}(T_A)$, $L_{A_2}(T_A)$) of (31°24.2′ N, 31°44.2′ N) to fall within the adaptive boundaries (LB_A, UB_A) of (31°24.2′ N, 63°03.0′ N) in order to establish the Sumner line. Then, the AVP can be determined. This shows that whether the Sumner line can be established depends on the ABT but not on the DR position.

(2) Once the two Sumner lines are established, as shown in Table 2, the real AVP (39°00.0′ N, 156°21.7′ W) is accurately obtained when the iteration method is introduced.

Table 3. Needed information for solving the AVP in Example 2.

Celestial body	Sun	Sun
DR	1975/05/31, ZT 12-24-00,	$\begin{cases} 20°17.4' \text{ N} \\ 050°07.4' \text{ W} \end{cases}$
ZT	12-15-15	12-24-13
H	88°09.2′	87°42.8′
d	21°53.1′ N	21°53.1′ N
G	049°25.6′	051°40.1′

Source: p. 569 of Bowditch (1984).

Table 4. Solving procedures in details by the MSM for Example 2.

Step	CB_A	Equations	Sun	Sun	Equations	CB_B	
1	$DR(T_A)$	(9a)-(9c) (10a), (10b)	$\begin{cases} 20°19.0' \text{ N} \\ 050°09.6' \text{ W} \end{cases}$	$\begin{cases} 20°17.4' \text{ N} \\ 050°07.3' \text{ W} \end{cases}$	(9a)-(9c) (10a), (10b)	$DR(T_B)$	
2	LB_A	(2a)	20°02.3′ N	19°35.9′ N	(4a)	LB_B	
	UB_A	(2b)	23°43.9′ N	24°10.3′ N	(4b)	UB_B	
	$L_{DR}(T_A)-I$	(2c)	20°09.0′ N	20°07.4′ N	(4c)	$L_{DR}(T_B)-I$	
	$L_{DR}(T_A)+I$	(2c)	20°29.0′ N	20°27.4′ N	(4c)	$L_{DR}(T_B)+I$	
	$L_{A_1}(T_A)$	(2c)	20°09.0′ N	20°07.4′ N	(4c)	$L_{B_1}(T_B)$	
	$L_{A_2}(T_A)$	(2c)	20°29.0′ N	20°27.4′ N	(4c)	$L_{B_2}(T_B)$	
	t_{A_1}	(3a)	000°40.6′ E	001°33.7′ W	(5a)	t_{B_1}	
	t_{A_2}	(3b)	001°17.3′ E	001°54.9′ W	(5b)	t_{B_2}	
1215	$A_1(T_A)$	(3c)	$\begin{cases} 20°09.0' \text{ N} \\ 050°06.2' \text{ W} \end{cases}$	$\begin{cases} 20°07.4' \text{ N} \\ 050°06.4' \text{ W} \end{cases}$	(5c)	$B_1(T_B)$	1224
	$A_2(T_A)$	(3d)	$\begin{cases} 20°29.0' \text{ N} \\ 050°42.9' \text{ W} \end{cases}$	$\begin{cases} 20°27.4' \text{ N} \\ 049°45.2' \text{ W} \end{cases}$	(5d)	$B_2(T_B)$	
1224 3	$A_1(T_F)$	(9a)-(9c) (10a), (10b)	$\begin{cases} 20°07.4' \text{ N} \\ 050°04.0' \text{ W} \end{cases}$	$\begin{cases} 20°07.4' \text{ N} \\ 050°06.5' \text{ W} \end{cases}$	(9a)-(9c) (10a), (10b)	$B_1(T_F)$	1224
	$A_2(T_F)$	(9a)-(9c) (10a), (10b)	$\begin{cases} 20°27.4' \text{ N} \\ 050°40.7' \text{ W} \end{cases}$	$\begin{cases} 20°27.4' \text{ N} \\ 049°45.3' \text{ W} \end{cases}$	(9a)-(9c) (10a), (10b)	$B_2(T_F)$	
4	F(0)	(7a), (7b)	(20°08.3′ N, 050°05.6′ W)		(7a), (7b)	F(0)	
5	F(1)	(8), (7a), (7b)	(20°08.8′ N, 050°05.1′ W)		(8), (7a), (7b)	F(1)	
	F(2)	(8), (7a), (7b)	(20°08.2′ N, 050°05.6′ W)		(8), (7a), (7b)	F(2)	
	F(3)	(8), (7a), (7b)	(20°08.1′ N, 050°05.7′ W)		(8), (7a), (7b)	F(3)	
	F(4)	(8), (7a), (7b)	(20°08.0′ N, 050°05.7′ W)		(8), (7a), (7b)	F(4)	
	F(5)	(8), (7a), (7b)	(20°08.0′ N, 050°05.7′ W)		(8), (7a), (7b)	F(5)	
Fix			(20°08.0′ N, 050°05.7′ W)				

Also, as shown in Fig. 7, the same results are reached by using the AVP-MSM Prog and validated in (Chen et al., 2003; Chen et al., 2014).

Example 2

On May 31, 1975, the ZT 12-24-00 the DR position of a vessel is L20°17.4′ N, λ050°07.4′ W. The ship is on course 127°, speed 18 knots. The navigator observes the lower limb of the ***Sun*** *twice. The first observation is made at 12-15-15. The second observation is made at 12-24-13. The navigator records the needed information and further reduces it from the Nautical Almanac for sight reduction as shown in Table 3.* (p. 569 of Bowditch (1984))

Required

Determine the AVP at ZT 12-24-00.

Table 5. Needed information for solving the AVP in Example 3.

Celestial body	Deneb	Antares	Vega
DR		1973/02/25, ZT 06-16-00, $\begin{cases} 45°10.0' \text{ N} \\ 030°15.0' \text{ W} \end{cases}$	
ZT	06-09-04	06-12-05	06-16-02
H	46°35.6′	18°46.9′	66°48.7′
d	45°10.9′ N	26°22.5′ S	38°45.2′ N
G	327°11.5′	031°07.6′	000°03.4′

Source: p.29 of NIMA (1981).

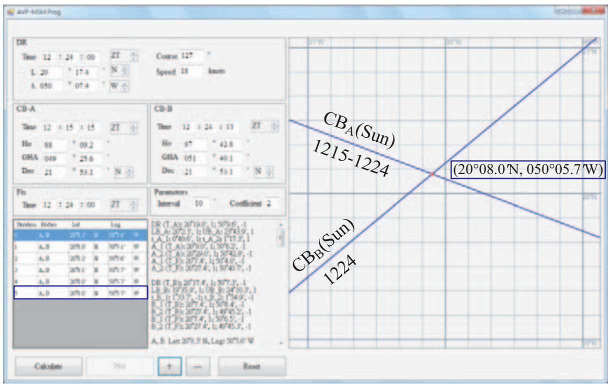


Fig. 8. Results of running the AVP-MSM Prog in Example 2.

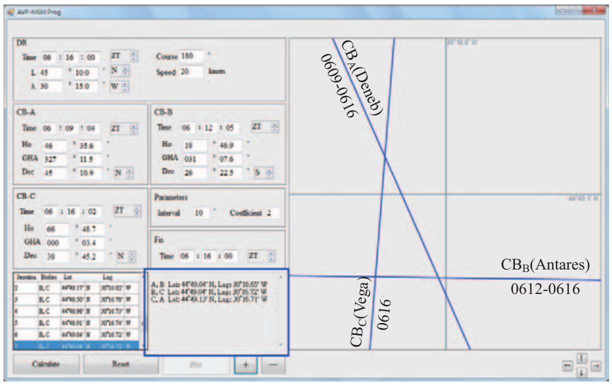


Fig. 9. Results of running the AVP-MSM Prog in Example 3.

Solution

The AVP (20°08.0′ N, 050°05.7′ W) is determined by using the MSM and the AVP-MSM Prog, respectively. Results and detailed information are shown in Table 4 and Fig. 8.

Remarks

(1) As shown in Table 4, two Sumner points, ($A_1(T_A)$, $A_2(T_A)$), obtained at T_A and another two Sumner points, ($B_1(T_B)$, $B_2(T_B)$), obtained at T_B, are translated to $A_1(T_F)$, $A_2(T_F)$, $B_1(T_F)$ and $B_2(T_F)$ at T_F by using middle-latitude sailing. Thus, by way of the four translated Sumner points, a set of two Sumner lines can be established to determine the AVP (L_F, λ_F).

(2) After iteration, the real AVP, (20°08.0′ N, 050°05.7′ W), is reached and validated in (Chen et al., 2014). Note that this example is a case of bodies at high altitudes; it shows that the proposed approach is quite accurate and widely applicable.

Example 3

On February 25, 1973, the ZT 06-16-00 the DR position of a ship is L45°10.0′ N, λ030°15.0′ W. The navigator observes Deneb, Antares, and Vega. The ship is on course 180° and speed 20 knots. Detailed information is recorded as shown in Table 5. (p.29 of NIMA (1981)).

Required

Determine the AVP at ZT 06-16-00.

Solution

The cocked hat is formed by three Sumner lines when the AVP-MSM Prog is run. Three points of the cocked hat are (44°49.04′ N, 030°16.65′ W), (44°49.04′ N, 030°16.72′ W) and (44°49.13′ N, 030°16.71′ W). Results and detailed information are shown in Fig. 9.

Remarks

(1) As shown in Fig. 9, since star Antares is observed near the time of meridian passage, the determined Sumner line is very close to the parallel of latitude in which the Sumner method cannot be used. However, once the ABT of the MSM is introduced, as shown in Table 6, the proposed ABT enforces the assume latitudes ($L_{A_1}(T_A)$, $L_{A_2}(T_A)$) of (44°30.6′ N, 44°50.6′ N) to fall within the adaptive boundaries (LB_A, UB_A) of (82°24.4′ S, 44°50.6′ N) in order to establish the Sumner line. Consequently, the proposed MSM effectively eliminates the shortcomings of the Sumner method in this example.

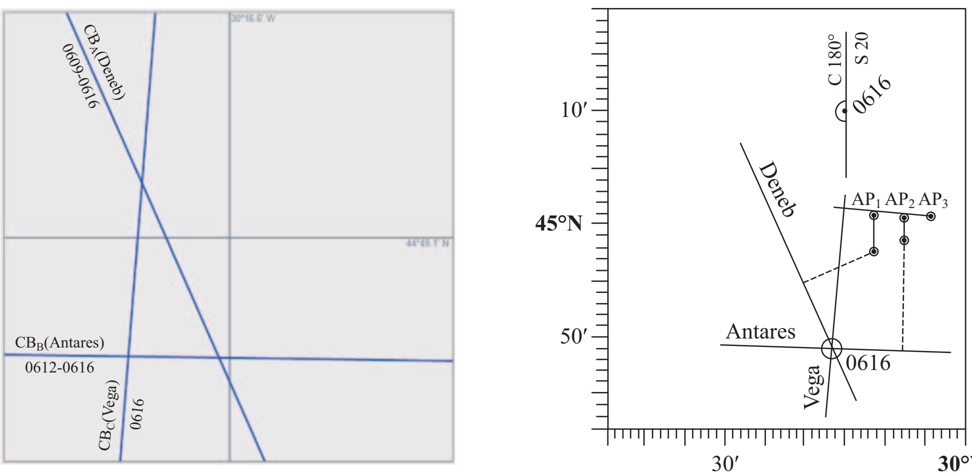
(2) Conventionally, the AVP is determined by the plotting method (NIMA, 1981) and thus requires tedious work, easily leading to errors; however, as shown in Fig. 10, the proposed MSM, using the developed AVP-MSM Prog can quickly and accurately calculate the AVP.

V. CONCLUSIONS

Based on the geometrical properties of celestial triangles,

Table 6. Solving procedures in details by the MSM for Example 3.

Step	CB_A	Equations	Antares	Vega	Equations	CB_B
1	$DR(T_A)$	(9a)-(9c) (10a), (10b)	45°11.3′ N 030°15.0′ W	45°10.0′ N 030°15.0′ W	(9a)-(9c) (10a), (10b)	$DR(T_B)$
2	LB_A	(2a)	82°24.4′ S	15°33.9′ N	(4a)	LB_B
	UB_A	(2b)	44°50.6′ N	61°56.5′ N	(4b)	UB_B
higher than UB_A	$L_{DR}(T_A)-I$	(2c)	45°01.3′ N	45°00.0′ N	(4c)	$L_{DR}(T_B)-I$
	$L_{DR}(T_A)+I$	(2c)	45°21.3′ N	45°20.0′ N	(4c)	$L_{DR}(T_B)+I$
	$L_{A_1}(T_A)$	(2c)	44°30.6′ N	45°00.0′ N	(4c)	$L_{B_1}(T_B)$
	$L_{A_2}(T_A)$	(2c)	44°50.6′ N	45°20.0′ N	(4c)	$L_{B_2}(T_B)$
	t_{A_1}	(3a)	007°31.5′ W	030°12.0′ E	(5a)	t_{B_1}
	t_{A_2}	(3b)	000°00.0′	030°09.4′ E	(5b)	t_{B_2}
0612	$A_1(T_A)$	(3c)	44°30.6′ N 023°36.1′ W	45°00.0′ N 030°15.4′ W	(5c)	$B_1(T_B)$ 0616
	$A_2(T_A)$	(3d)	44°50.6′ N 031°07.6′ W	45°20.0′ N 030°12.8′ W	(5d)	$B_2(T_B)$
3 0616	$A_1(T_F)$	(9a)-(9c) (10a), (10b)	44°29.3′ N 023°36.1′ W	45°00.0′ N 030°15.4′ W	(9a)-(9c) (10a), (10b)	$B_1(T_F)$ 0616
	$A_2(T_F)$	(9a)-(9c) (10a), (10b)	44°49.3′ N 031°07.6′ W	45°20.0′ N 030°12.8′ W	(9a)-(9c) (10a), (10b)	$B_2(T_F)$
4	F(0)	(7a), (7b)	(44°47.06′ N, 030°17.16′ W)		(7a), (7b)	F(0)
5	F(7)	(8), (7a), (7b)	(44°49.04′ N, 030°16.72′ W)		(8), (7a), (7b)	F(7)
Fix			(44°49.04′ N, 030°16.72′ W)			



(a) The AVPs determined by AVP-MSM Prog (b) The AVP determined by plotting method (NIMA, 1981)

Fig. 10. Comparison of the AVPs determined by AVP-MSM Prog and in the literature.

the ABT has been successfully derived. The Sumner method together with the ABT and the iteration method (MSM) effectively overcome the disadvantages of the Sumner method. Importantly, it is found that whether the Sumner line is successfully established is dependent on the ABT but not on the DR position. Due to the inclusion of the iteration method, the proposed approach is more accurate. Furthermore, when the non-simultaneous sights occur, middle-latitude sailing and the running fix are adopted, extending the application of the proposed approach. The three examples validate the applicability of the proposed method.

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