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# AN IMPROVED FUZZY TIME SERIES THEORY WITH APPLICATIONS IN THE SHANGHAI CONTAINERIZED FREIGHT INDEX

Ming-Tao Chou

Key words: fuzzy, Shanghai Containerized Freight Index, significance level, fuzzy time series.

## ABSTRACT

This study presents fuzzy time series based on the concept of long-term predictive significance level. Fuzzy time series theory and structural analysis are used to develop a long-term predictive significance level for evaluating the suitability of historical data. New triangular fuzzy numbers by  $S$  are subsequently obtained using the graded mean integration representation method. Finally,  $\Delta S$  can strengthen fuzzy time series data for a series and yield additional information. The Shanghai Containerized Freight Index is used to illustrate the forecasting process. The results indicate that the proposed definition can generate forecast levels that provide more information for analysis.

## I. INTRODUCTION

Since its creation in 1993 by Song and Chissom (1993a, 1994), fuzzy time series has achieved considerable success in both theory development (Song and Chissom, 1993a, 1993b, 1994, 1997; Chen, 1996; Liaw, 1997; Chen, 2002; Lee and Chou, 2004; Chou and Lee, 2006; Liang et al., 2006; Chou, 2008, 2009, 2011, 2013; Chou and Chou, 2013) and practical applications, i.e., education economics (Song and Chissom, 1993a, 1993b, 1994, 1997; Chen, 1996; Liaw, 1997; Chen, 2002; Lee and Chou, 2004), business economics (Chou, 2009; Chou and Chou, 2013), monetary economics (Teoh, 2008; Lai, 2009), etc. Each analysis mode has its own advantages and disadvantages and a different procedure or analysis process.

Currently, fuzzy time series is increasingly used to make long-term predictions, of which long-range forecasting methods (Chou, 2001), are a notable example. Fuzzy time series is no longer limited to short-term predictions but can be used to view and quantify long-term forecasting. The interval settings followed

by Chou in the traditional view of triangular fuzzy numbers and the fuzzy techniques developed by Chen and Hsieh (2000) to augment the original interval settings can yield accurate predicted values in the long term. These values can be used to determine long-term future standards and column numbers. The binding concept (Chou, 2001) uses fuzzy intervals to assess the range of an average predicted value, and the interactive use of these two methods can determine the scope of its predicted value. With the new paradigm and definition defined in this paper, we calculate and interpret predictions of the Shanghai Containerized Freight Index (SCFI) (Shanghai Shipping Exchange, 2015).

The remainder of this paper is organized as follows. Section 2 presents the definition and procedure of fuzzy time series, and Section 3 defines the long-term predictive significance level. A numerical example of SCFI is shown in Section 4, and concluding remarks are mentioned in conclusion.

## II. DEFINITION OF FUZZY TIME SERIES

Fuzzy sets, introduced by Zadeh (1965), has various applications, such as in fuzzy sets, fuzzy decision analysis, fuzzy regression, and fuzzy time series (Song and Chissom, 1993a, 1993b, 1994, 1997; Chen, 1996, 2002; Liaw, 1997; Lee and Chou, 2004; Chou and Lee, 2006; Liang et al., 2006; Chou, 2008, 2009, 2011, 2013; Duru and Yoshida, 2012; Chou and Chou, 2013; Chou, 2016). The theory is also widely applied in social science study and applications. Fuzzy time series is developed rapidly since their introduction by Song and Chissom (Song and Chissom, 1993a, 1994). Recent fuzzy time series methods have benefited from both theoretical developments as well as relevant applications in research (Song and Chissom, 1993a, 1993b, 1994, 1997; Chen, 1996; Liaw, 1997; Chen, 2002; Lee and Chou 2004; Chou and Lee, 2006; Liang et al., 2006; Chou, 2008, 2011; Duru and Yoshida, 2012; Chou, 2016), which has led to more diverse uses. This trend indicates that the development of fuzzy time series has markedly improved. The definitions and analytical procedures of the fuzzy time series used in this study are described as follows.

**Definition 1 (Song and Chissom, 1993a, 1994; Liaw, 1997):** A fuzzy number on the real line  $\mathfrak{R}$  is a fuzzy subset of  $\mathfrak{R}$  that is normal and convex.

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**Definition 2 (Song and Chissom, 1993a, 1994):** Let  $Y(t)$  ( $t = \dots, 0, 1, 2, \dots$ ), a subset of  $\mathfrak{R}$ , be the universe of discourse on which the fuzzy sets  $f_i(t)$  ( $t = 1, 2, \dots$ ) are defined, and let  $F(t)$  be the collection of  $f_i(t)$  ( $t = 1, 2, \dots$ ). Then,  $F(t)$  is called fuzzy time series on  $Y(t)$  ( $t = \dots, 0, 1, 2, \dots$ ).

**Definition 3 (Song and Chissom, 1993a, 1994):** Let  $I$  and  $J$  be the index sets for  $F(t-1)$  and  $F(t)$ , respectively. If for any  $f_j(t) \in F(t)$ , where  $j \in J$ , there then exists  $f_i(t-1) \in F(t-1)$ , where  $i \in I$ , such that there exists a fuzzy relation  $R_{ij}(t, t-1)$  and  $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$ , where ‘ $\circ$ ’ is the max-min composition. Then,  $F(t)$  is said to be caused by only  $F(t-1)$ . Denote this as  $f_i(t-1) \rightarrow f_j(t)$ , or equivalently,  $F(t-1) \rightarrow F(t)$ .

**Definition 4 (Song and Chissom, 1993a, 1994):** If, for any  $f_j(t-1) \in F(t)$ , where  $j \in J$ , there exists  $f_i(t-1) \in F(t-1)$ , where  $i \in I$ , and a fuzzy relation  $R_{ij}(t, t-1)$ , such that  $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$ . Let  $R(t, t-1) \cup_{ij} R_{ij}(t, t-1)$ , where  $\cup$  is the union operator. Then,  $R(t, t-1)$  is called the fuzzy relation between  $F(t)$  and  $F(t-1)$ . Thus, we define this as the following fuzzy relational equation:  $F(t) = F(t-1) \circ R(t, t-1)$ .

**Definition 5 (Song and Chissom, 1993a, 1994):** Suppose that  $R_1 = \cup_{ij} R^1_{ij}(t, t-1)$  and  $R_2(t, t-1) = \cup_{ij} R^2_{ij}(t, t-1)$  are two fuzzy relations between  $F(t)$  and  $F(t-1)$ . If, for any  $f_j(t) \in F(t)$ , where  $j \in J$ , there exists  $f_i(t-1) \in F(t-1)$ , where  $i \in I$ , and fuzzy relations  $R^1_{ij}(t, t-1)$  and  $R^2_{ij}(t, t-1)$  such that  $f_j(t) = f_i(t-1) \circ R^1_{ij}(t, t-1)$  and  $f_j(t) = f_i(t-1) \circ R^2_{ij}(t, t-1)$ , then define  $R_1(t, t-1) = R_2(t, t-1)$ .

**Definition 6 (Song and Chissom, 1993a, 1994):** Suppose that  $F(t)$  is only caused by  $F(t-1)$ ,  $F(t-2)$ , ..., or  $F(t-m)$  ( $m > 0$ ). This relation can be expressed as the following fuzzy relational equation:  $F(t) = F(t-1) \circ R_0(t, t-m)$ , which is called the first-order model of  $F(t)$ .

**Definition 7 (Song and Chissom, 1993a, 1994):** Suppose that  $F(t)$  is simultaneously caused by  $F(t-1)$ ,  $F(t-2)$ , ..., and  $F(t-m)$  ( $m > 0$ ). This relation can be expressed as the following fuzzy relational equation:  $F(t) = (F(t-1) \times F(t-2) \times \dots \times F(t-m)) \circ R_a(t, t-m)$ , which is called the  $m^{\text{th}}$ -order model of  $F(t)$ .

**Definition 8 (Chen, 1996):**  $F(t)$  is fuzzy time series if  $F(t)$  is a fuzzy set. The transition is denoted as  $F(t-1) \rightarrow F(t)$ .

**Definition 9 (Chou, 2009):** Let  $d(t)$  be a set of real numbers:  $d(t) \subseteq R$ . We define an exponential function where

- (1)  $y = \exp d(t) \Leftrightarrow \ln y = d(t)$  and
- (2)  $\exp(\ln d(t)) = d(t)$ ,  $\ln(\exp x) = d(t)$ .

**Definition 10 (Lee and Chou, 2004):** The universe of discourse  $U = [D_L, D_U]$  is defined such that  $D_L = D_{\min} - st_{\alpha}(n)/\sqrt{n}$  and  $D_U = D_{\max} + st_{\alpha}(n)/\sqrt{n}$  when  $n \leq 30$  or  $D_L = D_{\min} - \sigma Z_{\alpha}/\sqrt{n}$  and  $D_U = D_{\max} + \sigma Z_{\alpha}/\sqrt{n}$  when  $n > 30$ , where  $t_{\alpha}(n)$  is the 100(1- $\alpha$ )

percentile of the  $t$  distribution with  $n$  degrees of freedom.  $z_{\alpha}$  is the 100(1- $\alpha$ ) percentile of the standard normal distribution. Briefly, if  $Z$  is an  $N(0, 1)$  distribution, then  $P(Z \geq z_{\alpha}) = \alpha$ .

**Definition 11 (Lee and Chou, 2004):** Assuming that there are  $m$  linguistic values under consideration, let  $A_i$  be the fuzzy number that represents the  $i^{\text{th}}$  linguistic value of the linguistic variable, where  $1 \leq i \leq m$ . The support of  $A_i$  is defined as follows:

$$\begin{cases} D_L + (i-1) \frac{D_U - D_L}{m}, & D_L + \frac{i(D_U - D_L)}{m}, & 1 \leq i \leq m-1 \\ D_L + (i-1) \frac{D_U - D_L}{m}, & D_L + \frac{i(D_U - D_L)}{m}, & i = m. \end{cases}$$

**Definition 12 (Liaw, 1997):** For a test  $H_0$ : nonfuzzy trend against  $H_1$ : fuzzy trend, where the critical region  $C^* = \{C | C_2^k + C_2^{n-k} > C_{\lambda} = C_2^n \times (1 - \lambda)\}$ , the initial value of the significance level  $\alpha$  is 0.2.

**Definition 13 (Chou, 2011):** Let  $d(t)$  be a set of real numbers  $d(t) \subseteq R$ . An upper interval for  $d(t)$  is a number  $b$  such that  $x \leq b$  for all  $x \in d(t)$ . The set  $d(t)$  is said to be an interval higher if  $d(t)$  has an upper interval. A number, max, is the maximum of  $d(t)$  if max is an upper interval for  $d(t)$  and  $\max \in d(t)$ .

**Definition 14 (Chou, 2011):** Let  $d(t) \subseteq R$ . The least upper interval of  $d(t)$  is a number  $\overset{\rightarrow}{\max}$  satisfying:

- (1)  $\overset{\rightarrow}{\max}$  is an upper interval for  $d(t)$  such that  $x \leq \overset{\rightarrow}{\max}$  for all  $x \in d(t)$  and
- (2)  $\overset{\rightarrow}{\max}$  is the least upper interval for  $d(t)$ , that is,  $x \leq \overset{\rightarrow}{\max}$  for all  $x \in d(t) \Rightarrow \overset{\rightarrow}{\max} \leq b$ .

**Definition 15 (Chou, 2011):** Let  $d(t)$  be a set of real numbers  $d(t) \subseteq R$ . A lower interval for  $d(t)$  is a number  $b$  such that  $x \geq b$  for all  $x \in d(t)$ . The set  $d(t)$  is said to be an interval below if  $d(t)$  has a lower interval. A number, min, is the minimum of  $d(t)$  if min is a lower interval for  $d(t)$  and  $\min \in d(t)$ .

**Definition 16 (Chou, 2011):** Let  $d(t) \subseteq R$ . The least lower interval of  $d(t)$  is a number  $\overset{\leftarrow}{\min}$  satisfying:

- (1)  $\overset{\leftarrow}{\min}$  is a lower interval for  $d(t)$  such that  $x \geq \overset{\leftarrow}{\min}$  for all  $x \in d(t)$  and
- (2)  $\overset{\leftarrow}{\min}$  is the least lower interval for  $d(t)$ , that is,  $x \geq \overset{\leftarrow}{\min}$  for all  $x \in d(t) \Rightarrow \overset{\leftarrow}{\min} \leq b$ .

**Definition 17 (Chou, 2011):** The long-term predictive value interval  $(\min^{\leftarrow}, \max^{\rightarrow})$  is called the static long-term predictive value interval.

### III. DEFINITION OF LONG-TERM PREDICTIVE SIGNIFICANCE LEVEL AND PROCEDURE

This section proposes a method to forecast the long-term predictive significance level by using fuzzy time series, extending the method proposed by Chou (2011). Based on Chou’s predictive value interval  $\hat{d}(t)$ , many methods to construct triangular fuzzy numbers are developed using Chen’s technique (1996). These permit the prediction of long-term profit levels and the prediction of phase discrimination values for discriminating future trends.

**Definition 18 (Chen and Hsieh, 2000):** Let  $A_i = (\alpha_i, \beta_i, \gamma_i)$ ,  $i = 1, 2, \dots, n$ , be  $n$  triangular fuzzy numbers. By using the graded mean integration representation (GMIR) method, the GMIR value  $P(A_i)$  of  $A_i$  is  $P(A_i) = (\alpha_i + 4\beta_i + \gamma_i)/6$ .  $P(A_i)$  and  $P(A_j)$  are the GMIR values of the triangular fuzzy numbers  $A_i$  and  $A_j$ , respectively.

**Definition 19** Set up new triangular fuzzy numbers by  $S = (\min^{\leftarrow}, \hat{d}(t), \max^{\rightarrow})$ . After GMIR transformation,  $S$  becomes a real number  $\Delta S$ . This is called the long-term significance level with fuzzy time series. The  $\Delta S$  is a real number satisfying the following:

- (1)  $\Delta S$  is called a long-term significance level up, only if:  $\Delta S > \hat{d}(t)$ ;
- (2)  $\Delta S$  is called a long-term significance level down, only if:  $\Delta S < \hat{d}(t)$ ; and
- (3)  $\Delta S$  is called a long-term significance level stable, only if:  $\Delta S = \hat{d}(t)$ .

The stepwise procedure of the proposed method consists the following steps (Chou, 2016), illustrated as a flowchart in Fig. 1 (Chou, 2008; Chou, 2016).

**Step 1.** Let  $d(t)$  be the data under consideration and let  $F(t)$  be fuzzy time series. Following Definition 11, a difference test is performed to determine whether stability of the information. Recursion is performed until the information is in a stable state, where the critical region is

$$C^* = \left\{ C \mid C_2^k + C_2^{n-k} > C_\lambda = C_2^n \times (1 - \lambda) \right\}.$$

**Step 2.** Determine the universe of discourse  $U = [D_L, D_U]$ .

**Step 3.** Define  $A_i$  by letting its membership function be as follows:

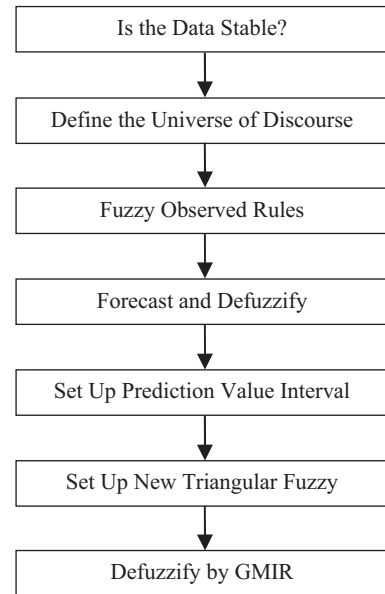


Fig. 1. Procedure of the proposed model.

$$u_{A_i}(x) = \begin{cases} 1 & \text{for } x \in [D_L + (i-1)\frac{D_U - D_L}{m}, D_L + \frac{i(D_U - D_L)}{m}] \\ & \text{where } 1 \leq i \leq m-1; \\ 1 & \text{for } x \in [D_L + (i-1)\frac{D_U - D_L}{m}, D_L + \frac{i(D_U - D_L)}{m}] \\ & \text{where } i = m; \\ 0 & \text{otherwise.} \end{cases}$$

**Step 4.** Then,  $F(t) = A_i$  if  $d(t) \in \text{supp}(A_i)$ , where  $\text{supp}(\cdot)$  denotes the support.

**Step 5.** Derive the transition rule from period  $t - 1$  to  $t$  and denote it as  $F(t-1) \rightarrow F(t)$ . Aggregate all transition rules. Let the set of rules be  $R = \{r_i \mid r_i : P_i \rightarrow Q_i\}$ .

**Step 6.** The value of  $d(t)$  can be predicted using the fuzzy time series  $F(t)$  as follows. Let  $T(t) = \{r_j \mid d(t) \in \text{supp}(P_j), \text{ where } r_j \in R\}$  be the set of rules fired by  $d(t)$ , where  $\text{supp}(P_j)$  is the support of  $P_j$ . Let  $\overline{\text{supp}(P_j)}$  be the median of  $\text{supp}(P_j)$ . The predicted value of  $d(t)$  is

$$\sum_{r_j \in T(t-1)} \frac{\overline{\text{supp}(Q_j)}}{|T(t-1)|}.$$

**Step 7.** The long-term predictive value interval for  $d(t)$  is given as  $(\min^{\leftarrow}, \max^{\rightarrow})$ .

**Step 8.** Set up new triangular fuzzy numbers by  $\Delta S = (\min^{\leftarrow}, \hat{d}(t), \max^{\rightarrow})$ .

**Step 9.** Defuzzify  $S$  to be  $\Delta S$ .

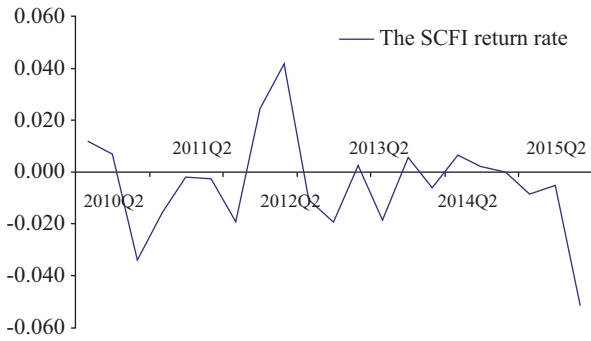


Fig. 2. Rate of return of the SCFI.

IV. NUMERICAL EXAMPLE OF SCFI DATA

In this study, the SCFI is used for a numerical example. The SCFI reflects the spot rates of the Shanghai export container transport market, including both freight rates (indices) of 15 individual shipping routes and a composite index (Shanghai Shipping Exchange, 2015). The SCFI data are sourced from the Shanghai Shipping Exchange (2015), the historical data for which is defined here as the SCFI, and season-averaged data for the period between Quarter 1, 2010, and Quarter 2, 2015, was collected.

Over these 22 data points, the analysis produces an average of 1124.70, with a standard deviation of 187.83, maximum value of 1514.51, and minimum value of 697.65. These descriptive statistics show that the SCFI has largely remained at the 1124.70 level. Compared with figures from Quarter 1, 2010, this value represents the peak level for recent years while China’s domestic demand has remained weak, because of the global financial crisis and European debt crisis. Therefore, the weak domestic demand has allowed China’s economic growth to remain crucial despite these negative influences. We discovered that although China is considerably affected by the SCFI, the consequent adjustments result in a synergetic change in the rate of return of the SCFI. As shown in Fig. 2, the SCFI has recovered from the effect of the financial crisis, although its current rate of return is negative.

The following steps in the procedure are performed when using fuzzy time series to analyze visitor arrivals.

- Step 1.** First, we take the logarithm of the SCFI data to reduce variation and improve the forecast accuracy, letting  $SCFI(\tilde{t}) = \ln SCFI(t)$ .
- Step 2.** Maintaining stationary data while forecasting helps to improve the forecast quality; therefore, we conduct a stationary test on the SCFI data. For fuzzy time series, a fuzzy trend test can measure whether the SCFI’s fuzzy trend moves upward or downward. Using this fuzzy trend test, the SCFI data can be converted into a stationary series. If the original SCFI data exhibited a fuzzy trend, it can be eliminated by taking the difference. We then repeat the test after taking the first difference to

Table 1. Fuzzy historical SCFI data and the forecasted results.

Year	ln(Actual)	Fuzzified	The forecast value
2010Q1	7.184	A <sub>6</sub>	7.194
2010Q2	7.270	A <sub>6</sub>	7.194
2010Q3	7.323	A <sub>7</sub>	7.194
2010Q4	7.074	A <sub>5</sub>	7.001
2011Q1	6.964	A <sub>4</sub>	6.872
2011Q2	6.950	A <sub>4</sub>	6.872
2011Q3	6.934	A <sub>4</sub>	6.872
2011Q4	6.801	A <sub>3</sub>	6.872
2012Q1	6.969	A <sub>4</sub>	6.872
2012Q2	7.268	A <sub>6</sub>	7.194
2012Q3	7.190	A <sub>6</sub>	7.194
2012Q4	7.052	A <sub>5</sub>	7.001
2013Q1	7.073	A <sub>5</sub>	7.001
2013Q2	6.940	A <sub>4</sub>	6.872
2013Q3	6.981	A <sub>4</sub>	6.872
2013Q4	6.935	A <sub>4</sub>	6.872
2014Q1	6.984	A <sub>4</sub>	6.872
2014Q2	6.996	A <sub>4</sub>	6.872
2014Q3	6.994	A <sub>4</sub>	6.872
2014Q4	6.933	A <sub>4</sub>	6.872
2015Q1	6.897	A <sub>4</sub>	6.872
2015Q2	6.548	A <sub>1</sub>	6.743

measure if the SCFI data exhibits a fuzzy trend. If a fuzzy trend is again observed, then we take the second difference, and so on.

Letting  $SCFI(t)$  be the historical data under consideration and fuzzy time series, a difference test is used (following Definition 11) to understand whether the stability of the information. Recursion is performed until the information is determined to be stable. Once the region

$$C^* = \{C | C = C_2^{13} + C_2^{21-13}\} = 106 < \{C | C_2^{21} \times (1-0.2)\} = 168$$

the SCFI data are considered in a stable state and are not rejected.

- Step 3.** According to the interval setting of the SCFI data, we define the upper and lower bounds, which facilitate dividing the linguistic value intervals later. From Definition 10, the discourse  $U = [D_L, D_U]$ . From Table 1,  $D_{min} = 6.548$ ,  $D_{max} = 7.323$ ,  $s = 0.170$ , and  $n = 22$  can be obtained. Letting  $\alpha = 0.55$ , since  $n$  is less than 30, a Student  $t$  distribution with 22 degrees of freedom was used as a substitute for the normal distribution. Thus,  $t_{\alpha}(n) =$

$t_{0.05}(22) = 1.717$ ,  $D_L = D_{\min} - st_{\alpha}/\sqrt{n} \approx 6.485$ , and  $D_U = D_{\max} + st_{\alpha}/\sqrt{n} \approx 7.386$ . That is,  $U = [6.485, 7.386]$ .

**Step 4.** After defining the upper and lower bounds of the SCFI data in Step 3, we can define the SCFI range by determining the membership function as well as the linguistic values. We can also define the range of the subinterval for each linguistic value, assuming that the following linguistic values are under consideration: extremely few, very few, few, some, many, very many, and extremely many. According to Definition 11, the supports of fuzzy numbers that represent these linguistic values are given as follows:

$$u_{A_i}(x) = \begin{cases} 1 & \text{for } x \in [6.485 + (i-1)(0.129), 6.485 + i(0.129)) \\ & \text{where } 1 \leq i \leq m-1; \\ 1 & \text{for } x \in [6.485 + (i-1)(0.129), 6.485 + i(0.129)] \\ & \text{where } i = m; \\ 0 & \text{otherwise.} \end{cases}$$

where  $A_1 =$  “extremely few,”  $A_2 =$  “very few,”  $A_3 =$  “few,”  $A_4 =$  “some,”  $A_5 =$  “many,”  $A_6 =$  “very many,” and  $A_7 =$  “extremely many.” Thus, the supports are  $\text{supp}(A_1) = [6.485, 6.614]$ ,  $\text{supp}(A_2) = [6.614, 6.743]$ ,  $\text{supp}(A_3) = [6.743, 6.872]$ ,  $\text{supp}(A_4) = [6.872, 7.001]$ ,  $\text{supp}(A_5) = [7.001, 7.130]$ ,  $\text{supp}(A_6) = [7.130, 7.259]$ , and  $\text{supp}(A_7) = [7.259, 7.386]$ .

**Step 5.** According to the subinterval setting of each linguistic value, we classified each historical dataset of the SCFI into its corresponding interval to measure the value corresponding to the linguistic value for each interval. The fuzzy time series  $F(t)$  was given by  $F(t) = A_i$  when  $d(t) \in \text{supp}(A_i)$ . Therefore,  $F(2010Q1) = A_6$ ,  $F(2010Q2) = A_6$ ,  $F(2010Q3) = A_7$ ,  $F(2010Q4) = A_5$ , ..., and  $F(2015) = A_1$ . Table 1 shows the comparison between the actual SCFI data and the fuzzy enrollment data.

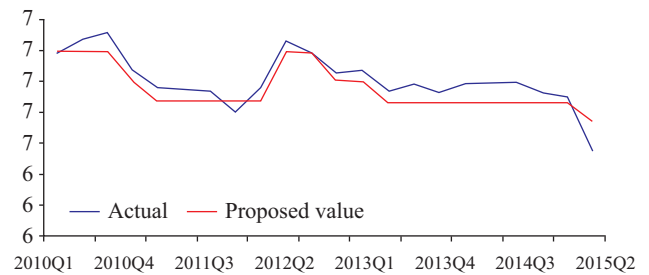
**Step 6.** We apply fuzzy theory to define the corresponding value for the intervals of the SCFI data, arrange the corresponding method for the SCFI data, and integrate the changes from all the rules to determine the rules for the SCFI quantity. The transition rules are derived from Table 1. For example,  $F(2010Q1) \rightarrow F(2010Q2)$  is  $A_6 \rightarrow A_6$ . Table 2 shows all transition rules obtained from Table 1.

**Step 7.** We calculate each rule by determining all the rules of the SCFI, and the calculation results can be used to forecast future values. Table 1 shows the forecasting results from 2010Q1 to 2015Q2.

**Step 8.** The calculated SCFI rules can define the intervals of the SCFI data; using these intervals, we can determine the variation in future long-term intervals. The long-term predictive value interval for the SCFI is given as  $(6.743, 7.194)$ . Thus, the long-term predictive interval

**Table 2. Fuzzy transitions derived from Table 1.**

$r_1 : A_3 \rightarrow A_4$	$r_5 : A_4 \rightarrow A_6$	$r_9 : A_6 \rightarrow A_6$
$r_2 : A_4 \rightarrow A_1$	$r_6 : A_5 \rightarrow A_4$	$r_{10} : A_6 \rightarrow A_7$
$r_3 : A_4 \rightarrow A_3$	$r_7 : A_5 \rightarrow A_5$	$r_{11} : A_7 \rightarrow A_5$
$r_4 : A_4 \rightarrow A_4$	$r_8 : A_6 \rightarrow A_5$	



**Fig. 3. Forecast SCFI and actual SCFI.**

for the SCFI is given as  $(848.005, 1331.198)$ . Therefore, the current long-term SCFI is bounded by this interval. According to Step 8, the fuzzy SCFI of 2015Q2 shown in Table 1 is  $A_1$ , and from Table 2, we can see that the rules are the fuzzy logical relationships in Rule 11 of Table 2, in which the current state of fuzzy logical relationships is  $A_6$ . Thus, the 2015Q3 SCFI predictive value is 848.005.

**Step 9.** Letting defuzzified  $S$  be  $\Delta S$ , the SCFI 2015Q3 forecast value based on our investigation is 848.005, and its trading range is between 848.005 and 1331.198. Thus, the new triangular fuzzy numbers by  $S = (848.005, 1331.198, 1331.198)$ . Thus, the defuzzified  $S$  is  $\Delta S = 928.537$ , and  $\Delta S = 928.537 > \hat{d}(t) = 848.005$ .

The result shows that based on the long-term significance level, the SCFI is currently oversold. This result and the risk-reward ratio are both related within the group. We used Table 1 data in our analysis according to the root mean square percentage error method, with an average prediction error of 0.278%. Fig. 3 shows the forecast visitor arrivals determined through fuzzy time series analysis and the actual SCFI values. Based on the fuzzy time series results, the average SCFI is estimated to be 848.005 in 2015Q3 (Fig. 3).

## V. CONCLUSION

In this paper, a long-term predictive value interval model is developed for forecasting the SCFI. This model facilitates mini-

mizing the uncertainties associated with fuzzy numbers. The method is examined by forecasting the SCFI by using data from which  $\Delta S = 928.537$  and  $\Delta S > \hat{d}(t)$  is obtained. For index returns, the current rate of return is negative and its volatility is increasing. The long-term predictive significance level of the SCFI is at the  $\Delta S$  level; the SCFI should thus exhibit extreme volatility.

The current model for the SCFI 2015Q3 forecast level deviates insignificantly from the actual values for an average of 848.005 and is within the group; the prediction error does not exceed 0.278% of the significance level. By constructing a fuzzy time series forecasting model for the SCFI with an error of less than 0.278%, with the traditional fuzzy time excluded from the single-point forecast comparison, this model provides a long-term predictive significance level.

Furthermore, the proposed method can be computerized. Thus, by improving fuzzy linguistic assessments as well as the evaluation of fuzzy time series, decision makers can automatically obtain the final long-term predictive significance level.

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