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IMPROVED ANALYSIS OF BOLTED SHEAR CONNECTION UNDER ECCENTRIC LOADS

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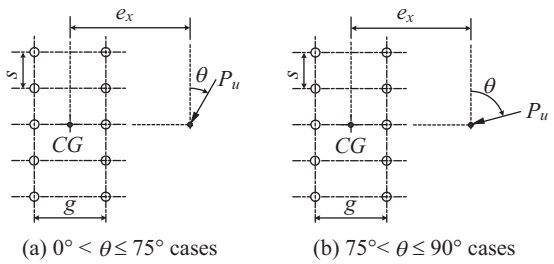
Key words: instantaneous center of rotation, eccentric load, bolted shear connections, ultimate analysis.

ABSTRACT

Almost all bolted connections are eccentrically loaded. The American Institute of Steel Construction (AISC) permits the use of elastic and instantaneous center of rotation (IC) methods to analyze eccentric bolted connections. The elastic method normally yields relatively conservative designs and the IC method, which provides more realistic analyses, is rather complex and tedious. The current AISC manual provides tables for determining coefficient *C*, which is used to obtain the design strength of bolt group patterns. However, the tables provide values for only six angles of inclination ($\theta = 0°$, 15°, 30°, 45°, 60°, and 75°). For other angles, a direct analysis using the IC method must be conducted. The straight-line interpolation between *C* values for loads at different angles may be non-conservative and it is not recommended by the AISC. This work develops an iterative algorithm for implementing the tedious IC method in the general analysis or design of eccentric bolted connections. To eliminate the tediousness of the IC method, a method is proposed to provide a reasonable result for all angles (between 0° and 90°) including those not being considered in the current AISC design tables. The proposed method is easy to implement but reasonably accurate, and replaces both the straight-line interpolation between *C* values for loads at various angles and direct analysis. This work eliminates the current limitations on AISC design. It provides a quick and reliable tool for preliminary design of eccentric bolted connection.

I. INTRODUCTION

Steel structures generally have eccentrically loaded joints.



(a) $0° < \theta \leq 75°$ cases (b) $75° < \theta \leq 90°$ cases

Fig. 1. Loading cases covered and not covered in the AISC manuals.

Some common examples of such joints are bracket-type connections and web splices in beams and girders. For a bolted connection, as shown in Fig. 1(a), both the eccentric load and the induced torsion contribute to bolt shear. The effect of the combined load is equivalent to a rotation of the bolts about a particular point, which is called the "instantaneous center of rotation" (IC). The exact position of IC is important in the analysis and design of eccentrically loaded joints. The location of an IC depends on the pattern of the group of bolts and the location and direction of the loading. Theoretically, the force equilibrium equations are satisfied at the true IC.

The American Institute of Steel Construction (AISC) design manuals (1986, 1989, 1993, 1999, 2005, and 2010) provide two practical methods for evaluating the design strength of an eccentrically bolted connection. The first method is essentially an elastic method and is regarded as conservative; the second is based on the concept of the instantaneous center of rotation (IC), and is a strength-based method that provides more realistic results. In the method of the AISC, the design strength of an eccentrically loaded bolt connection is evaluated using a tabulated coefficient *C*, which is proportional to the required strength of the bolt group. The AISC allowable stress design (ASD) manual (1989) contains the *C* coefficients for only vertical eccentric loads. Iwankiw (1987) proposed an approximate method to handle bolted connections under eccentric and inclined loading. The AISC load and resistance factor design (LRFD) manuals (1986, 1993, 1999, 2005, and 2010) provide the *C* coefficients for only six inclination angles of the load ($\theta = 0°$, 15°, 30°, 45°, 60°, and 75°), which were evaluated using the IC method. Design engineers tend to interpolate linearly the *C* coefficient for a non-specific θ value. However, doing so is not entirely justified. Additionally, the direct implementation of the IC method is dif-

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ficult because it involves a tedious trial-and-error process. Moreover, no design table or tool is available for $\theta > 75°$ (Fig. 1(b)), even though this angle exists in practice. A more effective and efficient method for analyzing an eccentrically bolted connection without the above limitations and shortcomings is therefore needed.

This work develops an iterative algorithm to locate the instantaneous center of rotation of an eccentrically bolted connection, and proposes a simple rational method for approximating the C coefficient for loads at any angle without direct analysis. The results thus obtained are compared with those obtained using other available methods, including the elastic method, the IC method, the AISC manual-based interpolation method, and the approximation method of Iwankiw (1987).

II. PROPOSED METHOD FOR ANALYZING ECCENTRIC BOLTED CONNECTIONS

Iwankiw (1987) presented a computationally simple but rather conservative method for approximating the coefficients C for bolt patterns under inclined loads. This work proposes an improved method that is based on the work of Iwankiw (1987) and yields sufficiently accurate results for loads at various angles between 0° and 90° without the complex iteration of the IC method.

According to the 2010 AISC design manual, the tabulated non-dimensional coefficient, C, represents the number of effective bolts that resist the eccentric force. C is proportional to the available strength (ϕR_n) of the eccentrically loaded bolts. The coefficient C_γ represents for the resistance at the six specified angles (0°, 15°, 30°, 45°, 60°, and 75°), as indicated in the AISC manuals. Any inclined load is conventionally split into vertical and horizontal components. To be consistent with the tabulated C coefficients in AISC design manuals, an applied inclined load (P_u) can be divided into two components (P'_γ and $P'_{\gamma+15}$), where P'_γ and $P'_{\gamma+15}$ are parts of the connection capacities of P_γ and $P_{\gamma+15}$. P_γ and $P_{\gamma+15}$ are proportional to C_γ and $C_{\gamma+15}$, which are listed in the manual in increments of 15°, as displayed in Fig. 2. Since the algebraic addition of two components is always greater than the vector addition thereof, these two components are added algebraically to provide a conservative estimate of the strength of bolted connections. To put this proposed approach into mathematical form which is compatible with the AISC Manual C tables, part of the connection capacity (C_γ) resists P'_γ, and the remainder resists $P'_{\gamma+15}$, based on the algebraic addition simplification. The magnitudes of P'_γ and $P'_{\gamma+15}$ are assumed to be as follows.

$$P'_\gamma = C'_\gamma r_n \text{ and } P'_{\gamma+15} = C'_{\gamma+15} r_n = \left(\frac{C_\gamma - C'_\gamma}{C_\gamma} \right) C_{\gamma+15} r_n \qquad (1)$$

where r_n is the nominal shear strength per bolt; C_γ and $C_{\gamma+15}$ are the coefficients at the six specified angles (0°, 15°, 30°, 45°, 60°, and 75°), as tabulated in the 2010 AISC manual, and C'_γ is a derived eccentricity coefficient to resist a part of P_γ.

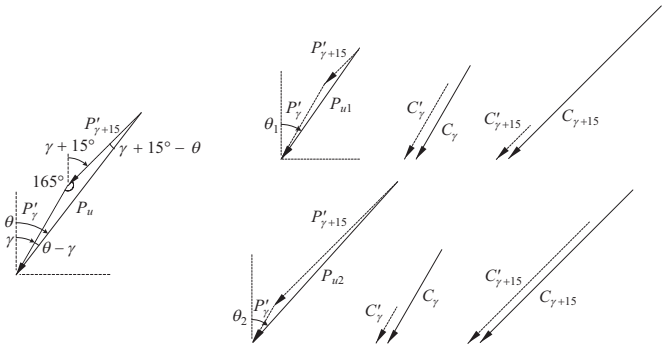


Fig. 2. Derivation of proposed method from trigonometric relationship (P'_γ and $P'_{\gamma+15}$).

From the simple trigonometric relationship between P'_γ and $P'_{\gamma+15}$, we have

$$\frac{P'_\gamma}{\sin(\gamma + 15 - \theta)} = \frac{P'_{\gamma+15}}{\sin(\theta - \gamma)} \qquad (2)$$

Substitution of Eq. (1) into Eq. (2) gives

$$C'_\gamma r_n = \frac{\sin(\gamma + 15 - \theta)}{\sin(\theta - \gamma)} \times \left(\frac{C_\gamma - C'_\gamma}{C_\gamma} \right) C_{\gamma+15} r_n \qquad (3)$$

Further simplification of Eq. (3), the equation then becomes

$$\begin{aligned} C'_\gamma &= \frac{C_{\gamma+15} \sin(\gamma + 15 - \theta)}{\sin(\theta - \gamma) + \dfrac{C_{\gamma+15}}{C_\gamma} \sin(\gamma + 15 - \theta)} \\ &= \frac{C_{\gamma+15}}{\dfrac{\sin(\theta - \gamma)}{\sin(\gamma + 15 - \theta)} + \dfrac{C_{\gamma+15}}{C_\gamma}} \end{aligned} \qquad (4)$$

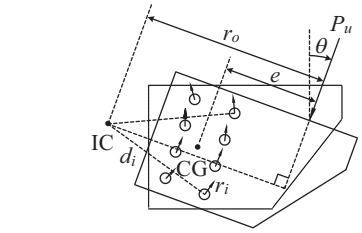
Hence the derived eccentricity coefficient C'_γ can be given as

$$C'_\gamma = \frac{C_{\gamma+15}}{\lambda + \dfrac{C_{\gamma+15}}{C_\gamma}} = \frac{C_\gamma C_{\gamma+15}}{C_\gamma \lambda + C_{\gamma+15}} \quad \text{if} \quad \lambda = \frac{\sin(\theta - \gamma)}{\sin(\gamma + 15 - \theta)} \qquad (5)$$

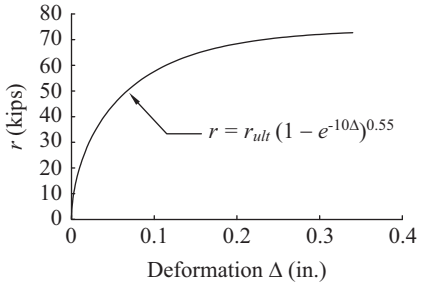
where C_γ is the AISC-tabulated C coefficient that corresponds to the angle γ(0°, 15°, 30°, 45°, 60°, or 75°) and $\gamma < \theta < \gamma + 15$ in degrees. From the law of cosines, we have

$$P_u^2 = P_\gamma'^2 + P_{\gamma+15}'^2 - 2P'_\gamma P'_{\gamma+15} \cos 165° \qquad (6)$$

Substituting $P'_\gamma = C'_\gamma r_n$ and $P'_{\gamma+15} = P'_\gamma \dfrac{\sin(\theta - \gamma)}{\sin(\gamma + 15 - \theta)} = \lambda C'_\gamma r_n$, we obtain



(a) IC method



(b) Load-deformation relationship of one ASTM A325 bolt under single shear

Fig. 3. Instantaneous center of rotation method.

$$C_u = \sqrt{\left(C'_\gamma\right)^2 + \left(C'_\gamma \lambda\right)^2 - 2C'_\gamma C'_\gamma \lambda \cos 165^\circ} = C'_\gamma \sqrt{1 + \lambda^2 - 2\lambda \cos 165^\circ} \quad (7)$$

Then the proposed method can be concluded as follows:

(1) Calculate C'_γ, which is based on Eq. (5).
(2) Compute C_u, which is calculated based on Eq. (7) and is the approximate coefficient for any angle (θ) of inclination between 0° and 90°.
(3) Obtain P_u, which is the maximum eccentric load of the bolt group.

$$P_u = C_u(\phi r_n) \quad (8)$$

To evaluate the accuracy of the proposed method, the exact result obtained by the IC method is developed as follows.

III. PROCEDURE FOR IMPLEMENTING THE IC METHOD

The combined effect of rotation and translation under the eccentric load is equivalent to a rotation about a particular location, which is called the instantaneous center of rotation (IC), as shown in Fig. 3(a). The exact location of the IC depends on the bolt patterns and the location and direction of the applied load. Fig. 3(b) plots the load-deformation relationship (Crawford and Kulak, 1971) of a high-strength bolt, which is given by

$$r = r_{ult}(1 - e^{-10\Delta})^{0.55} \quad (9)$$

where r denotes the nominal bolt shear strength (kips) (1 kip = 4.448 kN); r_{ult} is the ultimate bolt shear strength (kips), and Δ is the combined bolt shear, bearing, and bending deformation (inches) (1 inch = 2.54 cm).

Eq. (9) was obtained from an experiment on an ASTM A325 bolt with a diameter of ¾ inch under single shear with $r_{ult} = 74$ kips and $\Delta_{max} = 0.34$ in. The nominal shear strength (r) of the farthest bolt from the IC is obtained by applying a maximum deformation Δ_{max} to the bolt. The values of r of the other bolts are obtained by considering a deformation Δ that varies linearly with distance from the IC between zero (at the IC) and Δ_{max} (at the farthest bolt). The nominal shear strength of the bolt group is, then, the sum of the individual bolt strengths.

The AISC design manuals (1986, 1989, 1993, 1999, 2005, and 2010) used the IC method to analyze eccentric bolted connections by specifically evaluating C. Brandt (1982) presented a technique for locating the IC. However, none of the aforementioned design manuals provided detailed steps for the implementation of the tedious IC method. The present work develops a different algorithm for locating the IC, as follows.

In the IC method, three force equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$) are required to locate the instantaneous center of rotation. The exact solution to the problem is almost impossible to obtain directly, so an iterative algorithm that is based on the concept of gradient descent for line search (Nocedal and Stephen, 1999; Su and Siu, 2007) is developed to solve it.

Given $\Sigma M = 0$, the required trial strength (P_u) of the eccentrically loaded bolt group is $P_u = \Sigma M_u / r_o = \Sigma(r_i d_i)/r_o$, whose corresponding load components are P_{ux} and P_{uy}. At static equilibrium ($\Sigma F_x = 0$, $\Sigma F_y = 0$), the resultants are expressed as

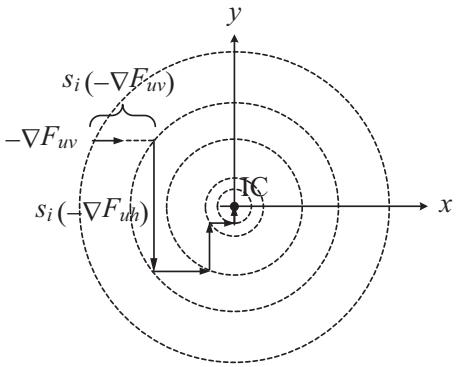
$$\Sigma F_x = F_{uh} = P_{ux} + \Sigma r_{ux} \text{ and } \Sigma F_y = F_{uy} = P_{uy} + \Sigma r_{uy} \quad (10)$$

where F_{uh} and F_{uv} are called unbalanced forces if $\Sigma F_x \neq 0$ or $\Sigma F_y \neq 0$.

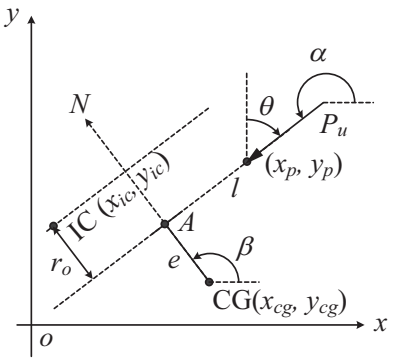
Let $F(x_0, y_0)$ be the magnitude of the resultant of the eccentrically loaded bolt group at the first trial point of the IC:

$$\nabla F(x,y)\Big|_{(x,y)=(x_0,y_0)} = \frac{\partial F}{\partial x}\mathbf{i} + \frac{\partial F}{\partial y}\mathbf{j}\Big|_{(x,y)=(x_0,y_0)} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = F_{uh}\mathbf{i} + F_{uv}\mathbf{j} \quad \text{where} \quad F = \sqrt{F_{uh}^2 + F_{uv}^2} \quad (11)$$

The magnitude of resultant $F(x, y)$ increases rapidly in the direction of positive gradient (∇F), and falls rapidly in the direction of negative gradient ($-\nabla F$). The negative gradient ($-\nabla F$) specifies the direction of descent of the resultant $F(x, y)$. In the iterative algorithm, this negative gradient is applied to reduce the x- and y-components of unbalanced forces, F_{uh} and F_{uv}, in each direction of descent. Gradient is perpendicular to the force



(a) Direction of descent with linear search



(b) Perpendicular distance (r_o) from IC to a line (l)

Fig. 4. Iterative algorithm to find the location of IC.

vector. Accordingly, the direction of descent opposes the normal to the force vector, so F_{uv} declines in the positive x direction and F_{uh} falls in the negative y direction. Then, step length, s_iF_{uv} or s_iF_{uh}, is adjusted as a shift along each direction of descent with reference to Fig. 4(a). The positive s_i is the steplength parameter, which can be set for each iterative process. Therefore, the iterated coordinates are given by

$$x_{i+1} = x_i + s_i(-\nabla F_{uy}) = x_i F_{uy} \text{ and}$$
$$y_{i+1} = y_i + s_i(-\nabla F_{uy}) = y_i F_{uh} \tag{12}$$

The algorithm requires that the initially guessed IC position is the centroid of the section; the iterative process generates the next point by moving one step length in the direction of negative gradient from the preceding IC point. The computational procedure is described in detail herein using an illustrative example.

The iterative process terminates when the unbalanced forces F_{uv} and F_{uh} have been obtained to the desired accuracy, which approaches zero. The required design force (P_u) and available bolt strength (ϕr_n) are then set, yielding the coefficient $C = P_u/(\phi r_n)$.

The detailed procedure for implementing the IC method is summarized as follows:

1. Determine the sectional properties and geometry of the bolt group and use the center of gravity (CG) of the bolt group as the first trial location of IC.
2. With reference to Fig. 4(b), find the normal form of the equation (Sisam and Atchison, 1955) along the load application line (l). The perpendicular distance (e) from the CG to l is expressed as

$$e = (x_p - x_{cg})\cos\beta + (y_p - y_{cg})\sin\beta \tag{13}$$

and the perpendicular distance (r_o) from the IC to l is calculated as

$$\begin{aligned} r_o &= (x_{ic} - x_{cg})\cos\beta + (y_{ic} - y_{cg})\sin\beta - e \\ &= (x_{ic} - x_p)\cos\beta + (y_{ic} - y_p)\sin\beta \end{aligned} \tag{14}$$

where β is the angle between the line that is normal to P_u and the horizontal axis.

3. Calculate the deformation (Δ_i) and resistance (r_i) from the load-deformation relationship that is given in the AISC manuals for each bolt:

$$\Delta_i = (d_i / d_{\max})\Delta_{\max} = 0.34(d_i / d_{\max}) \text{ and}$$
$$r_i = r_{ult}(1 - e^{-10\Delta_i})^{0.55} \tag{15}$$

4. Calculate the resultant moment (ΣM_u) and force components (Σr_{ux} and Σr_{uy}) for the bolt group.

$$\Sigma r_{ux} = \Sigma(r_i y_i)/d_i,\ \Sigma r_{uy} = \Sigma(r_i x_i)/d_i, \text{ and}$$
$$\Sigma M_u = \Sigma(r_i d_i) \tag{16}$$

where (x_i, y_i) are the coordinates of each bolt, r_i is the resistance of each bolt, and d_i is the radial distance from CG to the center of each bolt.

5. Calculate the corresponding applied load (P_u) and its components (P_{ux} and P_{uy}) by considering the static equilibrium,

$$P_u = \Sigma M_u / |r_o| = \Sigma(r_i d_i)/|r_o|,\ P_{ux} = P_u\cos\alpha, \text{ and}$$
$$P_{uy} = P_u\sin\alpha \tag{17}$$

where α is the angle of inclination of P_u with respect to the horizontal line and r_o is the load eccentricity, which is obtained using Eq. (14).

6. Confirm the force equilibrium.

$$\Sigma F_x = F_{uh} = P_{ux} + \Sigma r_{ux} \text{ and } \Sigma F_y = F_{uy} = P_{uy} + \Sigma r_{uy} \tag{18}$$

7. If an equilibrium condition is violated ($F_{uh} \neq 0$ or $F_{uv} \neq 0$),

Table 1. Comparisons of values of *C* obtained using proposed iterative algorithm.

Angle (θ)	0°	15°	30°	45°	60°	75°
Computational result (C)	3.554	3.618	3.921	4.554	5.710	7.902
2010 AISC manual (C)	3.550	3.620	3.920	4.550	5.710	7.900

(Based on 2010 AISC Tables 7 and 8 with $g = 5.5$ in., $s = 3$ in., $e_x = 16$ in., and $n = 6$; 1 in. = 2.54 cm)

then alter the location of IC to reduce the difference between the applied load (P_u) and the resultant force (Σr_u) of the bolt group. The next IC coordinates are adjusted to

$$x_{i+1} = x_i + s_i(-\nabla F_{uy}) = x_i F_{uy} \text{ and}$$
$$y_{i+1} = y_i + s_i(-\nabla F_{uy}) = y_i F_{uh} \quad (19)$$

where positive s_i is the step-length parameter for F_{uh} and F_{uv}.

8. Repeat Steps 2 to 7 until the convergence value of 0.1 percent for unbalanced forces F_{uh} and F_{uv} is reached.

A design example is presented below to illustrate the above procedure, whose results are compared with those of the proposed method. A computer program was developed to execute the above iterative process. The outputs (coefficients C) of the program were verified against the tables in the 2010 AISC design manual. Table 1, which presents a set of outputs, shows that the calculated C coefficients in the example compare favorably with those in the AISC design tables.

IV. ILLUSTRATIVE EXAMPLE

Fig. 5(a) shows a bolted connection under an applied load (P_u). The connection is supported by the bracket. Both the column and the bracket are made of steel with $F_y = 36$ ksi. A325-N bolts with a diameter of ⅞ in. are used in standard holes. Assume the column flange and the bracket plate have adequate strength. The objective is to evaluate the maximum load (P_u) using both the available methods and the proposed method.

1. Elastic Method

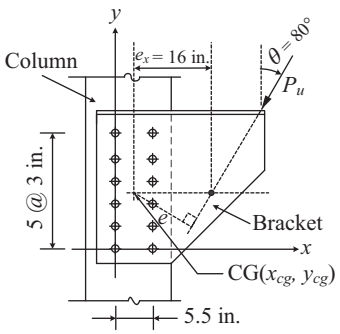
With reference to Fig. 5(a), the center of gravity (CG) of the given bolt group is located at

$$x_{cg} = (\Sigma x_i A_b)/\Sigma A_b = 2.75 \text{ in. and } y_{cg} = (\Sigma y_i A_b)/\Sigma A_b = 7.5 \text{ in.},$$

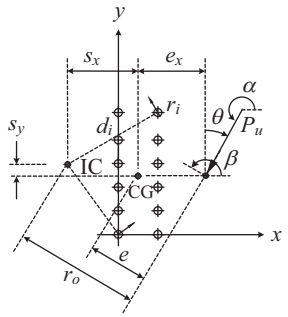
where A_b is the cross-sectional area of each bolt. By calculation, $\sum d^2 = 405.75$ in.^{2} with respect to the CG of the bolt group. The components of direct bolt shear are

$$r_{pux} = r_{pu}\sin\theta = P_u \sin\theta / n = P_u \sin 80°/n = -0.0821\, P_u \ (\leftarrow)$$

$$r_{puy} = r_{pu}\cos\theta = P_u \cos\theta / n = P_u \cos 80°/n = -0.0145\, P_u \ (\downarrow)$$



(a) Bolted connection



(b) Perpendicular distance (r_o) from trial IC to line along which loadis applied

Fig. 5. Illustrative example.

The torque at the CG is

$$M_{cg} = P_u \cos\theta \times (-0.17365\, P_u \times 16) = -2.7784\, P_u \text{ (clockwise)}$$

The components of torsion-induced bolt shear are

$$r_{mux} = \frac{M_{CG} d_y}{\sum d^2} = \frac{-2.7784 P_u \times 7.5}{405.75} = -0.05136\, P_u \quad (\leftarrow)$$

$$r_{muy} = \frac{M_{CG} d_x}{\sum d^2} = \frac{-2.7784 P_u \times 2.75}{405.75} = -0.01883\, P_u \quad (\downarrow)$$

Hence, the required strength per bolt, r_u, is

$$r_u = \sqrt{\left(r_{pux} + r_{mux}\right)^2 + \left(r_{puy} + r_{muy}\right)^2}$$
$$= \sqrt{\left(-0.0821P_u - 0.05136P_u\right)^2 + \left(-0.0145P_u - 0.01883P_u\right)^2}$$
$$= 0.1375\, P_u$$

According to the 2010 AISC LRFD, $r_u \le \phi r_n$. With $\phi r_n = 24.3$ kips, the obtained maximum P_u value is $(P_u)_{max} = 176.727$ kips

From $P_u \le \phi R_n = C_e \times \phi r_n$, $C_e = (P_u)_{max}/(\phi r_n) = 176.727/24.3 = 7.273$

2. IC Method

As stated above, the CG of the bolt group is located at x_{cg} = 2.75 in. and y_{cg} = 7.5 in. With reference to Fig. 5(b), the line along which the load is applied can be expressed in normal form. The angle between the horizontal line and the normal to the load direction is

$$\beta = 180° - \theta = 180° - 80° = 100°$$

The perpendicular distance from the CG to the line along which the load is applied is

$$e = (x_p - x_{cg})\cos\beta + (y_p - y_{cg})\sin\beta$$
$$= 16 \times \cos 100° + 0 \times \sin 100° = -2.778 \text{ in.}$$

where

$$(x_p, y_p) = (18.75 \text{ in.}, 7.5 \text{ in.}) \text{ and}$$
$$(x_{cg}, y_{cg}) = (2.75 \text{ in.}, 7.5 \text{ in.})$$

The initial guess of IC at CG is (x_0, y_0) = (2.75 in., 7.5 in.). The perpendicular distance from the initially guessed IC to the line along which the load is applied is

$$r_o = (x_0 - x_{cg})\cos\beta + (y_0 - y_{cg})\sin\beta - e$$
$$= (x_0 - 2.75)\cos 100° + (y_0 - 7.5)\sin 100° + 2.778$$
$$= 2.778 \text{ in.}$$

The AISC load-deformation relationship for one bolt is

$$r = r_{ult}(1 - e^{-10\Delta})^{0.55}, \Delta = 0.34(d_i / d_{\max}),$$
$$r_{ux} = r \times (d_y / d), \text{ and } r_{uy} = r \times (d_x / d)$$

From the developed program outputs, $\Sigma(r \times d)$ = 1501.396 k-in, $\Sigma r_{ux} = 0$, and $\Sigma r_{uy} = 0$

Therefore, $P_u = \Sigma M_u / |r_o| = \Sigma(r \times d)/2.778 = 1501.396/2.778 = 540.387$ kips

The equilibrium of the unbalanced forces in the horizontal and vertical directions is checked as follows. The angle between the line along which the load is applied and the horizontal axis is

$$\alpha = 270° - \theta = 270° - 80° = 190°$$

$$F_{uh} = \Sigma F_x = P_{ux} + \Sigma r_{ux} = 540.387 \times \cos 190° + 0$$
$$= -531.178 \text{ kips } (\leftarrow) \neq 0 \quad \text{NG.}$$

$$F_{uy} = \Sigma F_y = P_{uy} + \Sigma r_{uy} = 540.387 \times \sin 190° + 0$$
$$= -93.837 \text{ kips } (\downarrow) \neq 0 \quad \text{NG.}$$

In this example, the step-length parameter (s_i) is set to 5% in each iteration. The iterative algorithm generates the next trial location of IC as

$$x_{i+1} = x_i + s_i F_{uv} \Rightarrow x_1 = 2.75 + 0.05(-93.837) = -1.942 \text{ in.}$$
$$y_{i+1} = y_i + s_i F_{uh} \Rightarrow y_1 = 7.5 + 0.05(-532.178) = 34.109 \text{ in.}$$

The step length and direction of descent in this iteration are given by -4.692 in. in the negative x direction and 26.609 in. in the positive y direction, respectively. The iteration algorithm generates the next location (x_1, y_1) = (-1.942 in., 34.109 in.) from the current point (x_0, y_0) = (2.75 in., 7.5 in.). Repeating the above steps, as described above, yields a sequence of locations of IC.

At the correct IC (x_{ic}, y_{ic}), the force equilibrium equations are satisfied, ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$), and the final P_u value is thus determined. In this example, the correct IC coordinates, (x_{ic}, y_{ic}) = (1.3468 in., 15.8477 in.) are obtained after 25 iterations. The distance from the IC to the CG, S = 3.780 in. (with $S_x = -1.403$ in. leftward and $S_y = 8.3477$ in. upward). Table 2 presents in detail the calculations that are associated with, and the results that are obtained using, the iterative algorithm that is based on the IC method.

$$P_u = 2456.8452/[(1.3468 - 2.75)\cos 100°$$
$$+ (15.8477 - 7.5)\sin 100° + 2.778] = 218.522 \text{ kips}$$

The equilibrium of the horizontal forces is confirmed as follows.

$$P_{ux} = P_u \cos\alpha = 218.522 \times \cos 190° = -215.202 \text{ kips } (\leftarrow)$$

$$\Sigma F_x = P_{ux} + \Sigma R_{ux} = -215.202 + 215.203$$
$$= 0.001 \approx 0 \text{ kips} \quad \text{OK}$$

The equilibrium of the vertical forces is confirmed as follows.

$$P_{uy} = P_u \cos\alpha = 218.522 \times \sin 190° = -37.946 \text{ kips } (\downarrow)$$

$$\Sigma F_y = P_{uy} + \Sigma R_{uy} = -37.946 + 37.946 = 0 \text{ kips} \quad \text{OK}$$

ϕr_n = 24.3 kips as per the 2010 AISC LRFD, and $C = P_u/(\phi r_n)$ = 218.522/24.3 = 8.99

3. Straight-Line Interpolation

Tables 7-6 to 7-13 in the 2010 AISC manual provide the values of C for the six specified load inclination angles (θ = 0°, 15°, 30°, 45°, 60°, and 75°). For a non-tabulated θ value, straight-line interpolation between C values for loads at different angles may be non-conservative, so the AISC recommends direct analysis. In this case, linear interpolation yields C = 9.27 for θ = 80° where C = 7.90 for θ = 75° and C = 12 for θ = 90°.

Table 2. Results in illustrative example ($\theta = 80°$) obtained using the iterative algorithm.

Bolt No.	d_x (in.)	d_y (in.)	d (in.)	Δ (in.)	r (kip)	r_{ux} (kip)	r_{uy} (kip)	$R \times d$ (k-in.)
1	-1.347	-0.848	1.591	0.033	12.095	-6.443	-10.236	19.248
2	4.153	-0.848	4.239	0.088	18.096	-3.619	17.730	76.706
3	-1.347	-3.848	4.077	0.085	17.853	-16.851	-5.898	72.779
4	4.153	-3.848	5.662	0.118	19.833	-13.478	14.549	112.285
5	-1.347	-6.848	6.979	0.145	20.972	-20.578	-4.047	146.359
6	4.153	-6.848	8.009	0.166	21.645	-18.507	11.225	173.347
7	-1.347	-9.848	9.939	0.206	22.549	-22.342	-3.055	224.128
8	4.153	-9.848	10.688	0.222	22.808	-21.015	8.863	243.765
9	-1.347	-12.848	12.918	0.268	23.370	-23.243	-2.436	301.895
10	4.153	-12.848	13.502	0.280	23.478	-22.339	7.222	317.003
11	-1.347	-15.848	15.905	0.330	23.803	-23.718	-2.016	378.588
12	4.153	-15.848	16.383	0.340	23.851	-23.071	6.046	390.742
Σ						-215.203	37.946	2456.845

(1 kip = 4.448 kN; 1 in. = 2.54 cm)

Table 3. Differences among values of C in illustrative example ($\theta = 80°$).

	Elastic method (A)	IC method (B)	AISC Manual interpolated (C)	Method of Iwankiw (D)	Proposed method (E)
Coefficient C	7.27	8.99	9.27	7.63	8.85
P_u (kips)	176.73	218.52	225.26	185.47	215.08
Difference (%)	-19.12 (A – B)/B	0.00 (B – B)/B	3.05 (C – B)/B	-15.12 (D – B)/B	-1.58 (E – B)/B

The required strength of the bolt group is $P_u = C \times (\phi r_n) = 9.27 \times 24.3 = 225.26$ kips.

As expected, this result is an overestimate, relative to that obtained by the IC method.

4. Method of Iwankiw

C_{max} = 12 (total number of bolts) and C_o = 3.55 (AISC's tabulated C value for $\theta = 0°$).

$$A = \frac{C_{max}}{C_o} = \frac{12}{3.55} = 3.380 \geq 1.0$$

The approximate eccentricity coefficient for the inclined load (C_a) is given by

$$\frac{C_a}{C_o} = \frac{A}{\left(\sin\theta + A\cos\theta\right)} = \frac{3.38}{\left(\sin 80° + 3.38\cos 80°\right)} = 2.15 \geq 1.0$$

The required strength (P_u) is then calculated as

$$P_u = C_a(\phi r_n) = (2.15 \times 3.55) \times (24.3) = 185.47 \text{ kips}$$

5. Proposed Method

For $\gamma = 75°$, $C_\gamma = C_{75} = 7.90$, as obtained from the AISC table. Here, $C_{90} = 12$, so when $\theta = 80°$,

$$\lambda = \frac{\sin(\theta - 75°)}{\sin(90° - \theta)} = \frac{\sin(80° - 75°)}{\sin(90° - 80°)} = 0.5019$$

$$\Rightarrow C'_\gamma = \frac{C_{75}C_{90}}{C_{75}\lambda + C_{90}} = \frac{7.9 \times 12}{7.9 \times 0.5019 + 12} = 5.9380$$

$$C_u = C'_\gamma \sqrt{1 + \lambda^2 - 2\lambda\cos 165°} = 8.85$$

Therefore, $P_u = C_u\, \phi r_n = 8.85 \times 24.3 = 215.055$ kips

V. DISCUSSION

Table 3 presents the results that are obtained using the various methods for $\theta = 80°$. The values of C for other θ values can be computed similarly, and are shown in Table 4 and Fig. 6. Fig. 7 compares the values of C that are obtained using the various methods. The following observations are made.

(1) For the six specified values of θ (0°, 15°, 30°, 45°, 60° & 75°), the C coefficients that are calculated using the proposed iterative algorithm equal those that are tabulated in the 2010 AISC manual, which are presented in Table 1 and

Table 4. Comparisons of values of *C* obtained by various methods.

Angle θ (degree)	$C_{elastic}$ [1]	$C_{IC\ method}$ [2]	$C_{interpolated}$ [3]	$C_{Iwankiw}$ [4]	$C_{proposed}$ [5]	(%) $\frac{[1]-[2]}{[2]}$	(%) $\frac{[3]-[2]}{[2]}$	(%) $\frac{[4]-[2]}{[2]}$	(%) $\frac{[5]-[2]}{[2]}$
0	2.837	3.554	3.550	3.550	3.550	-20.16	0.00	0.00	0.00
15	2.787	3.618	3.620	3.405	3.620	-22.96	0.00	-5.88	0.00
30	2.933	3.921	3.920	3.501	3.920	-25.20	0.00	-10.70	0.00
45	3.329	4.554	4.550	3.874	4.550	-26.90	0.00	-14.92	0.00
60	4.166	5.710	5.710	4.695	5.710	-27.04	0.00	-17.79	0.00
75	6.072	7.902	7.900	6.519	7.900	-23.16	0.00	-17.50	0.00
76	6.277	8.102	8.173	6.711	8.069	-22.53	0.88	-17.17	-0.41
77	6.498	8.311	8.447	6.917	8.247	-21.81	1.64	-16.76	-0.76
78	6.736	8.528	8.720	7.139	8.436	-21.01	2.25	-16.29	-1.08
79	6.993	8.755	8.993	7.377	8.637	-20.12	2.72	-15.74	-1.35
80	7.273	8.993	9.267	7.635	8.850	-19.14	3.05	-15.10	-1.58
81	7.574	9.241	9.540	7.913	9.077	-18.03	3.24	-14.37	-1.77
82	7.903	9.499	9.813	8.215	9.319	-16.80	3.31	-13.51	-1.89
83	8.262	9.767	10.087	8.544	9.577	-15.41	3.27	-12.52	-1.94
84	8.655	10.045	10.360	8.903	9.853	-13.84	3.13	-11.37	-1.91
85	9.085	10.334	10.633	9.297	10.149	-12.08	2.90	-10.04	-1.79
86	9.558	10.632	10.907	9.730	10.466	-10.10	2.58	-8.49	-1.57
87	10.080	10.938	11.180	10.208	10.806	-7.85	2.21	-6.68	-1.21
88	10.656	11.248	11.453	10.740	11.174	-5.26	1.83	-4.52	-0.66
89	11.294	11.571	11.727	11.333	11.570	-2.39	1.59	-1.82	-0.01
90	12.000	12.000	12.000	12.000	12.000	0.00	0.00	0.00	0.00

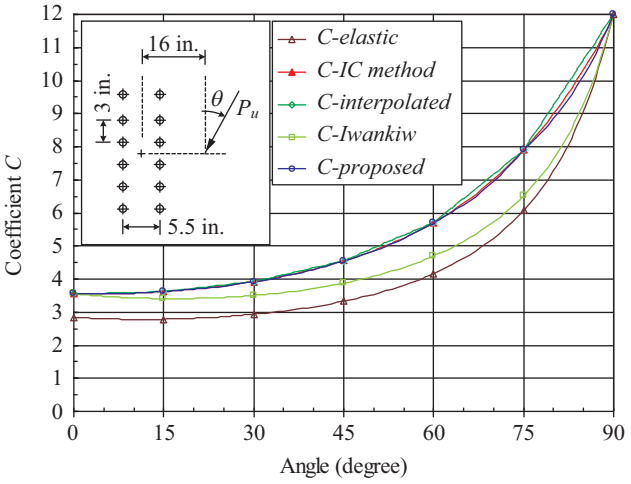


Fig. 6. Coefficients *C* computed using various methods.

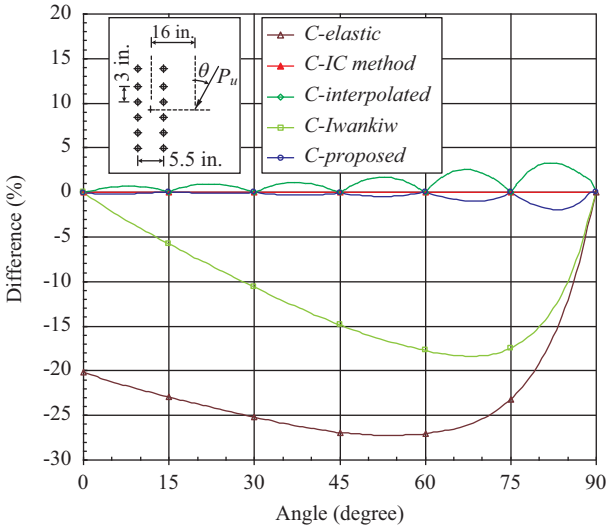


Fig. 7. Comparisons of *C* values obtained using various methods.

Fig. 6, verifying the very high accuracy of the proposed iterative algorithm.

(2) As indicated in Table 4 and Fig. 7 in this example, all methods except the linear interpolation method underestimate *C*, as determined by comparison with those obtained using the more exact IC method. The linear interpolation method overestimates the strength by 0-3%. For θ = 0-75°, the underestimations by the elastic method, the approximate method of Iwankiw (1987), and the proposed method are 27%, 18%, and 0.9%, respectively. For θ = 75-90°, not considered in any AISC manual, the maximum degrees of conservatism are 23%, 17%, and 1.91%, respectively. Apparently, the proposed method yields fairly accurate results.

Table 5. Bolt group geometry in 2010 AISC design tables.

AISC Tables	Column No.	Row No.	Horizontal eccentricity (in.)	Horizontal spacings (in.)	Vertical spacings (in.)
7-6	1	2-12	2-36	–	3, 6
7-7	2	1-12	2-36	3.0	3, 6
7-8			2-36	5.5	3, 6
7-9			2-36	8.0	3, 6
7-10	3	1-12	2-36	3.0	3, 6
7-11			2-36	6.0	3, 6
7-12	4	1-12	2-36	3.0	3, 6
7-13			2-36	4.0	3, 6

(1 in. = 2.54 cm)

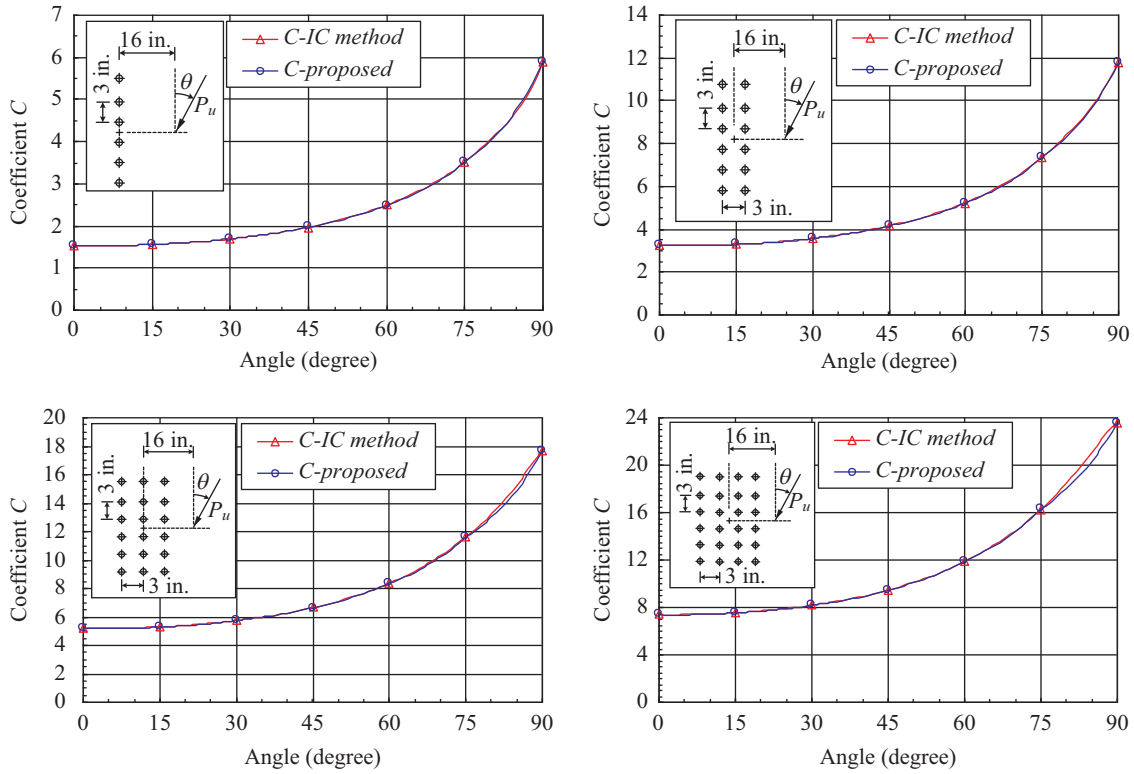


Fig. 8. Accuracy of the proposed method for other bolt groups in 2010 design manual.

(3) For all other bolt patterns in the 2010 AISC manual, shown in Table 5, the proposed approach yields results that are very close to those obtained using the IC method for $\theta =$ 0-75°, shown in Fig. 8. For $\theta = 75$-90°, not covered in any AISC manual, the proposed approach yields sufficiently accurate results.

(4) The proposed approach yields results with greater accuracy than straight-line interpolation between C values for loads at the specified angles; straight-line interpolation may be non-conservative and is not recommended by the AISC.

(5) The derived eccentricity coefficient C'_γ is obtained by a non-linear interpolation between C_γ and $C_{\gamma+15}$, which are listed in the AISC design tables. C'_γ decreases as the angle of inclination increases in range of each 15° interval for all bolt patterns in the 2010 AISC manual. This finding reflects that the assumption, as represented by Eq. (1), is a rational one. The proposed method is derived from a simple trigonometric relationship among the applied load and two components of force. Accordingly, the method can be reasonably applied to estimate the strength of an eccentrically loaded in-plane connection without any restriction on materials or a tedious iterative process.

VI. CONCLUSIONS

This work presents a rational procedure for determining the instantaneous center of rotation and strength of an eccentric group of bolts. The procedure is simple and reliable. This work

overcomes the limitations in current AISC design manuals. The results in this work support the following conclusions.

1. The proposed iterative algorithm constitutes a general procedure for implementing the tedious IC method to find the exact location of an IC. The iterative procedure yields identical eccentricity coefficients C at the six specified load inclination angles θ (0°, 15°, 30°, 45°, 60°, and 75°) being tabulated in the AISC design manuals, further demonstrating the accuracy and reliability of the iterative algorithm.
2. The proposed method yields fairly accurate results without the need for a tedious trial-and-error procedure for all angles of inclination ($0° \le \theta \le 90°$) as shown in Fig. 6. The accuracy is also applied to all other bolt patterns listed in the 2010 AISC design manual. Some examples of the accuracy are displayed as shown in Fig. 8.
3. The proposed method is a rational and reliable tool for approximating the eccentricity coefficients C for loads at angles between 0° and 90° instead of straight-line interpolation, which is not recommended by the AISC. The proposed model can get rid of engaging on a direct analysis or the C values for the next lower angle increment in the tables as recommended by the 2010 AISC design manual. The accuracy of the proposed method substantially exceeds the requirements of engineering. This work overcomes the design limitations (for $0° \le \theta \le 75°$ only) which are evident in current AISC design manuals.

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