AN INTEGRATED INVENTORY MODEL WITH DEPENDENTLY POLYNOMIAL CRASHING COSTS AND A FUZZY INTEREST RATE

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ABSTRACT

The purpose of this study is to minimize the present value of the joint expected total costs in an inventory model considering a fuzzy interest rate. In recent years, inventory policies have played a critical role in supply chain management in highly competitive environments. Therefore, this study aims at determining a suitable inventory policy to enhance the benefits of the supply chain. Reducing lead times and the associated inventory costs are vital concerns in supply chain management. However, most previous studies have not considered the effect of a time value. In addition, this study develops an integrated inventory model that considers crashing costs and time values to reduce inventory problems. In addition, this study applies a signed distance, a ranking approach used by fuzzy numbers to estimate the interest rate in order to represent real-world situations. Moreover, an algorithm is established to determine the optimal order quantity, the length of the lead time, and the number of lots that are delivered from the vendor to the buyer. Finally, a numerical example is provided to illustrate the solution procedure.

I. INTRODUCTION

Reducing total costs effectively and satisfying customer demands quickly are two critical time-based advantages. Both vendors and buyers must determine the optimal economic lot size. Consequently, integrated inventory policies might aid in determining the optimal order quantity and shipment size.

Wikner et al. (2007) indicated that postponement, a small lot size, and various customized items are generally applied in Just-in-Time (JIT) planning to address intense competition and that crashing cannot be classified as a lead-time reduction in a long-term time compression plan. However, using a short life cycle has been an obvious trend recently. Therefore, the basic assumption in this study is based on findings by Wikner et al. (2007).

Currently, the market is fully competitive, indicating that inventory investment in safety stock, the level of customer service, and the competitive ability of an enterprise are directly affected by the length of lead times.

Banerjee's (1986) model challenged the lot-for-lot assumptions and policies mandating that a supplier's economic production quantity should be an integer multiple of the purchase quantity of a buyer who supplies lower joint relevant costs. Regarding equal size shipments to the buyer, Lu (1995) presented an heuristic approach for a one-supplier-multiple-buyer integrated inventory case. Hill (1997) considered more general types of policy for determining successive shipment sizes to buyers.

In most inventory models, lead time is assumed to be constant or a random variable. In practice, additional crashing costs can reduce the lead time and the inventory in safety stocks. Over the last few decades, a wealth of research has considered various lead times. Tersine (1982) suggested that lead-time crashing typically consists of the following components: order transit, delivery time, order preparation, supplier lead time, and set-up time. Reducing lead times is essential to the success of JIT production.

The probabilistic inventory model, in which the lead time is a decision variable, was presented by Liao and Shyu (1991). This model assumes that demand follows a normal distribution and that the lead time, which consists of n components, has differing costs for lead time reduction. Yao and Wu (2000) added stock-out costs and assumed that shortages are allowed. Ouyang et al. (1999) investigated the impact of ordering cost reduction on a modified continuous review inventory system that involved a variable lead time and a mixture of back orders and lost sales.

Lead times can be reduced by paying crashing costs in many situations. A continuous review inventory system has been considered in which the shortage is allowed and the total quantity of the stock out is a combination of back orders and lost sales.

More recently, Pan and Hsiao (2005) investigated another integrated inventory system in which lead time and back ordering are negotiable. Sirinivas and Rao (2007) developed an inventory model that assumes the replenishment lead time to be dependent, focusing on reducing the lead time and total costs.

Most inventory systems overlook the factor of time values; nevertheless, inflation or deflation occurs frequently in the real world. Consequently, time values are critical factors. Day et al. (2008) demonstrated that deteriorating items with a time-dependent demand increased at a decreasing rate and interval-valued lead time over a finite time horizon. Yang (2006) provided an inventory model in which the lead-time demand follows a normal distribution and considers the time value.

Researchers have applied fuzzy theory to developing various solutions for integrated inventory problems. Park (1987) con sidered fuzzy inventory costs by using the arithmetic operations of the Extension Principle. Yang (2010) applied demand quantity fuzziness in a two-echelon inventory model. Building on the work of Pan and Yang (2002), the current paper presents an integrated inventory model that incorporates fuzzy setup costs and fuzzy ordering costs. The Yao and Wu ranking method and the cancroids method for fuzzy numbers were employed to estimate the joint total expected annual costs in the fuzzy sense, and the corresponding order quantity of the buyer was derived accordingly (George and Yuan, 1995; Yao and Wu, 2000).

In 2015, Brodetskiy developed a unique approach for optimizing such systems, considering the time value of money according to a simple interest scheme and modifying the economic order quantity formula. Frankel (2014) created a model that reflects the carry trade. In addition, Tsao and Sheen (2012) considered a multi-item supply chain involving a credit period and weight freight cost discounts. Moreover, Ma et al. (2012) investigated the inventory decisions of a capital-constrained retailer in relation to credit loaning.

Thus, the purpose of this research was to determine an optimal inventory strategy that could minimize the present value of the joint expected total costs over an infinite time horizon.

II. NOTATIONS AND ASSUMPTIONS

1. Notations

To establish the proposed inventory model, the following notations are applied as follows:

D: average demand per year,
P: production rate, and $P > D$,
Q: order quantity of the buyer,
A: buyer's ordering costs per order,
S: vendor's set-up costs per set-up,
L: length of the lead time,
C_V: unit production costs paid by the vendor,
C_P: unit purchase costs paid by the buyer,
m: an integer representing the number of lots in which the items are delivered from the vendor to the buyer,
r: annual inventory holding costs per dollar invested in stocks,
$\tilde{\imath}$: fuzzy interest rate per year that is compounded continuously

2. Umptions

According to the research of Pan and Yang (2002), the production cycle is $\frac{mQ}{D}$, which indicates that Q and m are decision variables. The JIT model has shown that there are advantages when venders and buyers cooperate effectively. In addition, the maximum profits for both venders and buyers are the global target in the JIT model. Our study extends the research of Pan and Yang (2002) and assumes that Q and m are independent decision variables.

- here is a single vendor and a single purchase of a single product in this model.
- The demand X during lead time L follows a normal distribution with a mean μL and a standard deviation $\sigma\sqrt{L}$.
- The reorder point (ROP) equals the sum of the expected demand during the lead time and the safety stock; that is, $\text{ROP} = \mu L + k\sigma\sqrt{L}$, where $K > 0$ is the safety factor.
- Inventory is continuously reviewed.
- The extra costs incurred by the vendor are fully transferred to the buyer if a shortened lead time is requested.

The lead-time crashing costs, determined by the length of the lead time and satisfying $R(L) = CL^{-a}$, is a polynomial, where C and a are positive constants. (This assumption is based on the research by Chandra and Grabis (2008)).

III. MATHEMATICAL MODEL

In this paper, we consider a problem involving a fuzzy interest rate per year by fuzzifying i to the triangular fuzzy number $\tilde{\imath}$, where $\tilde{\imath} = (i - \Delta_1, i, i + \Delta_2); 0 < \Delta_1 < i, 0 < \Delta_2$; and Δ_1, Δ_2 are both determined by decision makers.

The fuzzy interest rate defuzzified using the Yao and Wu (2000) ranking method, which is shown according to Definition 1 and in which the signed distance of the fuzzy number $\tilde{\imath}$ is changed to $\tilde{0}_1$, is

$$d(\tilde{\imath}, \tilde{0}_1) = \frac{1}{4}[(i - \Delta_1) + 2i + (i + \Delta_2)] = i + \frac{1}{4}(\Delta_2 - \Delta_1)$$

Definition 1. According to the research of Kaufmann and Gupta (1985), Zimmermann (1978), and Yao and Wu (2000), the fuzzy set $\tilde{B} \in \Omega$ and are [0, 1], the α-cut of the fuzzy set $\tilde{B}$, is $B(\alpha) = \{x \in \Omega | \mu_B(x) \geq \alpha\} = [B_L(\alpha), B_u(\alpha)]$, where $B_L(\alpha) = a + (b - d)\alpha$ and $B_u(\alpha) = c - (c - d)\alpha$. We can obtain the following equation. The signed distance of $\tilde{B}$ to $\tilde{0}_1$ is defined as

$$d(\tilde{B}, \tilde{0}_1) = \int_0^t d\left(\left[B_L(\alpha)_\alpha, B_u(\alpha)_\alpha\right], \tilde{0}_1\right) d\alpha$$
$$= \frac{1}{2}\int_0^1 (B_L(\alpha) + B_u(\alpha)) d\alpha$$

The equation is

$$d(\tilde{B}, \tilde{0}_1) = \frac{1}{2} \int_0^1 \left[B_L(\alpha)_\alpha, B_u(\alpha) \right] d\alpha = \frac{1}{4}(2b + a + c)$$

According to Definition 1, the signed distance of fuzzy number $\tilde{r}$ to $\tilde{0}_1$ is

$$d(\tilde{r}, \tilde{0}_1) = \frac{1}{4}\left[(r - \Delta_1) + 2r + (r + \Delta_2) \right] = r + \frac{1}{4}(\Delta_2 - \Delta_1)$$

Used to determine the fuzzy total expected costs for the buyer and vendor, the notations and assumptions of the total expected costs of the buyer and vendor are as follows:

TRC_P = ordering costs + holding costs + crashing costs

TRC_V = set-up costs + holding costs

We obtain the following equations based on the mathematical model by Yang (2010):

$$TRC_p(\tilde{m}, Q, L) = mA + \frac{mrC_P}{\tilde{i}}\left[(Q + k\sigma\sqrt{L})(1 - e^{-\frac{Q\tilde{i}}{D}}) \right.$$
$$\left. + Qe^{-\frac{Q\tilde{i}}{D}} + \frac{D}{\tilde{i}}(e^{-\frac{Q\tilde{i}}{D}} - 1) \right] + mCL^{-a} \quad (1)$$

$$TRC_V(\tilde{m}, Q) = S + \frac{rC_V}{\tilde{i}}(1 - e^{-\frac{mQ\tilde{i}}{D}})\frac{Q}{2}\left[m(1 - \frac{D}{p}) \right.$$
$$\left. -1 + \frac{2P}{D} \right] \quad (2)$$

In this case, the joint expected total relevant costs for the first cycle $TRC_{joint}(\tilde{m}, Q, L)$ are as follows:

$$TRC_{joint}(\tilde{m}, Q, L) = TRC_P(\tilde{m}, Q, L) + TRC_V(\tilde{m}, Q) = (1) + (2)$$
$$= mA + \frac{mrC_P}{\tilde{i}}\left[(Q + k\sigma\sqrt{L})(1 - e^{-\frac{Q\tilde{i}}{D}}) \right.$$
$$\left. + Qe^{-\frac{Q\tilde{i}}{D}} + \frac{D}{\tilde{i}}(e^{-\frac{Q\tilde{i}}{D}} - 1) \right] + mCL^{-\alpha} + S$$
$$+ \frac{rC_V}{\tilde{i}}(1 - e^{-\frac{mQ\tilde{i}}{D}})\frac{Q}{2}\left[m(1 - \frac{D}{p}) - 1 + \frac{2P}{D} \right] \quad (3)$$

$PVC(\tilde{m}, Q, L)$, which is obtained using the time value concepts approach of Yang et al. (2005), over an infinite time horizon is as follows:

$$PVC(\tilde{m}, Q, L) = \frac{1}{1 - e^{-\frac{mQ\tilde{i}}{D}}}[TRC_{joint}(\tilde{m}, Q, L)]$$
$$= \frac{1}{1 - e^{-\frac{mQ\tilde{i}}{D}}}\left\{ mA + \frac{mrC_P}{\tilde{i}}\left[(Q + k\sigma\sqrt{L})(1 \right.\right.$$
$$\left. - e^{-\frac{Q\tilde{i}}{D}}) + Qe^{-\frac{Q\tilde{i}}{D}} + \frac{D}{\tilde{i}}(e^{-\frac{Q\tilde{i}}{D}} - 1) \right] \quad (4)$$
$$+ mCL^{-\alpha} + S$$
$$\left. + \frac{rC_V}{\tilde{i}}(1 - e^{-\frac{mQ\tilde{i}}{D}})\frac{Q}{2}[m(1 - \frac{D}{p}) - 1 + \frac{2P}{D}] \right\}$$

IV. SOLUTION PROCEDURE

We take the first and second partial derivation of $PVC(\tilde{m}, Q, L)$ and fixed Q and m, subsequently obtaining

$$\frac{\partial PVC(\tilde{m}, Q, L)}{\partial L} = \frac{1}{1 - e^{-\frac{mQ\tilde{i}}{D}}}\left[\frac{mrC_P}{2\tilde{i}\sqrt{L}}k\sigma(1 - e^{-\frac{Q\tilde{i}}{D}}) - maCL^{-a-1} \right] \quad (5)$$

and

$$\frac{\partial^2 PVC(\tilde{m}, Q, L)}{\partial^2 L} = \frac{1}{1 - e^{-\frac{mQ\tilde{i}}{D}}}\left[\frac{mrC_P}{4\tilde{i}\sqrt{L^3}}k\sigma(1 - e^{-\frac{Q\tilde{i}}{D}}) - ma(a+1)CL^{-a-2} \right] \quad (6)$$

Because (5) > 0, and (6) > 0, we set Q and m. We observe that $PVC(\tilde{m}, Q, L)$ is a convex function in L, indicating that there is a unique value for L that minimizes $PV(\tilde{m}, Q, L)$.

L can be obtained by solving the equation $\partial PVC(\tilde{m}, Q, L)/\partial_L = 0$ in (5). Let the result be B. Subsequently, fix m, obtaining the optimal order quantity of the buyer Q by taking the first partial derivation of $PVC(\tilde{m}, Q)$ with respect to Q and setting the result to zero:

$$\frac{\partial PVC(\tilde{m}, Q)}{\partial Q} = \frac{1}{1 - e^{-\frac{mQ\tilde{i}}{D}}}\left\{ \frac{mrC_P}{\tilde{i}}\left[\frac{2a}{2a+1}k\sigma\sqrt{B}(1 - e^{-\frac{Q\tilde{i}}{D}})^{-\frac{1}{2a+1}}(\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}}) \right.\right.$$
$$\left. + (1 - e^{-\frac{Q\tilde{i}}{D}}) \right] + \frac{2am}{2a+1}CB^{-a}(1 - e^{-\frac{Q\tilde{i}}{D}})^{-\frac{1}{2a+1}}(\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}})$$
$$\left. + \frac{rC_P}{2\tilde{i}}(1 - e^{-\frac{mQ\tilde{i}}{D}})\left[m(1 - \frac{D}{P}) - 1 + \frac{2P}{D} \right] \right\}$$
$$- \frac{1}{(1 - e^{-\frac{mQ\tilde{i}}{D}})^2}\left(\frac{mi}{D}e^{-\frac{mQ\tilde{i}}{D}} \right)\left\{ mA + \frac{mrC_P}{\tilde{i}}\left[Q + k\sigma\sqrt{B}(1 \right.\right.$$
$$\left.\left. - e^{-\frac{Q\tilde{i}}{D}})^{\frac{2a}{2a+1}} + \frac{D}{\tilde{i}}(e^{-\frac{Q\tilde{i}}{D}} - 1) \right] + mCB^a(1 - e^{-\frac{Q\tilde{i}}{D}})^{\frac{2a}{2a+1}} + S \right\} \quad (7)$$

Moreover, we must assess the second-order condition for concavity; that is,

$$
\frac{\partial^2 PVC(\tilde{m},Q)}{\partial^2 Q} = \frac{1}{1-e^{-\frac{mQ\tilde{i}}{D}}}\left\{\frac{mrC_P}{\tilde{i}}\left[\frac{2a}{2a+1}k\sigma\sqrt{B}\left(\frac{-1}{2a+1}(1-e^{-\frac{Q\tilde{i}}{D}})^{-\frac{2a-2}{2a+1}}(\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}})^2\right.\right.\right.
$$

$$
-(1-e^{-\frac{Q\tilde{i}}{D}})^{\frac{1}{2a+1}}(\frac{\tilde{i}^2}{D^2}e^{-\frac{Q\tilde{i}}{D}})\Big)+\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}}\bigg]
$$

$$
+\frac{2am}{2a+1}CB^{-a}\left[\frac{-1}{2a+1}(1-e^{-\frac{Q\tilde{i}}{D}})^{-\frac{2a-2}{2a+1}}(\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}})^2\right.
$$

$$
\left.\left.-(1-e^{-\frac{Q\tilde{i}}{D}})^{\frac{1}{2a+1}}(\frac{\tilde{i}^2}{D^2}e^{-\frac{Q\tilde{i}}{D}})\right]\right\}
$$

$$
-\frac{1}{(1-e^{-\frac{mQ\tilde{i}}{D}})^2}\left(\frac{mi}{D}e^{-\frac{mQ\tilde{i}}{D}}\right)\left\{\frac{mrC_P}{\tilde{i}}\left[\frac{2a}{2a+1}k\sigma\sqrt{B}(\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}})(1-e^{-\frac{Q\tilde{i}}{D}})^{-\frac{1}{2a+1}}\right.\right.
$$

$$
+\left(1-e^{-\frac{Q\tilde{i}}{D}}\right)+\frac{2am}{2a+1}CB^{-a}(\frac{\tilde{i}}{D}e^{-\frac{Q\tilde{i}}{D}})(1-e^{-\frac{Q\tilde{i}}{D}})^{-\frac{1}{2a+1}}\bigg]\bigg\}
$$

$$
+\frac{1-e^{-\frac{mQ\tilde{i}}{D}}}{(1-e^{-\frac{mQ\tilde{i}}{D}})^2}\left(\frac{m^2\tilde{i}^2}{D^2}e^{-\frac{mQ\tilde{i}}{D}}\right)\left\{mA+\frac{mrC_P}{\tilde{i}}\Big[Q+k\sigma\sqrt{B}(1
$$

$$
-e^{-\frac{mQ\tilde{i}}{D}})^{\frac{2a}{2a+1}}+\frac{\tilde{D}}{\tilde{i}}(e^{-\frac{Q\tilde{i}}{D}}-1)\Big]+mCB^a(1-e^{-\frac{Q\tilde{i}}{D}})^{\frac{2a}{2a+1}}+S\right\}
$$

(8)

The results identify $PV(\tilde{m},Q)$ as a convex function in Q for the fixed m. Thus, it is reduced to find a local optimal solution to a local minimum.

1. Algorithm

Summarizing the above processes, we establish the following algorithm to obtain the optimal values of m, Q, and L:

Step 1. Obtain Δ_1 and Δ_2 from the decision maker.
Step 2. Set $m = 1$.
Step 3. Determine Q by solving (8).
Step 4. If Q exists and satisfies (9), we could determine an L by (6). Then $(Q^{(m)}, L^{(m)})$ is the optimal solution for the given m.
Step 5. Obtain $PVC(m, \tilde{Q}^{(m)}, L^{(m)})$ by using (3).
Step 6. If $PVC(m, \tilde{Q}^{(m)}, L^{(m)}) \leq PVC(m-1, \tilde{Q}^{(m-1)}, L^{(m-1)})$, then set $m = m + 1$ and repeat Steps 2-4; otherwise, proceed to Step 7.
Step 7. Set $PVC(\tilde{m}^*, \tilde{Q}^*, L^*) = PVC(m-1, \tilde{Q}^{(m-1)}, L^{(m-1)})$. Then, (m^*, Q^*, L^*) is the optimal solution.

V. NUMERICAL

To demonstrate the results of the proposed model, we consider an inventory system with a fuzzy interest rate per year. All data are based on Yang's research (2010). The data related to our proposed model are listed below: $D = 1000$ unit/year, $P = 1250$ unit/year, $A = \$25$/order, $S = \$500$/set-up, $CP = \$15$/unit, $CV = \$25$/unit, $r = 0.2$, $I = 0.5$, $k = 2.33$, $\sigma = 7$ unit/week, and the lead-time crashing costs $R(L) = 1000L - 3$. We fuzzy the interest rate $\tilde{i}$, subsequently performing a sensitive analysis for the Δ_1 and Δ_2 that affect the fuzzy number $\tilde{i}$. (Please see Table 1).

Table 1. Sensitive analysis of $\tilde{i}$.

Δ_1	Δ_2	$m*$	$Q*$	$L*$	Integrated PVC
0.1	0.1	11	77.66	8.25	4343.85
0.2	0.1	11	77.90	8.23	4542.72
0.3	0.1	11	78.13	8.23	4763.84
0.4	0.1	11	78.37	8.22	5011.12
0.1	0.2	11	77.42	8.26	4164.06
0.1	0.3	11	77.18	8.26	4000.704
0.1	0.4	11	76.94	8.27	3851.68
0.1	0.5	11	76.69	8.28	3715.17

*The minimum present value of the joint expected total relevant costs.
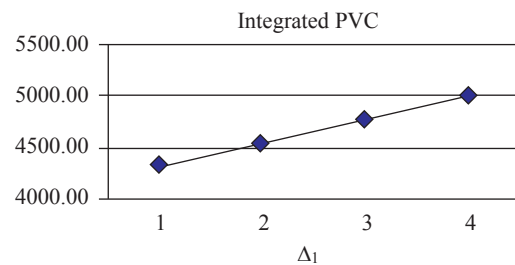
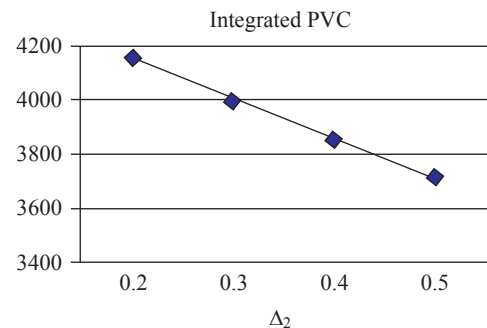**Fig. 1. Total expected joint costs by changes in Δ_1.**

Fig. 2. Total expected joint costs by changes in Δ_2.

When Δ_1 rises (decreasing the upper bound of the fuzzy i), the optimal order quantity of the buyer Q^* remains stable, but the integrated PVC increases dramatically. When Δ_2 increases (increasing the lower bound of the fuzzy i), the results show that the integrated PVC decreases. The results and graphs illustrate that the interest rate per year i is sensitive to the value of PVC. (Please see Figs. 1 and 2.).

According to the numerical example, we observe that the factor of the interest rate is sensitive to the integrated PVC. Basically, the lead-time reduction is insufficient yet substantial in the integrated fuzzy interest model. However, the total costs and integrated PVC are important, showing 4343.85-3715.17 in the entire integrated model.

Consequently, we note that the factor of interest rate is sensitive to the integrated PVC. Decision makers consider interest rates as part of professional decisions. In other words, we can

determine the importance of interest rates to decision makers within the entire supply chain system.

VI. CONCLUSION

In summary, this study incorporates time value into an integrated inventory model featuring variable lead times and considering polynomial-dependent crashing costs. In addition, the effect of the time value is one of the vital factors in the inventory system. Moreover, the fuzziness of the interest rate per year is investigated in this paper. Based on our example, the findings from our proposed model show that the interest rate per year i is sensitive to the value of *PVC*. When the upper bound of the fuzzy i rises, the optimal order quantity of buyer Q^* remains stable, even though the integrated *PVC* increases substantially. Moreover, when the lower bound of the fuzzy i increases, the integrated *PVC* decreases.

The interest rate must be considered by decision makers and managers. Thus, the views of decision makers strongly affect the entire model.

Although it is difficult to say that our model and example are superior to other models, we consider the factor of crashing costs; in addition, including the time value is more practical for real-world applications. We expect our research to provide an effective tool for both decision makers and related researchers in furthering research. Nevertheless, this paper is not without limitations. Future research is required, and we expect that our research will encourage future study in related areas that incorporate additional real-world situations. We will improve our future studies by reworking imperfect items, by determining the delay permissible in payment strategies, and by applying other fuzzy parameters to the proposed models.

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