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NONLINEAR HYDROSTATIC ANALYSIS OF THE FLOATING STRUCTURE CONSIDERING THE LARGE ANGLE OF INCLINATION

Kwang-Phil Park[1], Ju-Hwan Cha[2], and Namkug Ku[3]

Key words: nonlinear hydrostatic analysis, large inclination, floating structure.

ABSTRACT

When ships are flooded or equipped with a heavy object, the floating structures are excessively inclined, and the immersion, heel, and trim affecting the hydrostatic restoration performance are very large. In those cases, the assumption that the position and orientation are independent of each other is not valid. Therefore, nonlinear governing equations were formulated by sequential linearization to calculate the static equilibrium position of floating structures with excessive inclination. The equations are represented using a plane area, a primary moment, and a moment of inertia of the water plane area, and the immersion, heel, and trim are fully coupled. The position and orientation of the floating structure are obtained by iterative calculation. This paper describes the derivation procedure of the equations and proves the accuracy and efficiency of the equations by comparing with a commercial S/W, which uses a numerical method for determining hydrostatic restoring coefficients.

I. INTRODUCTION

Most problems related with the wave-induced motions or sea loads are solved assuming that the initial position of the floating structures is on an even keel. When the floating structures are flooded or the floating crane is lifting a heavy cargo as shown in Figs. 1(a) and (b), the assumption of the even keel condition is not valid anymore because the flooded floating structure or floating crane lifting a heavy cargo will be inclined at a large angle owing to the flooding or the weight of the cargo. This large angle of inclination results in motions that are quite different from those of the floating structure on an even keel. It is for this reason that we need to determine the initial position and orientation of the floating cranes with large angles of inclination.

(a) A flooded ship

(b) A floating crane lifting a heavy cargo

Fig. 1. Examples of large inclined floating structures.

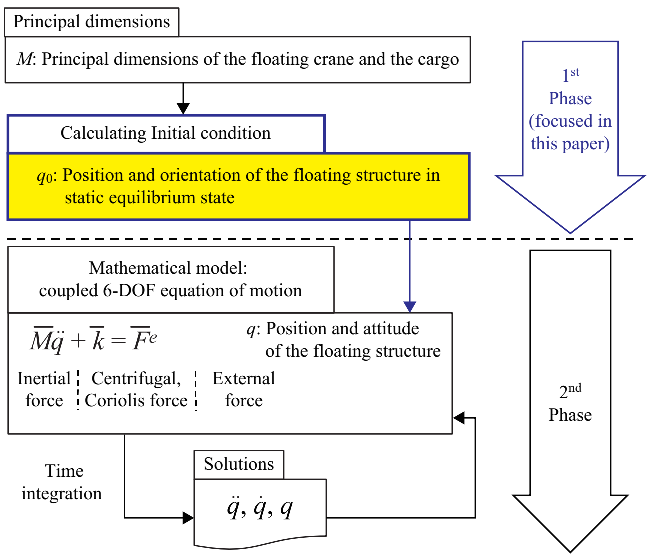

Fig. 2. Flow diagram of the dynamic response analysis of a floating structure (Cha et al., 2010a, 2010b).

Fig. 2 shows the flow diagram of the dynamic analysis program developed by the authors of this paper (Cha et al., 2010a, 2010b). The position and orientation, such as the immersion, heel, and trim, of the floating offshore structures in static equilibrium should be determined in advance before dynamic analysis is conducted. In the first phase of dynamic analysis, therefore,

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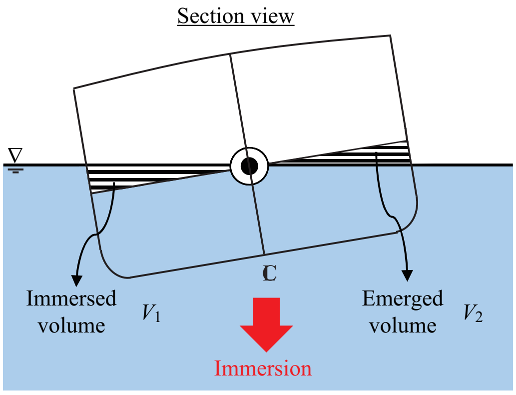

Fig. 3. An example of coupled heel and immersion of a floating structure in the case of a large inclination of the heel.

the initial position and orientation are determined by static analysis. Then, dynamic analysis of the floating offshore structures is performed by solving the equations of motion at every time step in the second phase. In this study, the focus is on the first phase of the static analysis.

When the immersion, heel, and trim are large, they are coupled to each other. For example, when the floating structure heels with a large angle and the deck are immersed, as shown in Fig. 3, the floating structure does not only heel, but also changes the draft, because the immersed volume and emerged volume are different. Therefore, we cannot assume that the immersion, heel, and trim are uncoupled, and we focus on this nonlinear hydrostatic analysis of the floating structure considering the large angle of inclination.

II. RELATED WORK

Singh and Sen (2007) considered the wave amplitude for calculating the hydrostatic force. However, they did not conduct hydrostatic analysis, as the initial position of the ship was assumed as an even keel state. Moreover, they did not consider the large angle of inclination.

Kim and Kim (2009) also considered the wave amplitude for calculating the hydrostatic force and large angle of inclination. However, they did not conduct hydrostatic analysis as the initial position of the ship was assumed to be in an even keel state.

Lee and Roh (2009), and EzCOMPART (2010), which is a commercial program for naval architectural calculation, can determine the static equilibrium state of the floating structure. This program calculates the static equilibrium state considering the hydrostatic forces. However, hydrostatic analysis is carried out using a purely numerical method that is slower than the analytic method.

Bronsart (2008) considered hydrostatic analysis. He derived the equations for calculating the changes in the hydrostatic forces caused by immersion, heel, and trim using an analytic method and used them to calculate the static equilibrium state, which was faster than carrying out hydrostatic analysis using a purely numerical method. However, he derived the changes in the hydrostatic forces when the ship was in an upright position and assumed that the heel and trim angles were small.

Lee (2015) considered the attitude of the floating structure. He applied the orifice flow model. However, he did not focus on the final attitude of the floating structure and computing time, but focused on the whole flooding procedure.

Bogner (2013) also considered the attitude of the floating structure. He used the lattice Boltzmann method to solve the problem. However, he focused on the dynamic motion of the floating structure rather than on the static equilibrium state and computing time.

Lee and Han (2002b) presented the optimal layout design method for the compartment. They applied the "Efficient Global-Local Hybrid Optimization Method (Lee and Cho, 2002a)" to find the optimal solution. Furthermore, they considered hydrostatic analysis, using a numerical method, and it is slower than the analytic method.

In this study, we consider hydrostatic analysis. The equations for calculating the changes in the hydrostatic forces caused by immersion, heel, and trim are derived using an analytic method, and it is used to calculate the static equilibrium state. In addition, the changes in the hydrostatic forces are derived when the ship is inclined with a large angle.

III. GOVERNING EQUATION

1. Derivation of Governing Equation

The governing equation for calculating the static equilibrium state can be derived from the equations of motion for one rigid body (Eqs. (1) and (2)).

$$m\,{}^{n}\ddot{\mathbf{r}}_{O/E} + m\,{}^{n}\mathbf{R}_{b}\,{}^{b}\dot{\tilde{\boldsymbol{\omega}}}_{b/n}\,{}^{b}\mathbf{r}_{G/O} + m\,{}^{n}\mathbf{R}_{b}\,{}^{b}\tilde{\boldsymbol{\omega}}_{b/n}\,{}^{b}\tilde{\boldsymbol{\omega}}_{b/n}\,{}^{b}\mathbf{r}_{G/O} = {}^{b}\mathbf{F}_{O} \tag{1}$$

$$m\,{}^{n}\tilde{\mathbf{r}}_{G/O}\,{}^{n}\mathbf{R}_{b}\,{}^{b}\ddot{\mathbf{r}}_{O/E} + {}^{b}\mathbf{I}_{O}\,{}^{b}\dot{\boldsymbol{\omega}}_{b/n} + {}^{b}\tilde{\boldsymbol{\omega}}_{b/n}\,{}^{b}\mathbf{I}_{O}\,{}^{b}\boldsymbol{\omega}_{b/n} = {}^{b}\boldsymbol{\tau}_{O} \tag{2}$$

where m is the mass of the body, ${}^{n}\mathbf{R}_{b}$ is the rotational transform matrix from the vector decomposed with the unit vectors of the b-frame to the vector decomposed with the unit vectors of the n-frame, ${}^{b}\boldsymbol{\omega}_{b/n}$ is the angular velocity vector of the b-frame, which is the body fixed frame, with respect to the n-frame, which is the inertial reference frame, ${}^{b}\mathbf{r}_{G/O}$ is the position vector of center of mass G with respect to point O, which is the origin of the body fixed frame, ${}^{b}\mathbf{I}_{O}$ is the inertia of mass of the body about the point O, ${}^{n}\mathbf{F}_{O}$ is a force vector exerted on point O, and ${}^{b}\boldsymbol{\tau}_{O}$ is the moment exerted on point O. The force and moment are divided into gravitational, hydrostatic, and hydrodynamic forces as follows.

$$ {}^{n}\mathbf{F}_O = \mathbf{F}_{Gravity} + \mathbf{F}_{Hydrostatic} + \mathbf{F}_{Hydodynamic} \tag{3} $$

$$ {}^{b}\boldsymbol{\tau}_O = \boldsymbol{\tau}_{Gravity} + \boldsymbol{\tau}_{Hydrostatic} + \boldsymbol{\tau}_{Hydodynamic} \tag{4} $$

As the velocity, acceleration, and hydrodynamic force are zero at static equilibrium, Eqs. (5) and (6) can be obtained from Eqs. (1)-(4).

$$ \mathbf{0} = \mathbf{F}_{Gravity} + \mathbf{F}_{Hydrostatic} \tag{5} $$

$$ \mathbf{0} = \boldsymbol{\tau}_{Gravity} + \boldsymbol{\tau}_{Hydrostatic} \tag{6} $$

The gravitational and hydrostatic forces are vertical, and the horizontal force and moment about the z-axis are zero. Therefore, we can obtain Eq. (7) from Eqs. (5) and (6) by decomposing the force and moment vectors.

$$ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} F_z\left(T^*, \phi^*, \theta^*\right) \\ \tau_T\left(T^*, \phi^*, \theta^*\right) \\ \tau_L\left(T^*, \phi^*, \theta^*\right) \end{bmatrix}, \tag{7} $$

where T is the horizontal displacement of the body, ϕ is the heel angle, θ is the trim angle, F_z is the resultant force along the z-axis, and τ_T and τ_L are the resultant moments about the x-axis and y-axis, respectively. The solutions of Eq. (13), T^*, ϕ^*, and θ^*, are the immersion, heel, and trim of the floating structures in the equilibrium state, respectively.

2. Linearization of the Governing Equation

As we discussed in the introduction, hydrostatic forces are nonlinear coupled functions of the immersion, heel, and trim of the floating body, and it is difficult to obtain an analytic solution for Eq. (7).

To find the solution for Eq. (7), the Taylor series expansion can be applied. The Taylor series is a representation of a nonlinear function that infinitely sums the terms calculated from the values of the function's derivatives at the instantaneous state. In an infinite sum series of the Taylor series expansion, only the first order is considered, and the series after the first series are neglected as it is assumed that the displacement is small. Therefore, after the linearization, the series simplifies to

$$ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} F_z\left(T, \phi, \theta\right) \\ \tau_T\left(T, \phi, \theta\right) \\ \tau_L\left(T, \phi, \theta\right) \end{bmatrix} + \mathbf{A}\left(T, \phi, \theta\right) \begin{bmatrix} \delta T \\ \delta\phi \\ \delta\theta \end{bmatrix} \tag{8} $$

where the elements of the matrix $\mathbf{A}\left(T, \phi, \theta\right)$ are

$$ \mathbf{A}\left(T, \phi, \theta\right) = \begin{bmatrix} \dfrac{\partial F_z}{\partial T} & \dfrac{\partial F_z}{\partial \phi} & \dfrac{\partial F_z}{\partial \theta} \\ \dfrac{\partial \tau_T}{\partial T} & \dfrac{\partial \tau_T}{\partial \phi} & \dfrac{\partial \tau_T}{\partial \theta} \\ \dfrac{\partial \tau_L}{\partial T} & \dfrac{\partial \tau_L}{\partial \phi} & \dfrac{\partial \tau_L}{\partial \theta} \end{bmatrix}_{T,\phi,\theta} \tag{9} $$

The matrix $\mathbf{A}(T, \phi, \theta)$ is called the hydrostatic restoring coefficients matrix. The elements of this matrix mean change in force and moment with respect to the change in the corresponding variables, and the equations for calculating the elements can be derived through partial derivation, which will be explored in chapter 4. Those equations are the functions of the immersion, heel, and trim of the floating body.

Because Eq. (8) is a linearized equation with an assumption of a small displacement, the equation can result in an inaccurate solution. Therefore, iterations are needed to obtain an accurate solution.

3. Iterative Calculation

The sequence of iterative calculations using Eq. (8) is as follows.

(1) The initial immersion $T^{(k)}$, heel $\phi^{(k)}$, and trim $\theta^{(k)}$ are given when $k = 0$.
(2) As we are given $T^{(k)}$, $\phi^{(k)}$, and $\theta^{(k)}$, calculate $F_z(T^{(k)}, \phi^{(k)}, \theta^{(k)})$, $\tau_T(T^{(k)}, \phi^{(k)}, \theta^{(k)})$, and $\tau_L(T^{(k)}, \phi^{(k)}, \theta^{(k)})$. To check whether these values allow a steady condition of the floating body, check if Eq. (7) is satisfied.
(3) When Eq. (7) is satisfied, T^*, ϕ^*, and θ^* are set as $T^{(k)}$, $\phi^{(k)}$, and $\theta^{(k)}$, respectively, and the iteration is completed. If Eq. (7) is not satisfied go to next step.
(4) Determine the hydrostatic restoring coefficients matrix $\mathbf{A}(T^{(k)}, \phi^{(k)}, \theta^{(k)})$ using Eq. (9).
(5) Determine $\delta T^{(k)}$, $\delta\phi^{(k)}$, and $\delta\theta^{(k)}$ using Eq. (8).
(6) $T^{(k+1)}$, $\phi^{(k+1)}$, and $\theta^{(k+1)}$ are replaced with $T^{(k)} + \delta T^{(k)}$, $\phi^{(k)} + \delta\phi^{(k)}$, and $\theta^{(k)} + \delta\theta^{(k)}$ respectively. We set $k = k + 1$, and go back to step (2).

IV. DERIVATION OF HYDROSTATIC RESTORING COEFFICIENTS MATRIX FOR THE FLOATING STRUCTURE IN THE EVEN KEEL STATE

For ease of understanding, the hydrostatic restoring coefficients matrix will be derived when the floating structure is in an even keel state. The elements of the restoring coefficient matrix $\mathbf{A}(T, \phi, \theta)$ are the change in F_z, τ_T, and τ_L with respect to changes in corresponding variables T, ϕ, and θ, and are derived using partial derivatives. In this chapter, the steps for deriving each element in the matrix will be explored.

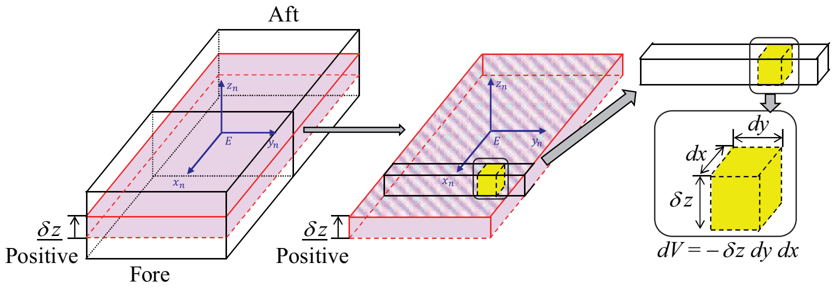

Fig. 4. Infinitesimal volume for floating structures in immersion.

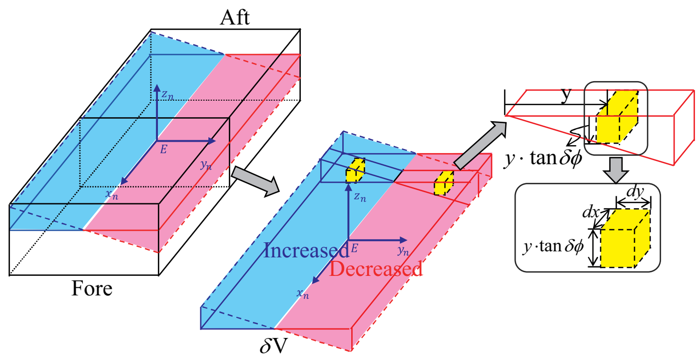

Fig. 5. Infinitesimal volume for floating structures in the heel.

1. Change in Displaced Volume with Respect to Immersion, Heel, and Trim

It is known that the hydrostatic force, especially the buoyant force, acting on a body is proportional to the weight of the fluid displaced by the body, which is Archimedes' principle, and the weight is proportional to the volume in general. Therefore, the change in displaced volume with respect to the immersion, heel, and trim will be discussed.

To derive the change in volume δV with respect to immersion, an infinitesimal volume dV is taken in the changing volume and is integrated along the x and y direction as seen in Fig. 4 and Eq. (10).

$$\delta V = \iint dV \tag{10}$$

If the floating structure rises by an amount δz, then the submerged volume will decrease; thus, the infinitesimal volume dV can be expressed with a negative sign as $dV = -\delta z\ dy\ dx$. Therefore, the change in volume δV is

$$\begin{aligned} \delta V &= -\iint \delta z\ dy\ dx \\ &= -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{y_{port}} \delta z\ dy\ dx \end{aligned} \tag{11}$$

To derive the change in volume δV with respect to the heel, an infinitesimal volume dV is integrated along the x and y direction as seen in Fig. 5 and Eq. (12).

Similar to the case of immersion, an infinitesimal volume dV can be expressed with the negative sign as $dV = -y \cdot \tan\delta\phi\ dy\ dx$. Therefore, the change in volume δV is

$$\begin{aligned} \delta V &= -\iint y \cdot \tan\delta\phi\ dy\ dx \\ &= -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{y_{port}} y \cdot \tan\delta\phi\ dy\ dx \end{aligned} \tag{12}$$

To derive the change in volume δV with respect to the trim, an infinitesimal volume dV is integrated along the x and y direction as seen in Fig. 6 and Eq. (13).

The infinitesimal volume dV can be expressed as $dV = x \cdot \tan\delta\theta\ dx\ dy$. Therefore, the change in volume δV is

$$\begin{aligned} \delta V &= \iint x \cdot \tan\delta\theta\ dx\ dy \\ &= \int_{y_{star}}^{y_{port}} \int_{x_{aft}}^{x_{fore}} x \cdot \tan\delta\theta\ dx\ dy \end{aligned} \tag{13}$$

2. Partial Derivatives of Restoring Force and Moment with Respect to Immersion, Heel, and Trim

The elements of the restoring coefficient matrix $\mathbf{A}(T, \phi, \theta)$ are derived through partial derivatives of F_z, τ_T, and τ_L with respect to T, ϕ, and θ.

(1) Change in buoyant force with respect to immersion, heel, and trim

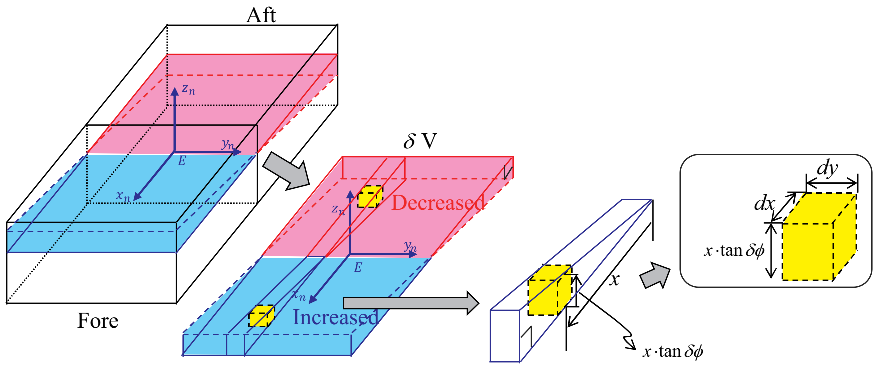

Fig. 6. Infinitesimal volume for floating structures in the trim.

In the previous section, we derived the change in volume in terms of immersion, as seen in Eq. (11). Using the equation, therefore, the change in buoyant force can be equated in the following way.

$$\begin{aligned}\delta F_B &= \iint dF_B = \rho g \iint dV = -\rho g \cdot \iint \delta z \; dx \; dy \\ &= -\rho g \delta z \cdot \iint \; dx \; dy = -\rho g \delta z \cdot A_{WP}\end{aligned} \tag{14}$$

where A_{WP} is the waterplane area.

Dividing both sides of Eq. (14) with ∂z, we can obtain the partial derivative of a buoyant force with respect to the immersion, which is the element (1, 1) of the restoring coefficient matrix.

$$\frac{\partial F_B}{\partial z} = -\rho g \cdot A_{WP} \tag{15}$$

Similarly, we derived the change in volume in terms of the heel and trim, as seen in Eqs. (12) and (13). Using the equation, therefore, the change in buoyant force can be equated in the following way.

$$\begin{aligned}\delta F_B &= -\rho g \iint y \cdot \tan \delta\phi \; dx \; dy \\ &= -\rho g \tan \delta\phi \iint y \cdot \; dx \; dy \\ &= -\rho g \cdot \tan \delta\phi \cdot T_{WP}\end{aligned} \tag{16}$$

$$\begin{aligned}\delta F_B &= \rho g \iint x \cdot \tan \delta\theta \; dx \; dy \\ &= \rho g \tan \delta\theta \iint x \cdot \; dx \; dy \\ &= \rho g \cdot \tan \delta\theta \cdot L_{WP}\end{aligned} \tag{17}$$

where T_{WP} is the transverse moment of the waterplane area, and L_{WP} is the longitudinal moment of waterplane area.

Dividing both sides of Eqs. (16) and (17) with $\partial\phi$ and $\partial\theta$, respectively, we can obtain the partial derivatives of the buoyant force with respect to the heel and trim, which are elements (1, 2) and (1,3) of the restoring coefficient matrix.

$$\frac{\partial F_B}{\partial \phi} = -\rho g \cdot T_{WP} \tag{18}$$

$$\frac{\partial F_B}{\partial \theta} = \rho g \cdot L_{WP} \tag{19}$$

(2) Change in transverse restoring moment with respect to immersion, heel, and trim

The change in the transverse restoring moment is equal to the moment arm y times the change in the restoring force, and the force can be expressed with the change in volume. Therefore, the change in moment is

$$\begin{aligned}\delta\tau_T &= \iint d\tau_T = \iint y \cdot dF \\ &= \iint y \cdot \rho g \cdot dV\end{aligned} \tag{20}$$

In the previous section, we derived the change in volume in terms of immersion and trim, as seen in Eqs. (11) and (13). Using the equations, therefore, the change in buoyant force can be equated in the following way.

$$\begin{aligned}\delta\tau_T &= -\iint y \cdot \rho g \cdot \delta z \cdot dx \; dy \\ &= -\rho g \delta z \cdot \iint y \cdot dx \; dy \\ &= -\rho g \delta z \cdot T_{WP}\end{aligned} \tag{21}$$

$$\begin{aligned}\delta\tau_T &= \iint y \cdot \rho g \cdot x \cdot \tan \delta\theta \cdot dx \; dy \\ &= \rho g \cdot \tan \delta\theta \cdot \iint xy \cdot dx \; dy \\ &= \rho g \cdot \tan \delta\theta \cdot I_P \\ &\approx \rho g \cdot \delta\theta \cdot I_P\end{aligned} \tag{22}$$

where T_{WP} is the transverse moment of the waterplane area, and I_P is a product of inertia of the waterplane area.

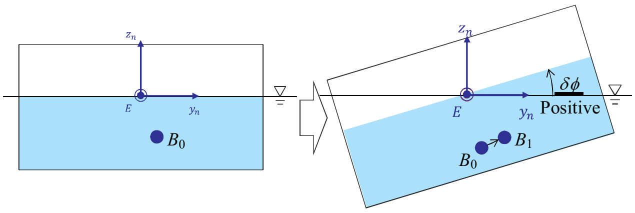

Fig. 7. The change in the moment of the current displaced volume due to the floating structures' heel.

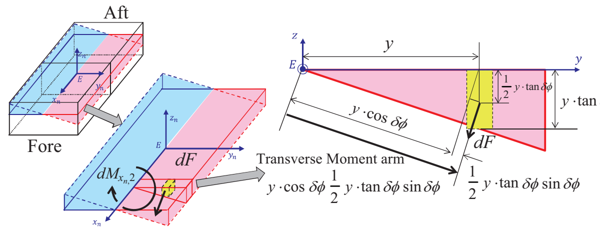

Fig. 9. The change in the moment due to change in the displaced volume and its moment arm.

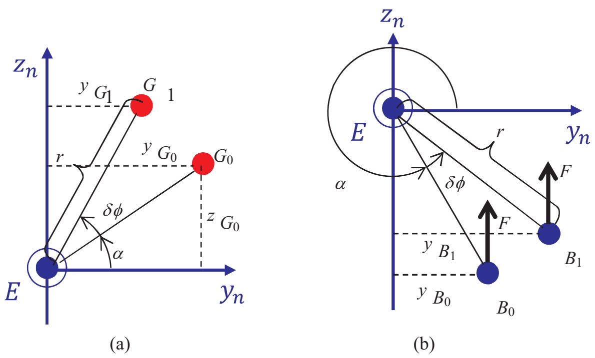

Fig. 8. The change in the moment arm of (a) gravity and (b) buoyancy force.

Dividing both sides of Eqs. (21) and (22) with ∂z and $\partial\theta$, respectively, we can obtain the partial derivatives of the transverse restoring moment with respect to the immersion and trim, which are elements (2, 1) and (2, 3) of the restoring coefficient matrix.

$$\frac{\partial \tau_T}{\partial z} = -\rho g \cdot T_{WP} \tag{23}$$

$$\frac{\partial \tau_T}{\partial \theta} = \rho g \cdot I_P \tag{24}$$

To find the change in the transverse restoring moment due to the restoring force with respect to the heel, it is important to comprehend that it is made up of two different components: one is the change in the moment of the current displaced volume, $\delta\tau_{T,1}$, due to the change in the moment arm, and the other is the change in the displaced volume $\delta\tau_{T,2}$.

The change in the moment of the current displaced volume is caused by the shift in gravity and buoyancy force owing to the floating structures' rotation as seen in Fig. 7.

The moment arm of the gravity and buoyancy force can be obtained from a given position in terms of the heeling angle as seen in Fig. 8.

The changed moment arm of gravity y_{G_1} and moment arm of buoyancy y_{B_1} are as follows.

$$\begin{aligned} y_{G_1} &= r\cos(\alpha+\delta\phi) \\ &= r(\cos\alpha\cdot\cos\delta\phi - \sin\alpha\cdot\sin\delta\phi) \\ &= r\cos\alpha\cdot\cos\delta\phi - r\sin\alpha\cdot\sin\delta\phi \\ &= y_{G_0}\cdot\cos\delta\phi - z_{G_0}\cdot\sin\delta\phi \end{aligned} \tag{25}$$

$$\begin{aligned} y_{B_1} &= r\cos(\alpha+\delta\phi) \\ &= r(\cos\alpha\cdot\cos\delta\phi - \sin\alpha\cdot\sin\delta\phi) \\ &= r\cos\alpha\cdot\cos\delta\phi - r\sin\alpha\cdot\sin\delta\phi \\ &= y_{B_0}\cdot\cos\delta\phi - z_{B_0}\cdot\sin\delta\phi \end{aligned} \tag{26}$$

With the obtained equation for y_{B_1}, the change in the moment of the current displaced volume can be found as follows.

$$\begin{aligned} \delta\tau_{T,1} &= \tau_{T,B_1} - \tau_{T,B_0} \\ &= y_{B_1}\cdot F - y_{B_0}\cdot F \\ &= \left\{ y_{B_0}(\cos\delta\phi - 1) - z_{B_0}\sin\delta\phi \right\}\cdot\rho g V \end{aligned} \tag{27}$$

Next, the moment due to a change in the displaced volume is found by calculating the sum of the moment caused by the submerged and immersed volume. The calculation can be processed by taking an infinitesimal volume and integrating it through these volumes as seen below.

The transverse moment arm is as shown in Fig. 9. The moment then can be equated as follows.

$$\begin{aligned} \delta\tau_{T,2} &= \iint d\tau_{T,2} \\ &= \iint\left(y\cdot\cos\delta\phi + \frac{1}{2} y\cdot\tan\delta\phi\sin\delta\phi \right)\cdot dF \\ &= \rho g \iint\left(y\cdot\cos\delta\phi + \frac{1}{2} y\cdot\tan\delta\phi\sin\delta\phi \right)\cdot dV \end{aligned} \tag{28}$$

In the previous section, we derived the change in volume in terms of the heel, as seen in Eq. (12). Using the equations, therefore, $\delta\tau_{T,2}$ can be equated in the following way.

$$\begin{aligned}\delta\tau_{T,2} &= \rho g \iint \left(y \cdot \cos\delta\phi + \frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi \right) \cdot dV \\ &= -\rho g \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2 \delta\phi \right) \iint y^2 \; dx \; dy \right\} \\ &= -\rho g \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2 \delta\phi \right) I_T \right\}\end{aligned} \quad (29)$$

where I_T is transverse moment of inertia of waterplane area.

As we obtained both $\delta\tau_{T,1}$ and $\delta\tau_{T,2}$, we can now add both moments to find the change in the transverse moment due to the buoyant force with respect to the heel.

$$\begin{aligned}\delta\tau_T &= \delta\tau_{T,1} + \delta\tau_{T,2} \\ &= \left\{ y_{B_0} (\cos\delta\phi - 1) - z_{B_0} \sin\delta\phi \right\} \cdot \rho g V \\ &\quad - \rho g \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2 \delta\phi \right) I_T \right\} \\ &= \rho g V \left[\left\{ y_B (\cos\delta\phi - 1) - z_B \sin\delta\phi \right\} \right. \\ &\quad \left. - \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2 \delta\phi \right) \frac{I_T}{V} \right\} \right]\end{aligned} \quad (30)$$

As discussed earlier, the angle is assumed to be considerably less than 1, and thus, the trigonometry function is simplified.

$$\begin{aligned}\delta\tau_T &= \rho g V \left[\left\{ y_B (\cos\delta\phi - 1) - z_B \sin\delta\phi \right\} - \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2 \delta\phi \right) \frac{I_T}{V} \right\} \right] \\ &= \rho g V \left[\left\{ y_B (1 - 1) - z_B \delta\phi \right\} - \left\{ \delta\phi (1 + 0) \frac{I_T}{V} \right\} \right] \\ &= \rho g V \left[-z_B \delta\phi - \delta\phi \frac{I_T}{V} \right]\end{aligned} \quad (31)$$

Dividing both sides of Eq. (31) with $\partial\phi$, we can obtain the partial derivative of the transverse restoring moment with respect to the heel, which is element (2, 2) of the restoring coefficient matrix.

$$\frac{\partial\tau_T}{\partial\phi} = -\rho g V \left[z_B + \frac{I_T}{V} \right] \quad (32)$$

However, Eq. (32) does not include the effect due to the change in the moment arm of the gravity. Considering gravity, Eq. (33) was obtained.

$$\frac{\partial\tau_T}{\partial\phi} = mgz_G - \rho g V \left[z_B + \frac{I_T}{V} \right] \quad (33)$$

where m is the mass of the floating structures.

(3) Change in longitudinal restoring moment with respect to immersion, heel, and trim

In the hydrostatic restoring coefficients matrix $\mathbf{A}(T, \phi, \theta)$, element (1, 1), (1, 2), (1, 3) respectively represent the partial derivatives of the restoring force along the z-axis with respect to immersion, heel, and trim. Element (2, 1), (2, 2), (2, 3) respectively represents the partial derivatives of transverse restoring moment with respect to immersion, heel, and trim. The derivation of the first and second row is explained in the previous chapter. The third row represents the partial derivatives of the longitudinal restoring moment with respect to immersion, heel, and trim. The elements in the third row correspond to the partial derivatives of the transverse restoring moment with respect to immersion, trim, and heel, respectively, and can be obtained in the same manner.

Therefore, the elements (1, 1)-(3, 3) of the hydrostatic restoring coefficients matrix $\mathbf{A}(T, \phi, \theta)$ are as follows.

$$\frac{\partial F_z}{\partial T} = -\rho g \cdot A_{WP}(T, \phi, \theta) \quad (34)$$

$$\frac{\partial F_z}{\partial \phi} = -\rho g T_{WP}(T, \phi, \theta) \quad (35)$$

$$\frac{\partial F_z}{\partial \theta} = \rho g \cdot L_{WP}(T, \phi, \theta) \quad (36)$$

$$\frac{\partial \tau_T}{\partial T} = -\rho g \cdot T_{WP}(T, \phi, \theta) \quad (37)$$

$$\begin{aligned}\frac{\partial \tau_T}{\partial \phi} &= mg \cdot z_G(T, \phi, \theta) - \rho g \cdot V(T, \phi, \theta) \cdot z_B(T, \phi, \theta) \\ &\quad - \rho g \cdot I_T(T, \phi, \theta)\end{aligned} \quad (38)$$

$$\frac{\partial \tau_T}{\partial \theta} = \rho g \cdot I_P(T, \phi, \theta) \quad (39)$$

$$\frac{\partial \tau_L}{\partial T} = \rho g \cdot L_{WP}(T, \phi, \theta) \quad (40)$$

$$\frac{\partial \tau_L}{\partial \phi} = \rho g \cdot I_P(T, \phi, \theta) \quad (41)$$

$$\begin{aligned}\frac{\partial \tau_L}{\partial \theta} &= mg \cdot z_G(T, \phi, \theta) - \rho g \cdot V(T, \phi, \theta) \cdot z_B(T, \phi, \theta) \\ &\quad - \rho g \cdot I_L(T, \phi, \theta)\end{aligned} \quad (42)$$

For ease of understanding, a small inclination was used for the explanation. However, the derived equations are all valid for

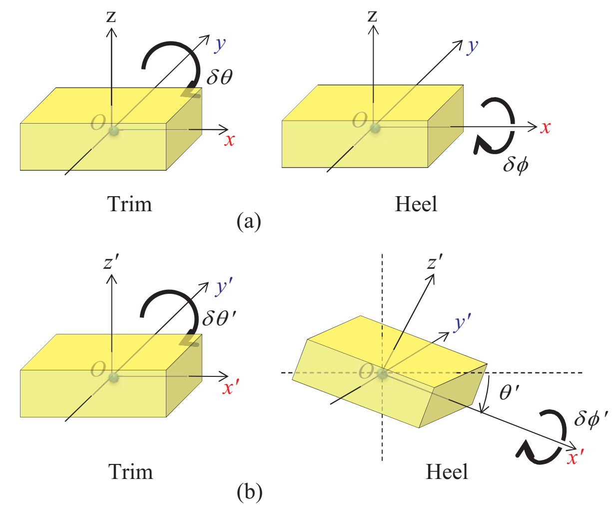

Fig. 10. Trim and heel of the floating structure about (a) water surface frame and (b) body fixed frame.

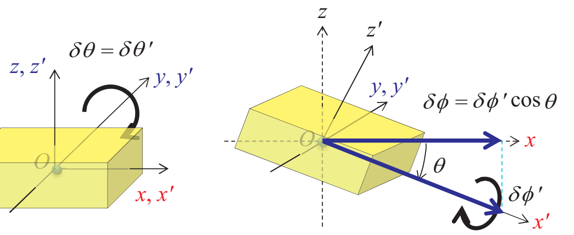

Fig. 11. The relationship between the rotation about the water surface fixed frame and body fixed frame.

any attitude of the floating structure, as the restoring coefficients matrix A is composed of the instantaneous water surface properties, which can be calculated in any attitude of the ship.

V. DERIVATION OF THE HYDROSTATIC RESTORING COEFFICIENTS MATRIX FOR THE FLOATING STRUCTURE WITH THE LARGE ANGLE OF INCLINATION

In chapter 5, the hydrostatic restoring coefficients matrix was derived when the floating structure was in an even keel state. In other words, the elements of the matrix were derived assuming the floating structure's trims and heels about the *y*- and *x*-axis of the water surface fixed frame, as shown in Fig. 10(a).

However, the floating structure may not be in an even keel state. To define the inclination of the floating structure with a large angle, body fixed local coordinates, named *x'y'z'*-coordinates, are used. When the orientation of the floating structure is defined in terms of local coordinates, the inclined floating structure trims and heels not about *y*- and *x*-axis, but about *y'*- and *x'*-axis of the body fixed frame as shown in Fig. 10(b). Therefore, the hydrostatic restoring coefficients matrix should be transformed. The relationship between the rotation about the water surface fixed frame and body fixed frame is shown in Fig. 11 and Eq. (43).

$$\begin{bmatrix} \delta\phi \\ \delta\theta \end{bmatrix} = \begin{bmatrix} \delta\phi' \cos\theta \\ \delta\theta' \end{bmatrix} \tag{43}$$

Including the immersion, Eq. (43) can be expressed as a matrix multiplication as follows.

$$\begin{bmatrix} \delta T \\ \delta\phi \\ \delta\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta T' \\ \delta\phi' \\ \delta\theta' \end{bmatrix} \tag{44}$$

Substituting Eq. (44) into the second term of the right-hand side of Eq. (8), the hydrostatic restoring coefficients matrix can be finally derived as follows.

$$\begin{aligned} \begin{bmatrix} \frac{\partial F_z}{\partial T} & \frac{\partial F_z}{\partial \phi} & \frac{\partial F_z}{\partial \theta} \\ \frac{\partial \tau_T}{\partial T} & \frac{\partial \tau_T}{\partial \phi} & \frac{\partial \tau_T}{\partial \theta} \\ \frac{\partial \tau_L}{\partial T} & \frac{\partial \tau_L}{\partial \phi} & \frac{\partial \tau_L}{\partial \theta} \end{bmatrix} \begin{bmatrix} \delta T \\ \delta\phi \\ \delta\theta \end{bmatrix} &= \begin{bmatrix} \frac{\partial F_z}{\partial T} & \frac{\partial F_z}{\partial \phi} & \frac{\partial F_z}{\partial \theta} \\ \frac{\partial \tau_T}{\partial T} & \frac{\partial \tau_T}{\partial \phi} & \frac{\partial \tau_T}{\partial \theta} \\ \frac{\partial \tau_L}{\partial T} & \frac{\partial \tau_L}{\partial \phi} & \frac{\partial \tau_L}{\partial \theta} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta T' \\ \delta\phi' \\ \delta\theta' \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial F_z}{\partial T} & \frac{\partial F_z}{\partial \phi}\cos\theta & \frac{\partial F_z}{\partial \theta} \\ \frac{\partial \tau_T}{\partial T} & \frac{\partial \tau_T}{\partial \phi}\cos\theta & \frac{\partial \tau_T}{\partial \theta} \\ \frac{\partial \tau_L}{\partial T} & \frac{\partial \tau_L}{\partial \phi}\cos\theta & \frac{\partial \tau_L}{\partial \theta} \end{bmatrix} \begin{bmatrix} \delta T' \\ \delta\phi' \\ \delta\theta' \end{bmatrix} \end{aligned} \tag{45}$$

where the partial derivatives are defined in Eqs. (34)-(42).

VI. MEANING OF THE LINEARIZED GOVERNING EQUATION FOR STATIC ANALYSIS

Eq. (8) is the linearized governing equation of motion for the floating structure, and the equation is rewritten as follows.

$$-\begin{bmatrix} F_z(T,\phi,\theta) \\ \tau_T(T,\phi,\theta) \\ \tau_L(T,\phi,\theta) \end{bmatrix} = \mathbf{A}(T,\phi,\theta) \begin{bmatrix} \delta T \\ \delta\phi \\ \delta\theta \end{bmatrix} \tag{46}$$

F_z, τ_T, and τ_L in Eq. (46) mean the force along the *z*-axis, the moment about the *x*-axis, and the moment about the *y*-axis, respectively. δT, $\delta\phi$, and $\delta\theta$ in Eq. (46) mean the infinitesimal immersion along the *z*-axis, the infinitesimal heel about the *x*-axis, and the infinitesimal trim about the *y*-axis, respectively. As mentioned in section 5, however, we want to calculate the

Table 1. Parameters of 4800TEU container carrier for case study.

Parameters	Values	Unit
Length	274.7	(m)
Breadth	32.2	(m)
Depth	27.1	(m)
Draft	12	(m)
COG	(-40.0, 5.0, 9.0)	((m), (m), (m)) from midship, centerline, and keel
Load	110,000	(Mg)

Table 2. Computer specification for case study.

Components	Specification
CPU	Intel Core i7 (3.50 GHz)
RAM	8 GB
OS	Windows 8 64 bit

following using Eq. (46): the infinitesimal immersion $\delta T'$, heel $\delta\phi'$, and trim $\delta\theta'$, which correspond to the z'-axis, x'-axis, and y'-axis; herein, $\delta\phi'$ and $\delta\theta'$ are Euler angles. Therefore, we derive Eq. (44) using the relationship between the rotation about the water surface fixed frame and body fixed frame, and substituting Eq. (44) into Eq. (46), we obtain Eq. (47).

$$
\begin{aligned}
-\begin{bmatrix} F_z(T,\phi,\theta) \\ \tau_T(T,\phi,\theta) \\ \tau_L(T,\phi,\theta) \end{bmatrix} &= \mathbf{A}(T,\phi,\theta)\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \delta T' \\ \delta\phi' \\ \delta\theta' \end{bmatrix} \\
&= \mathbf{A}'(T,\phi,\theta)\begin{bmatrix} \delta T' \\ \delta\phi' \\ \delta\theta' \end{bmatrix}
\end{aligned} \tag{47}
$$

Using the hydrostatic restoring coefficients matrix A', the position and orientation of the inclined floating structure in the static equilibrium state can be determined. Then, this position and orientation are utilized as the initial condition for the second phase of the dynamic analysis. To calculate the elements of the hydrostatic restoring coefficients matrix, polyhedron integration, which approximates a hull surface to a set of polyhedrons, is used. This method gives more accurate volume values because the B-spline curve is used for defining the hull surface (Lee et al., 2007; Lee et al., 2010).

VII. CASE STUDY

To prove the efficiency of the equations derived in this paper, hydrostatic analysis was performed for a 4800TEU Container Carrier, and main dimensions of the container carrier is shown in table 1. In fact, although the title of this paper includes "floating crane," it is easy to calculate the static equilibrium of the barge of a floating crane. Therefore, a container carrier was chosen as an example, and for that example, it is more difficult to calculate the equilibrium position than for the barge, since the hull shape is more complicated. The hull surface geometry is defined by B-Spline curves in 30 section points in the longitudinal direction. The parameters used in the calculation are tabulated as follows.

The specification of the computer in which the case study was performed is described in Table 2.

Table 3. Calculation results of hydrostatic analysis.

Position and Orientation	Results
Immersion	2.75 (m)
Heel	36.37 (Degree)
Trim	4.94 (Degree)

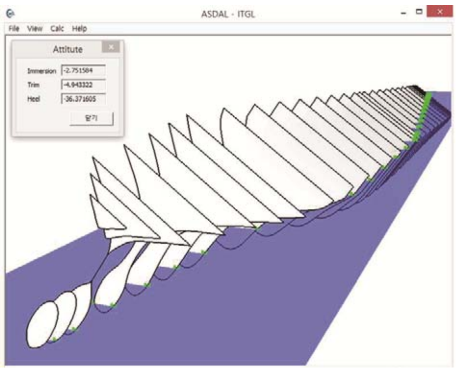

Fig. 12. The position and attitude of the 4800TEU Container Carrier in static equilibrium state.

1. Static Analysis Results

The static equilibrium position and orientation for the case study are calculated as shown in Fig. 12. The total calculation time was approximately 0.987 s after eight iterations, and the results are tabulated in Table 3.

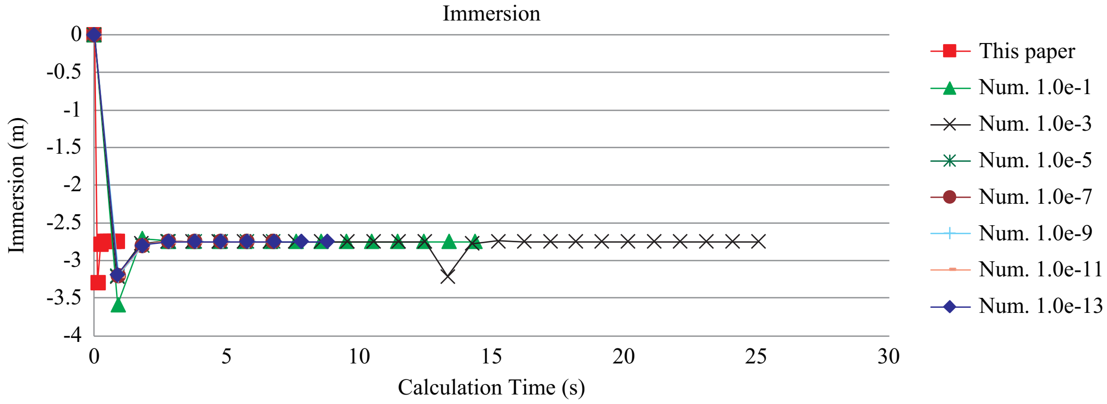

Fig. 13. **Comparison of calculation time obtained in this paper and that of a the related study (EzCompart, 2010), the immersion of the 4800TEU Container Carrier.**

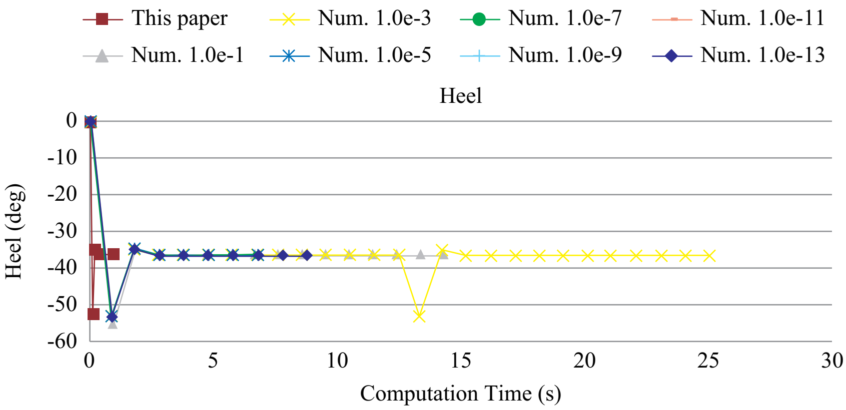

Fig. 14. **Comparison of the computation time obtained in this paper and that of a the related study (EzCompart, 2010), the heel of the 4800TEU Container Carrier.**

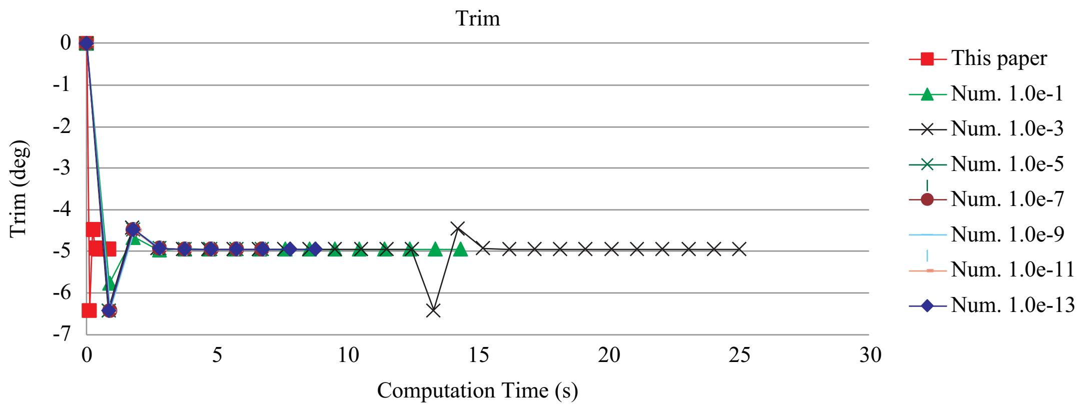

Fig. 15. **Comparison of the computation time obtained in this paper and that of a related study (EzCompart, 2010), the trim of the 4800TEU Container Carrier.**

2. Calculation Performance Analysis

The calculation performance of the equations was analyzed by comparison with a commercial program (EzCOMPART, 2010), which obtains the partial derivatives of the hydrostatic force using a numerical method including complex mesh generation algorithm for calculating the immersed volume under waterplane surface (Otomo at al., 2014). The numerical calculation was tested by varying the incremental value by 10^{-1}, 10^{-3}, 10^{-5}, 10^{-7}, 10^{-9}, 10^{-11}, and 10^{-13}. The variation in calculation time was observed from 6.703 to 25.049 s.

The calculation time using the equation derived in this study was approximately 1/7 to 1/25 of that of the numerical method for the same results, as shown in Figs. 13-15.

The method proposed in this paper did not cause any numerical problem as the hydrostatic restoring coefficients are derived analytically. However, the required iteration numbers in the compared program were different according to the chosen incremental value, and the calculation was not converged when the incremental values were less than 10^{-13} or larger than 10^{-3}. From the above graph, it can be seen that the calculated final values of the immersion, heel, and trim using the proposed algorithm are the same as those obtained using a commercial program. From this, it was confirmed that the exactness of the proposed algorithm is the same as that of the commercial program.

VIII. CONCLUSIONS

In this paper, nonlinear governing equations were derived using sequential linearization to calculate the static equilibrium position of floating structures with excessive inclination. The immersion, heel, and trim were fully coupled in the equations, and a plane area, primary moment, and the moment of inertia of the water plane area were also included. To verify the equations, static analysis for a 4800TEU Container Carrier was performed. The case study result shows that the calculation with the equations is more efficient than other algorithms that use numerical methods for determining partial derivatives of the hydrostatic force. In future, the design of a compartment will be considered for calculation of the hydrostatic equilibrium by applying the optimization algorithm (Lee et al., 2002a, Yazdani et al., 2016).

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