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OPTIMAL IMPERFECT PREVENTIVE MAINTENANCE LOGISTIC INVENTORY MODEL WITH BACKORDER AND MINIMAL REPAIR

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Key words: integrated inventory model, preventive maintenance, rework, backorder.

ABSTRACT

This study developed and evaluated an integrated inventory model incorporating production programs and maintenance to model an imperfect process of a deteriorating production system in firm's activities of inbound logistics and production. Two preventive maintenance activities are performed during each production run period: perfect preventive maintenance and imperfect preventive maintenance. The perfect preventive maintenance's probability depends on the number of imperfect maintenance operations performed resulting from the last renewal cycle. The occurrence of a failure causes defective products which have a certain number of the ability of rework and not to be rework, and those cannot rework will lead to shortages. Experiments showed that the model optimizes the number of shipments and costs. The model is applied in various special cases to evaluate failure rate, including Weibull, geometric and learning effect. Finally a numerical example is presented.

I. INTRODUCTION

To determine what activities enhance firm performance and customer value, Porter (1985) suggested a value chain analysis that groups firm activities into primary activities and support activities. Primary activities that directly create value for customers include inbound logistics, operation/production, outbound logistics, marketing/sales and service. Support activities, including procurement, technology development, human resource management and infrastructure, enhance primary activities for better coordination and process improvement. However, to maximize the customer value and minimize the firm cost, cooperation with the activities of upstream vendor and downstream retailer is required (Porter, 1985). That is, a supply chain must be constructed.

A supply chain is a complex system that consists of component/raw material suppliers, manufactures, wholesalers/distributors and retailers involved directly or indirectly to fulfill customer requests (Chopra and Meindl, 2004). In today's business, a close cooperation with each supply chain member is necessary to decrease cost, especially the joint total inventory cost. Just-in-time (JIT) manufacturing is a useful technique for achieving cooperation target. A JIT system is characterized by high quality, small lot sizes, frequent delivery, short lead time, and close supplier ties.

In the context of supply chain management, Bowersox et al. (2002) reported that managers can minimize total costs by building integrated logistics models that include order processing, inventory, transportation, warehousing, materials handling, packaging and facility network. This study investigated inventory cost in activities of inbound logistics and production in a supply chain. The inbound logistic activities begin from moving raw materials from venders to firm's storage place and then move again to the plant for manufacturing. Accordingly, inventory costs include inbound logistic costs of material handling, transportation-in, order processing, storage space, and carrying in addition to the cost of purchasing. Coyle et al. (2003) indicated that a classic interface area between inbound logistics and production relates to the length of the production run, which in turn decides the production lot quantity in each run. Lambert and Stock (1999) showed that the costs of production lot quantities include the costs of setup time, inspection, capacity lost due to changeover as well as materials handling, scheduling, and expediting. Therefore, the cost of production lot quantities will become part of inventory cost.

Although high product quality is the main concern of customers, the literature on the Integrated Inventory Model usually assume that all products produced by the vender have perfect qualities. However, failure process often occurs in every workplace. Accordingly, it is realistic to assume that production sometimes is imperfect. Such a production process is called imperfect production.

A stable production quantity requires good condition of the

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whole production system. For a good production system, the manufacturer must perform preventive maintenance strategy. In reliability engineering, the optimal determination of preventive maintenance (PM) strategy is important because it can slow degradation of the system during operation and extend the system lifetime.

Unexpected breakdown of production equipment is inevitable. Following each failure, one of two breakdown policies is taken: (1) perform a major repair or (2) take a minimal repair, considering minimization of maintenance cost. The major repairs reset the system failure intensity and are very expensive. Therefore, the main goals should be performing minimal repairs and restoring the system to operational status.

We divided the literature review into three parts: (1) JIT system, (2) Integrated inventory and (3) Imperfect production and preventive maintenance policy.

1. JIT System

In the highly competitive globalized environment of today, supply members with a quick customer response are likely to gain market share. Accordingly, supply chain members attempt to manage their supply chain effectively. A JIT system is an effective way to achieve this target (Hahn et al., 1983). With a close ties relationship with suppliers, Banerjee (1986) proposed a model which incorporated with JIT purchasing and JIT manufacturing and found that a joint integrated inventory replenishment policy had significantly higher efficiency compared to independently derived policies for a buyer and a supplier. Martinich (1997) also described the substantial benefits of establishing a long-term sole-supplier relationship with a supplier. Ha and Kim (1997) addressed the necessity of integration between buyer and supplier for effective implementation of the JIT system. They developed an integrated lot-splitting model to facilitate multiple shipments in small lots. Comparisons with the existing approach in a simple JIT environment showed that the integrated approach can reduce the total cost for the vendor and the buyer over the existing approaches. Khan and Sarker (2002) proposed a two-stage integrated inventory system to incorporate the JIT concept in the conventional joint batch-sizing problem. Pan and Yang (2002) presented an integrated model with controllable lead time in a JIT environment. Yang (2007) proposed a single buyer and a single vendor integrated inventory model order policy and used fuzzy theory to forecast productivity and demand.

2. Integrated Inventory

The advantages of the integrated inventory model include improved quality, lowered inventory cost, technology sharing and reduction of lead time. Harris (1913) proposed the Economic Order Quantity (EOQ) model to minimize total inventory cost, including purchasing cost, carrying cost, ordering cost and stock-out cost. Many researchers then extended this model to fit in with the conditions in the actual business environment. Correspondingly, this paper will research the EOQ and economic production quantity (EPQ) in manufacturers of integrated supply chain model.

The first integrated inventory model was published by Goyal (1976), who deduced that the optimal order time interval and production cycle time can be obtained by supposing that the supplier's production cycle time is an integer multiple of the customer's order time interval. In 1988, Goyal extended the Banerjee (1986) model by relaxing the lot-for-lot policy and assumed that the economic production quantity of the vendor must be an integer multiple of the purchase quantity of the buyer, which results in a lower joint total relevant cost. Ramasesh (1990) separated the total order cost of the EOQ model into the cost of placing a contact order with multiple small lots shipments. Lu (1995) developed a one-vender multi-buyer integrated inventory model with the objective of minimizing the total annual costs incurred by the vendor subject to the maximum cost that the buyer may be prepared to incur. Lu also proposed a heuristic solution for the single-vendor multi-buyer integrated inventory problem. Ha and Kim (1997) presented an integrated lot-splitting model of facilitating multiple shipments in small lots, one-vender, one-buyer, under deterministic conditions for a single product, and compared it with existing models of the JIT environment. Yang et al. (2013) considered the time of the inventory model with single buyer and single vendor. In their model, the inventory cost changes with inventory cycle time. Yang and Lin (2012) proposed a single-vendor and multiple buyer integrated inventory model with a normal distribution of lead time demand.

3. Imperfect Production and Preventive Maintenance Policy

To maintain global competitiveness, manufacturers require a production policy that effectively controls inventory levels in the face of uncertainty regarding production failure and demand. Porteus (1990) assumed that the probability of a shift from the "in-control" state to the "out-of-control" state has a given value for each production item. This study developed models for two maintenance activities: (1) performing a major repair, and (2) performing a minor repair, considering minimization of maintenance cost.

Lam and Yeh (1993) presented algorithms for deriving optimal maintenance policies that minimize the mean long-run cost-rate in continuous-time Markov deteriorating systems. Five maintenance strategies were considered, including failure replacement, age replacement, sequential inspection, periodic inspection, and continuous inspection. Tseng (1996) demonstrated a Perfect maintenance that can increase the reliability of a deteriorating system. Through perfect maintenance, the production system is returned to a like-new state after following each PM action. Sheu et al. (2006) considered periodic preventive maintenance policies, which maximizes the availability of a repairable system with major repair at failure. The three categories of preventive maintenance are imperfect preventive maintenance (IPM), perfect preventive maintenance (PPM) and failed preventive maintenance (FPM). The probability that preventive maintenance is perfect depends on the number of imperfect maintenance activities performed since the previous renewal cycle and on the probability that preventive maintenance remains imperfect is not increasing. Liao et al. (2009) presented an integrated EPQ model that incorporated EPQ and maintenance programs. This model considered imperfect repair, preventive maintenance and rework on the damage of a deteriorating production system as well as various special cases, such as the maintenance learning effect. The model in Liao et al. (2009) was then extended in Liao (2012) by relaxing the model of a backorder owing to rejection of defective parts after a failure. This study found that the optimal policy condition demonstrated was more flexible than previously described policies. The authors also discussed the effects of number of non-reworkable defective products, minimal repair cost, and other factors. Khan et al. (2011a) developed an EOQ model that considers (a) cost of inspection; (b) cost of Type I errors and; (c) cost of Type II errors. After being classified by the inspector and buyer, the defective items would be salvaged as a single batch and sold at a lower price. In addition, Khan et al. (2011b) presented a paper that reviewed literature relevant to extensions of the EOQ model for items of imperfect quality. The review herein provides a useful resource for researchers currently to engage in the work of inventory systems with imperfect item.

II. GENERAL MODEL

To construct the model, relevant notations are defined as follows:

D	average demand per year
P	production rate, P > D
Q	order quantity
T	time of inventory cycle
C_h	inventory cost rate per unit per year
C_v	vendor's production cost per unit
C_p	purchaser's purchase cost per unit
Q_d	Number of non-re workable defective products at each failure
R_b	Backorder cost per unit
Q_{rw}	Number of re-workable defective products at each failure
R_{rw}	Rework cost per unit
C_m	Minimal repair cost at each failure
C_o	purchaser's ordering cost
C_s	vendor's set-up cost
C_{pm}	cost of each PM
R	breakdown rate of unit
M	integer number of lots of items delivered form vender to purchaser
$\overline{P_j}$	probability of j PM are imperfect maintenance
P_j	probability of j PM is perfect maintenance which following the (j-1) imperfect PM: $p_j = \overline{p}_{j-1} - \overline{p}_j$ production run period.

Moreover, the following assumptions are made.

(1) The demand rate, setup cost, ordering cost and holding cost are known constants.
(2) Backorder is permitted during the inventory depletion period.
(3) The original system begins operating at time 0. The production process begins in an in-control state and produces perfect items.
(4) Setup cost C_s is incurred at the start of each inventory cycle. PM is performed following the production run period. The cost of each PM is C_{pm}.
(5) A system has two types of PM at cumulative production run time j. $T(j = 1, 2, 3 \ldots\ldots)$ based on outcome.
   - type-I PM (imperfect PM) results in the system having the same failure rate as before PM, with probability $\overline{p}_j$.
   - type-II PM (perfect PM) makes the system as good as new, with probability $p_j = \overline{p}_{j-1} - \overline{p}_j$.
(6) Following a perfect PM, the system returns to age 0.
(7) If failure occurs before the scheduled PM, the system shifts into the "out-of-control" state, then minimal repair can be made immediately. Minimal repair merely restores the system to a functioning state following failure, so the production process returns to the in-control condition. The backorder occurs because of insufficient production following rejection of defective parts. The minimal repair cost at each failure is Cm while the backorder cost per unit is R_b. The number of defective products at each failure is Q_d.
(8) The repair times are negligible.

Let D_j denote the maintenance and backorder cost, including the backorder and minimal repair cost among the (j-1)-th PM and j-th PM (production run period of inventory cycle j), and the PM cost of the j-th PM (inventory depletion period of inventory cycle j). Let Y_1, Y_2, … denote independent copies of Y. Finally, let $\sum_{j=1}^{\infty} E\left[D_j\right]$ denote the expected minimal repair and backorder cost. The expected failure number of periodic time T is :

$$\sum_{j=1}^{\infty} \frac{\overline{P}_{j-1}}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} \int_{j-1}^{jT} r(t)dt \tag{1}$$

Proof:

The

$$\sum_{j=1}^{\infty} \frac{\overline{P}_{j-1}}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} \int_{j-1}^{jT} r(t)dt$$

can be rewritten as

$$\frac{\sum_{j=1}^{\infty} \overline{P}_{j-1} \ln\left(\frac{\overline{F}\left((j-1)T\right)}{\overline{F}(jT)}\right)}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} \int_{j-1}^{jT} r(t)dt$$

Forming T is finite, $\frac{\bar{F}((j-1)T)}{\bar{F}(jT)}$ is finite, and there exists a finite number V that satisfies $1<\frac{\bar{F}((j-1)T)}{\bar{F}(jT)}\leqq V$, for $j = 1, 2, \ldots\ldots$

Notably,

$$\frac{\sum_{j=1}^{\infty}\bar{P}_{j-1}\ln\left(\frac{\bar{F}((j-1)T)}{\bar{F}(jT)}\right)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}<\frac{\sum_{j=1}^{\infty}\bar{P}_{j-1}\ln V}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}=\ln V,$$

therefore, this series

$$\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_{j-1}^{jT}r(t)dt$$

The proof is complete.

We have

$$\sum_{j=1}^{\infty}E\left[D_j, Y_1\geq jT\right]=C_m \tag{2}$$

and

$$\begin{aligned}&\sum_{j=1}^{\infty}E\left[D_j,(j-1)T<Y_1<jT\right]\\&=\left(C_m+Q_dR_b\right)\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_{j-1}^{jT}r(t)dt\end{aligned} \tag{3}$$

From Eq. (3), we get

$$\begin{aligned}&\sum_{j=1}^{\infty}E\left[D_j,(j-1)T<Y_1<jT\right]\\&=\left(C_m+Q_dR_b\right)\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}-\bar{P}_j}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_{0}^{jT}r(t)dt\end{aligned} \tag{4}$$

1. The Vendor's Total Expected Cost

Fig. 1 shows the inventory level of this model. Once the vender receives an order, the vender produces the items immediately until quantity reach to mQ. The item delivered from vender to buyer by each Q unit, and there are m lots will deliver in an inventory cycle. The vendor average inventory can be evaluated as follows:

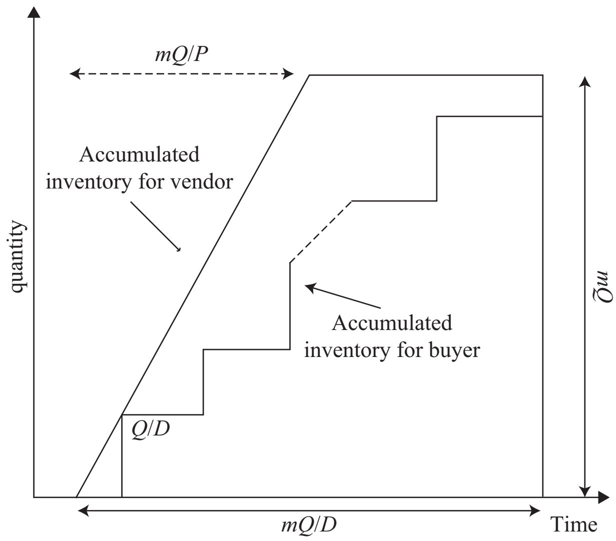

Fig. 1. Inventory model for vendor.

$$\begin{aligned}I_v&=\left\{\left[mQ\left(\frac{Q}{P}+(m+1)\frac{Q}{D}\right)-\frac{m^2Q^2}{2P}\right]\right.\\&\left.-\left[\frac{Q^2}{D}\left(1+2+\ldots+(m-1)\right)\right]\right\}/\left(\frac{mQ}{D}\right)\\&=\frac{Q}{2}m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\end{aligned} \tag{5}$$

Hence, the vendor has an expected annual holding cost of:

$$C_hC_v\left(\frac{Q}{2}m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right) \tag{6}$$

According to the assumptions and notations, the total expected annual cost for vendor is

$$\begin{aligned}TEC_V=&\text{ hoding cost}+\text{set up cost}+\text{PM cost}\\&+\text{minimal repair cost}\\&+\text{backorder cost}+\text{rework cost.}\end{aligned}$$

The various costs of vender model are derived as follows:

(a) Holding cost:

$$\begin{aligned}&C_hC_v\left[\frac{Q}{2}m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right]\\&-\frac{C_hQ_d}{2}\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}-\bar{P}_j}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt\end{aligned} \tag{7}$$

We have

$$T=\frac{mQ}{D}$$

and

$$\frac{TD}{2m}C_hC_v\left[m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right] \\ -\frac{C_hQ_d}{2}\sum_{j=1}^{\infty}\frac{\overline{P}_{j-1}-\overline{P}_j}{\sum_{j=1}^{\infty}\overline{P}_{j-1}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt \tag{8}$$

(b) Set up cost:

$$\frac{C_s}{T}.$$

(c) PM cost is:

$$\frac{C_{pm}}{T}.$$

(d) Minimal repair cost is:

$$\frac{C_m}{T}\sum_{j=1}^{\infty}\frac{\overline{P}_{j-1}-\overline{P}_j}{\sum_{j=1}^{\infty}\overline{P}_{j-1}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt \quad during\ T.$$

(e) Backorder cost:

$$\frac{Q_dR_b}{T}\sum_{j=1}^{\infty}\frac{\overline{P}_{j-1}-\overline{P}_j}{\sum_{j=1}^{\infty}\overline{P}_{j-1}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt.$$

(f) Rework cost:

$$\frac{Q_{rw}R_{rw}}{T}\sum_{j=1}^{\infty}\frac{\overline{P}_{j-1}-\overline{P}_j}{\sum_{j=1}^{\infty}\overline{P}_{j-1}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt.$$

In the proposed production system strategy, the cycle time for each production lot is T. At time (D/P)T, the machines stop producing products, and delivery of lot size m begins during time (1-D/P)T. The PM is performed after production run period during time (1-D/P)T. If failure occurs before the scheduled PM, the system shifts into the "out-of-control" state, and an immediate repair is made.

Fig. 2 shows the production System of the PM Strategy.

The total cost incurred by the vendor can be calculated by the following equation:

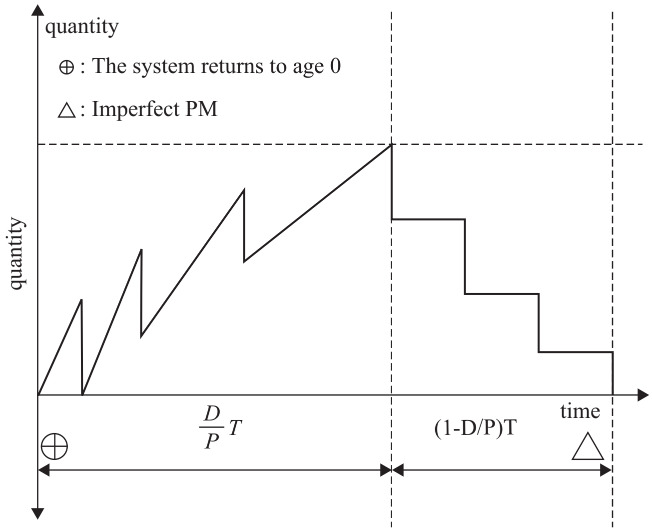

Fig. 2. Production System of PM Strategy.

$$TEC_V=\frac{C_s}{T}+\frac{C_{pm}}{T}+\frac{TD}{2m}C_hC_v\left[m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right] \\ +\left[\left(\frac{C_m}{T}+\frac{Q_dR_b}{T}+\frac{Q_{rw}R_{rw}}{T}-\frac{C_hQ_d}{2}\right)\sum_{j=1}^{\infty}\frac{\overline{P}_{j-1}-\overline{P}_j}{\sum_{j=1}^{\infty}\overline{P}_{j-1}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt\right] \tag{9}$$

2. The Purchaser's Total Expected Cost

$$TEC_P=\text{ordering cost}+\text{holding cost}$$

The various costs of buyer model are derived as follows:

(a) Ordering cost of each cycle is:

$$\frac{C_om}{T}$$

(b) Holding cost:

$$C_hC_p\frac{Q}{2}$$

In addition we have

$$T=\frac{mQ}{D}$$

and

$$C_hC_p\frac{TD}{2m} \tag{10}$$

Therefore, the above equation obtains the cost expected by the buyer as follows:

$$TEC_p = \frac{C_o m}{T} + C_h C_p \frac{TD}{2m} \tag{11}$$

Adding TEC_V and C_P obtains the joint expected annual cost as follows:

$$\begin{aligned}
&JTEC(T,m) = \\
&\frac{C_s}{T} + \frac{C_{pm}}{T} + \frac{TD}{2m} C_h C_v \left[m\left(\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \right] \\
&+\left[\left(\frac{C_m}{T} + \frac{Q_d R_b}{T} + \frac{Q_{rw} R_{rw}}{T} - \frac{C_h Q_d}{2} \right) \sum_{j=1}^{\infty} \frac{\bar{P}_{j-1} - \bar{P}_j}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)\,dt \right] \\
&+\frac{C_0 m}{T} + C_h C_p \frac{TD}{2m}
\end{aligned} \tag{12}$$

Using the partial derivatives of $JTEC(T, m)$ to optimize inventory run time T and m as described in Sheu et al. (2006) and Yang (2010) reveals a finite and unique optimal solution that minimizes JTEC.

$$\begin{aligned}
&\frac{\partial JTEC(T,m)}{\partial T} = \\
&-\frac{\left(C_o m + C_s + C_{pm} + \left[\left(Q_d R_b + C_m + Q_{rw} R_{rw} \right) \sum_{j=1}^{\infty} \frac{\bar{P}_{j-1} - \bar{P}_j}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt \right] \right)}{T^2} \\
&+\left[\left(\frac{C_m}{T} + \frac{Q_d R_b}{T} + \frac{Q_{rw} R_{rw}}{T} - \frac{C_h Q_d}{2} \right) \sum_{j=1}^{\infty} \frac{\bar{P}_{j-1} - \bar{P}_j}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} j\frac{D}{P} r\left(jT\frac{D}{P} \right) \right] \\
&+\frac{C_h C_p D}{2m} + \frac{C_h C_v D}{2m} \left[m\left(\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \right]
\end{aligned} \tag{13}$$

Let

$$\sum_{j=1}^{\infty} \frac{\bar{P}_{j-1} - \bar{P}_j}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt = Z$$

$$\begin{aligned}
&\frac{\partial JTEC(T,m)}{\partial T} = \\
&-\frac{\left(C_o m + C_s + C_{pm} + \left[\left(Q_d R_b + C_m + Q_{rw} R_{rw} \right) Z \right] \right)}{T^2} \\
&+\left[\left(\frac{C_m}{T} + \frac{Q_d R_b}{T} + \frac{Q_{rw} R_{rw}}{T} - \frac{C_h Q_d}{2} \right) Z' \right] \\
&+\frac{C_h C_p D}{2m} + \frac{C_h C_v D}{2m} \left[m\left(\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \right]
\end{aligned} \tag{14}$$

For second order partial derivatives, if Eq. (15) > 0, than JTEC will be a local minimal solution.

$$\begin{aligned}
\frac{\partial^2 JTEC(T,m)}{\partial^2 T} &= \frac{2\left(C_o m + C_s + C_{pm} \right)}{T^3} \\
&+\frac{2Z\left(Q_d R_b + C_m + Q_{rw} R_{rw} \right)}{T^3} \\
&-\frac{2Z'\left(Q_d R_b + C_m + Q_{rw} R_{rw} \right)}{T^2} \\
&+\frac{Z'\left(Q_d R_b + C_m + Q_{rw} R_{rw} \right)}{T} - \frac{C_h Q_d Z''}{2}
\end{aligned} \tag{15}$$

Proof: See Appendix 1.

III. PROBABILITY MODEL

1. Geometrically Distribution

This study applied the Geometrically Distribution type II PM proposed by Nakagawa (1979). $\bar{P}_0 = 1$; $\bar{P}_1 = q^j$, $0 \le q < 1$, $\bar{q} = 1 - q$. The break down cost is as follows:

$$C_m \bar{q}^{-2} \sum_{j=1}^{\infty} q^{j-1} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt$$

Therefore, JTEC is

$$\begin{aligned}
JTEC &= \frac{C_s}{T} + \frac{C_{pm}}{T} + \frac{TD}{2m} C_h C_v \left[m\left(\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \right] \\
&+\left[\left(\frac{C_m}{T} + \frac{Q_d R_b}{T} + \frac{Q_{rw} R_{rw}}{T} - \frac{C_h Q_d}{2} \right) \bar{q}^{-2} \sum_{j=1}^{\infty} q^{j-1} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt \right] \\
&+\frac{C_o m}{T} + C_h C_p \frac{TD}{2m}
\end{aligned} \tag{16}$$

Let

$$\bar{q}^{-2} \sum_{j=1}^{\infty} q^{j-1} \int_0^{j\left(\frac{D}{P}\right)T} r(t)dt = Z \tag{17}$$

$$\begin{aligned}
\frac{\partial JTEC(T,m)}{\partial T} &= -\frac{\left(C_o m + C_s + C_{pm} + \left[\left(Q_d R_b + C_m + Q_{rw} R_{rw} \right) Z \right] \right)}{T^2} \\
&+\left[\left(\frac{C_m}{T} + \frac{Q_d R_b}{T} + \frac{Q_{rw} R_{rw}}{T} - \frac{C_h Q_d}{2} \right) Z' \right] + \frac{C_h C_p D}{2m} \\
&+\frac{C_h C_v D}{2m} \left[m\left(\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P} \right) \right]
\end{aligned} \tag{18}$$

For second order partial derivatives, if Eq. (19) obtains a value larger than 0, than JTEC will exist a local minimal solution.

$$\frac{\partial^2 JTEC(T,m)}{\partial^2 T}=\frac{2\left(C_o m+C_s+C_{pm}\right)}{T^3}+\frac{2Z\left(Q_d R_b+C_m+Q_{rw}R_{rw}\right)}{T^3}-\frac{2Z'\left(Q_d R_b+C_m+Q_{rw}R_{rw}\right)}{T^2}+\frac{Z'\left(Q_d R_b+C_m+Q_{rw}R_{rw}\right)}{T}-\frac{C_h Q_d Z''}{2} \quad (19)$$

Proof: See Appendix 2.

2. Weibull Distribution

In the Weibull distribution, $\overline{P}_0=1;\ \overline{P}_J=q^{j\beta}$ $(j=1,2,\ldots\ldots)$, $0\le q\le 1$, $0<\beta<1$. The costs are broken down as follow:

$$C_m\sum_{j=1}^{\infty}\frac{q^{(j-1)^\beta}-q^{j\beta}}{\sum_{j=1}^{\infty}q^{(j-1)^\beta}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt$$

So, the JTEC will goes to:

$$JTEC(T,m)=\frac{C_s}{T}+\frac{C_{pm}}{T}+\frac{TD}{2m}C_hC_v\left[m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right]+\left[\left(\frac{C_m}{T}+\frac{Q_dR_b}{T}+\frac{Q_{rw}R_{rw}}{T}-\frac{C_hQ_d}{2}\right)\sum_{j=1}^{\infty}\frac{q^{(j-1)^\beta}-q^{j\beta}}{\sum_{j=1}^{\infty}q^{(j-1)^\beta}}\int_0^{j\left(\frac{D}{P}\right)T}r(t)dt\right]+\frac{C_om}{T}+C_hC_p\frac{TD}{2m} \quad (20)$$

Let

$$\sum_{j=1}^{\infty}\frac{q^{(j-1)^\beta}-q^{j\beta}}{\sum_{j=1}^{\infty}q^{(j-1)^\beta}}\,j\frac{D}{P}r\left(jT\frac{D}{P}\right)=Z \quad (21)$$

$$\frac{\partial JTEC(T,m)}{\partial T}=-\frac{\left(C_om+C_s+C_{pm}+\left[\left(Q_dR_b+C_m+Q_{rw}R_{rw}\right)Z\right]\right)}{T^2}+\left[\left(\frac{C_m}{T}+\frac{Q_dR_b}{T}+\frac{Q_{rw}R_{rw}}{T}-\frac{C_hQ_d}{2}\right)Z'\right]+\frac{C_hC_pD}{2m}+\frac{C_hC_vD}{2m}\left[m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right] \quad (22)$$

For second order partial derivatives, if Eq. (23) obtains a value larger than 0, than JTEC will exist a local minimal solution.

$$\frac{\partial^2 JTEC(T,m)}{\partial^2 T}=\frac{2\left(C_o m+C_s+C_{pm}\right)}{T^3}+\frac{2Z\left(Q_d R_b+C_m+Q_{rw}R_{rw}\right)}{T^3}-\frac{2Z'\left(Q_d R_b+C_m+Q_{rw}R_{rw}\right)}{T^2}+\frac{Z'\left(Q_d R_b+C_m+Q_{rw}R_{rw}\right)}{T}-\frac{C_h Q_d Z''}{2} \quad (23)$$

Proof: See Appendix 3.

3. Learning Effect

In this case, the learning effect decreases as the number of PMs increase. Following the discussion, a probability model is developed, and the following assumptions are made:

(1) $\overline{P}_{j-1}>\overline{P}_j\left(j=1,2,\ldots\ldots\right)$

(2) $\overline{P}_0=1;\ \overline{P}_1\ne 0;\ \overline{P}_j=\overline{P}_1\times j^b\left(j=1,2,\ldots\ldots\right)$, b is learning rate.

(3) $b=\dfrac{\log r}{\log 2}$

The break down cost as follow (Sheu et al., 2006):

$$C_m\left[\frac{1-\overline{P}_1}{1+\overline{P}_1\sum_{j=2}^{\infty}(j-1)^b}\int_0^{j\frac{D}{P}T}r(t)dt+\overline{P}_1\sum_{j=2}^{\infty}\frac{(j-1)^b-j^b}{1+\overline{P}_1\sum_{j=1}^{\infty}(j-1)^b}\int_0^{j\frac{D}{P}T}r(t)dt\right]$$

So, the JTEC will goes to:

$$JTEC(T,m)=\frac{C_s}{T}+\frac{C_{pm}}{T}+\frac{TD}{2m}C_hC_v\left[m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right]+\left[\left(\frac{C_m}{T}+\frac{Q_dR_b}{T}+\frac{Q_{rw}R_{rw}}{T}-\frac{C_hQ_d}{2}\right)Z\right]+\frac{C_om}{T}+C_hC_p\frac{TD}{2m} \quad (24)$$

Let

$$\left[\frac{1-\overline{P}_1}{1+\overline{P}_1\sum_{j=2}^{\infty}(j-1)^b}\int_0^{j\frac{D}{P}T}r(t)dt+\overline{P}_1\sum_{j=2}^{\infty}\frac{(j-1)^b-j^b}{1+\overline{P}_1\sum_{j=1}^{\infty}(j-1)^b}\int_0^{j\frac{D}{P}T}r(t)dt\right]=Z \quad (25)$$

$$\frac{\partial JTEC(T,m)}{\partial T}=-\frac{\left(C_om+C_s+C_{pm}+\left[\left(Q_dR_b+C_m+Q_{rw}R_{rw}\right)Z\right]\right)}{T^2}+\left[\left(\frac{C_m}{T}+\frac{Q_dR_b}{T}+\frac{Q_{rw}R_{rw}}{T}-\frac{C_hQ_d}{2}\right)Z'\right]+\frac{C_hC_pD}{2m}+\frac{C_hC_vD}{2m}\left[m\left(\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)\right] \quad (26)$$

For second order partial derivatives, if Eq. (27) obtains a value larger than 0, then JTEC will be a local minimal solution.

Table 1. Parameter setting.

Parameter	Cost/Number	Parameter	Cost/Number
D	600 unit/year	C_{PM}	\$20 per run
P	1000 unit/year	C_m	\$10 each time
C_h	0.2 per unit	Q_{rw}	4
C_v	\$20 per unit	R_{rw}	\$5 per unit
C_p	\$25 per unit	Q_d	1
C_o	\$20 each order	R_b	\$6
C_s	\$30 per run		
$r(t)$	Following a uniform distribution with a = 0.1, b = 0.4		
$g(t)$	Following a Gamma distribution with α = 0.02, β = 2		

Table 2. Basic model with uniform distribution.

m	p = 0.01		p = 0.05		p = 0.1		p = 0.15	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	839.1	0.177	837.1	0.177	834.5	0.177	832.0	0.177
2	747.5	0.258	745.5	0.258	743.0	0.258	740.4	0.258
3	**733.1**	**0.322**	**731.1**	**0.322**	**728.5**	**0.322**	**726.0**	**0.322**
4	739.7	0.377	737.7	0.377	735.2	0.376	732.6	0.377
5	754.6	0.426	752.6	0.426	750.1	0.425	747.5	0.426
6	773.2	0.471	771.2	0.471	768.7	0.469	766.2	0.470
7	793.5	0.512	791.5	0.512	789.0	0.510	786.5	0.511
8	814.7	0.550	812.7	0.550	810.2	0.548	807.7	0.550
9	836.1	0.586	834.1	0.586	831.6	0.584	829.2	0.585
10	857.7	0.620	855.7	0.620	853.2	0.618	850.7	0.619

$$\frac{\partial^2 JTEC(T,m)}{\partial^2 T} = \frac{2\left(C_o m + C_s + C_{pm}\right)}{T^3} + \frac{2Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{T^3} - \frac{2Z'\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{T^2} + \frac{Z'\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{T} - \frac{C_h Q_d Z''}{2} \tag{27}$$

Proof: See Appendix 4.

IV. NUMERCIAL EXAMPLE

The following procedure is used to find the optimal values for T and m.

Step 1 Let m be equal to the minimum feasible value, 1.
Step 2 Calculate T using relation.
Step 3 Calculate JTEC by embedding the last calculates T and m. If $TC_m < TC_{m-1}$ let $m = m + 1$ and go to step 2; otherwise go to step 4.
Step 4 Find the minimum JTEC and the corresponding value of decision variables T and m as the optimal solution.

Table 3. Geometrically model with uniform distribution.

m	p = 0.01		p = 0.05		p = 0.1		p = 0.15	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	827.6	0.177	823.9	0.177	823.8	0.177	813.2	0.177
2	736.2	0.258	732.4	0.258	732.3	0.258	721.4	0.258
3	**721.8**	**0.322**	**718.1**	**0.322**	**718.0**	**0.322**	**706.8**	**0.323**
4	728.6	0.377	724.9	0.377	724.8	0.377	713.3	0.378
5	743.5	0.426	739.8	0.426	739.7	0.426	728.1	0.427
6	762.2	0.470	758.5	0.470	758.4	0.470	746.5	0.471
7	782.7	0.511	778.9	0.511	778.9	0.511	766.8	0.512
8	803.9	0.549	800.2	0.549	800.1	0.549	787.9	0.551
9	825.4	0.585	821.7	0.585	821.6	0.585	809.2	0.587
10	847.0	0.619	843.3	0.619	843.3	0.619	830.7	0.621

Table 4. Weibull model with uniform distribution.

m	p = 0.01		p = 0.05		p = 0.1		p = 0.15	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	834.9	0.169	830.7	0.177	825.8	0.177	822.1	0.177
2	743.6	0.244	739.1	0.258	734.2	0.258	730.5	0.258
3	**729.3**	**0.322**	**724.7**	**0.322**	**719.8**	**0.322**	**716.0**	**0.322**
4	736.0	0.355	731.4	0.377	726.4	0.377	722.6	0.377
5	750.9	0.401	746.3	0.426	741.3	0.426	737.5	0.426
6	769.5	0.443	764.9	0.470	760.0	0.470	756.1	0.471
7	788.5	0.510	785.3	0.511	780.3	0.511	776.4	0.512
8	811.1	0.518	806.5	0.550	801.5	0.550	797.6	0.550
9	832.5	0.552	827.9	0.585	823.0	0.585	819.2	0.574
10	852.8	0.626	849.5	0.619	844.5	0.619	840.6	0.620

Table 5. Learning curve model with uniform distribution.

m	r = 0.8		r = 0.75		r = 0.7		r = 0.65	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	809.8	0.177	813.3	0.177	820.7	0.177	826.7	0.177
2	718.4	0.258	721.9	0.258	729.6	0.258	735.2	0.258
3	**704.1**	**0.322**	**707.6**	**0.322**	**715.5**	**0.322**	**720.8**	**0.322**
4	710.9	0.377	714.4	0.377	722.5	0.376	727.6	0.377
5	725.9	0.426	729.4	0.426	737.6	0.425	742.5	0.426
6	744.6	0.471	748.1	0.471	756.5	0.469	761.1	0.470
7	765.1	0.512	768.6	0.512	777.1	0.510	781.5	0.511
8	786.3	0.550	789.8	0.550	798.5	0.548	802.8	0.550
9	807.8	0.586	811.3	0.586	820.1	0.584	824.2	0.585
10	829.5	0.620	833.0	0.620	841.9	0.618	845.8	0.619

The data in Table 1 are used to demonstrate how the above steps are used to solve an inventory problem.

1. Numerical Results Sensitive Analysis

This section shows how the uniform and gamma distributions are used to calculate the shot down rate. Tables 2-9 and Figs. 3-10 show the results.

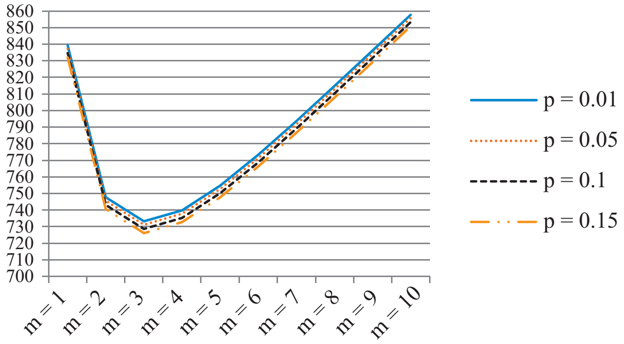

Fig. 3. Basic model with uniform distribution.

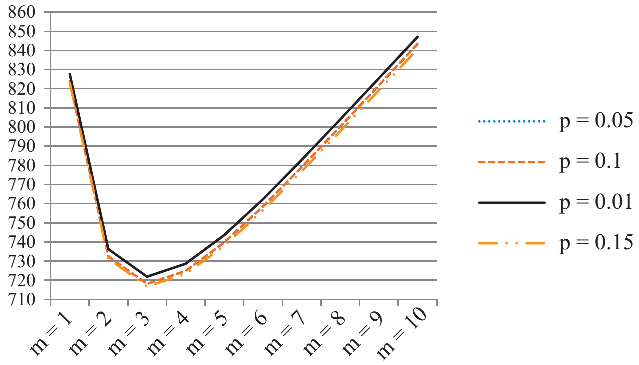

Fig. 4. Geometrically model with uniform distribution.

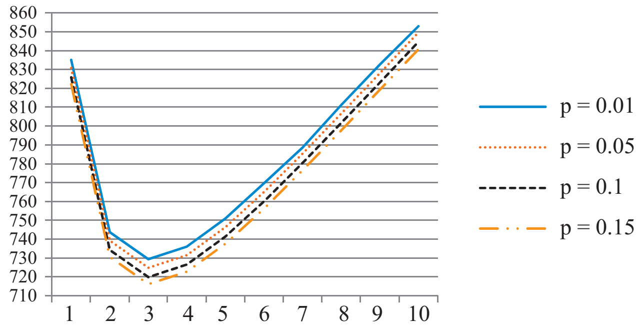

Fig. 5. Weibull model with uniform distribution.

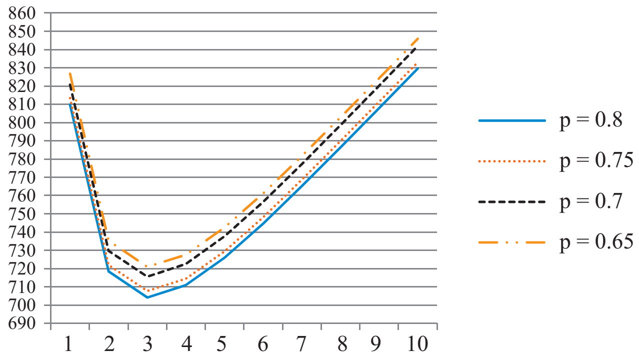

Fig. 6. Learning curve model with uniform distribution.

Table 6. Basic model with gamma distribution.

m	$p = 0.01$		$p = 0.05$		$p = 0.1$		$p = 0.15$	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	823.4	0.177	822.1	0.177	820.3	0.177	818.5	0.177
2	732.0	0.258	730.6	0.258	728.9	0.258	727.1	0.258
3	**717.7**	**0.322**	**716.3**	**0.322**	**714.6**	**0.322**	**712.8**	**0.322**
4	724.4	0.377	723.1	0.377	721.3	0.377	719.6	0.377
5	739.4	0.426	738.0	0.426	736.3	0.426	734.5	0.426
6	758.0	0.470	756.7	0.470	755.0	0.470	753.2	0.470
7	778.5	0.511	777.2	0.511	775.4	0.511	773.7	0.511
8	799.7	0.549	798.4	0.549	796.7	0.549	794.9	0.549
9	821.2	0.585	819.9	0.585	818.1	0.585	816.4	0.585
10	842.8	0.619	841.5	0.619	839.8	0.619	838.1	0.619

Table 7. Geometrically model with gamma distribution.

m	$p = 0.01$		$p = 0.05$		$p = 0.1$		$p = 0.15$	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	816.7	0.177	813.8	0.177	813.3	0.177	812.8	0.177
2	725.4	0.258	722.4	0.258	721.9	0.258	721.2	0.258
3	**711.2**	**0.322**	**708.1**	**0.322**	**707.6**	**0.322**	**706.8**	**0.322**
4	718.1	0.376	714.9	0.377	714.3	0.377	713.5	0.377
5	733.1	0.425	729.9	0.425	729.3	0.426	728.4	0.426
6	751.9	0.469	748.7	0.470	748.0	0.470	747.1	0.470
7	772.4	0.510	769.1	0.511	768.4	0.511	767.5	0.511
8	793.7	0.548	790.4	0.549	789.6	0.549	788.7	0.549
9	815.2	0.584	811.9	0.584	811.1	0.585	810.1	0.585
10	836.9	0.618	833.6	0.618	832.8	0.619	831.7	0.619

Table 8. Weibull model with gamma distribution.

m	$p = 0.01$		$p = 0.05$		$p = 0.1$		$p = 0.15$	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	819.2	0.177	817.6	0.177	816.4	0.177	813.1	0.177
2	727.8	0.258	726.2	0.258	725.0	0.258	721.6	0.258
3	**713.5**	**0.322**	**711.9**	**0.322**	**710.6**	**0.322**	**707.3**	**0.322**
4	720.3	0.377	718.7	0.377	717.4	0.377	714.0	0.377
5	735.2	0.426	733.7	0.426	732.4	0.426	729.0	0.426
6	753.9	0.470	752.3	0.470	751.1	0.470	747.7	0.470
7	774.4	0.511	772.8	0.511	771.5	0.511	768.1	0.511
8	795.6	0.549	794.1	0.549	792.8	0.549	789.3	0.549
9	817.1	0.585	815.5	0.585	814.3	0.585	810.8	0.585
10	838.8	0.619	837.2	0.619	835.9	0.619	832.5	0.619

Tables 2-5 and Figs. 3-6 exhibit the results of four models with uniform distribution and show that, when lot size m is 3, JTEC decreases. Therefore, the corresponding values for the decision variables T and m are the optimal solution, and the range of JTEC is 704-733.

Tables 6-9 and Figs. 7-10 exhibit the results of our models with gamma distribution and show that, when lot size m is at 3,

Table 9. Learning curve model with gamma distribution.

m	$r = 0.8$		$r = 0.75$		$r = 0.7$		$r = 0.65$	
	JTEC	T	JTEC	T	JTEC	T	JTEC	T
1	809.6	0.177	813.2	0.177	818.1	0.178	824.2	0.178
2	718.3	0.258	721.8	0.258	725.9	0.259	731.7	0.259
3	**704.0**	**0.322**	**706.5**	**0.322**	**710.9**	**0.323**	**716.5**	**0.324**
4	710.8	0.377	713.3	0.377	717.1	0.379	722.6	0.380
5	725.8	0.425	728.3	0.425	731.6	0.428	736.9	0.429
6	744.5	0.470	747.0	0.470	749.9	0.473	755.0	0.474
7	765.0	0.511	767.5	0.511	769.9	0.514	774.9	0.516
8	786.2	0.549	788.7	0.549	790.8	0.553	795.6	0.554
9	807.7	0.585	810.2	0.585	811.9	0.589	816.6	0.591
10	829.4	0.618	831.9	0.618	833.2	0.623	837.8	0.625

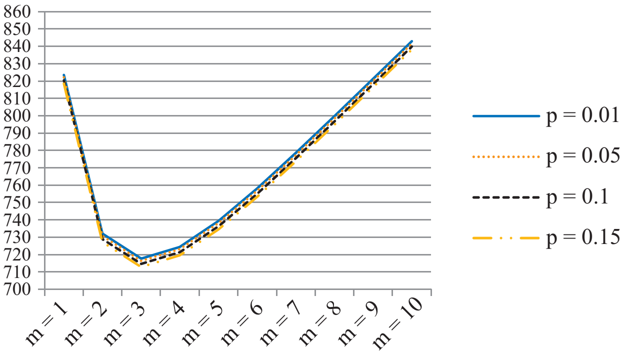

Fig. 7. Basic model with gamma distribution.

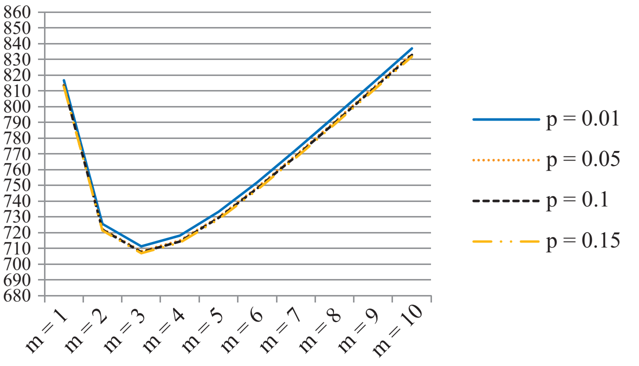

Fig. 8. Geometrically model with gamma distribution.

JTEC also decreases. Therefore, the corresponding values for the decision variables T and m are the optimal solution, and the range of JTEC is 704-717.

The results essentially show that, in the Weibull model with either uniform or gamma distribution, an increase in the maintenance factor causes a decrease in JTEC. However, in learning curve model, the basic value of maintenance factor differs from other model. That is, if the maintenance factor decreases, JTEC increases. The maintenance factor increases can lead to a decrease in system error rates and a reduction in preventive maintenance costs.

Table 10. Eight different cases for sensitive analysis.

Case	Full Name
1	Basic model with uniform distribution
2	Geometrically model with uniform distribution
3	Weibull model with uniform distribution
4	Learning curve model with uniform distribution
5	Basic model with gamma distribution
6	Geometrically model with gamma distribution
7	Weibull model with gamma distribution
8	Learning curve model with gamma distribution

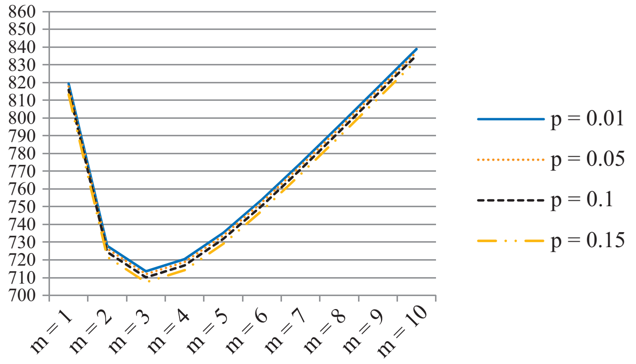

Fig. 9. Weibull model with gamma distribution.

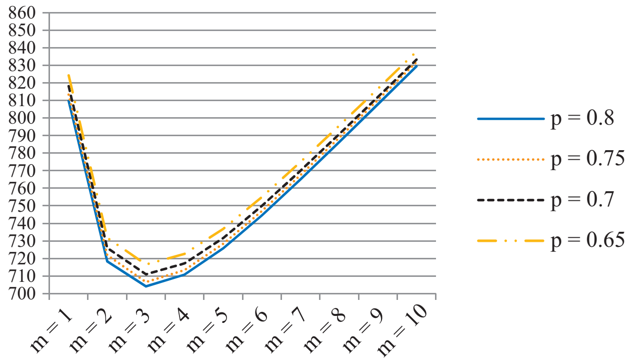

Fig. 10. Learning curve model with gamma distribution.

2. Sensitivity Analysis

This section uses sensitivity analysis to analyze our integrated inventory mode with three key impact factors: D/P, C_{pm} and Q_d.

Since P > D, we want to understand the impact of D/P on the integrated inventory cost and the size of production scale. As the scale of production increases, economic efficiency increases. The C_{pm} is preventive maintenance cost. We use sensitive analysis by considering how preventive maintenance cost affects the integrated inventory cost. Q_d is the quantity of non-reworkable defective products at each failure. Sensitive analysis reveals how the quantity of non-re workable defective products affects integrated inventory cost.

Table 11. Sensitive analysis of D/P ratio for JTEC.

	Ratio				
Case	0.4	0.5	0.6	0.7	0.8
1	636.3152	686.8782	733.1380	775.0945	812.7478
2	625.2388	675.7124	721.8903	763.7726	801.3593
3	638.1779	685.7748	729.3210	768.8163	804.2609
4	607.4517	657.9488	704.1482	746.0050	783.6542
5	620.9754	671.490	717.7055	759.6219	797.2391
6	614.6158	665.0889	711.2665	753.1484	790.7348
7	616.8299	667.3389	713.5493	755.4610	793.0741
8	607.3481	657.8448	704.0439	745.9455	783.5494

Table 12. Sensitive analysis of PM cost for JTEC.

	C_{pm}				
Case	15	20	25	30	35
1	717.6458	733.1380	748.6302	764.1244	779.6147
2	706.3720	721.8903	737.4086	752.9269	768.4453
3	712.8633	729.3210	745.7786	762.2362	778.6939
4	688.6357	704.1482	719.6606	735.1731	750.6855
5	702.1984	717.7055	733.2126	748.7196	764.2267
6	695.7467	711.2665	726.7863	742.3061	757.8259
7	698.0405	713.5493	729.0581	744.5668	760.0756
8	688.5314	704.0439	719.5565	735.0690	750.5816

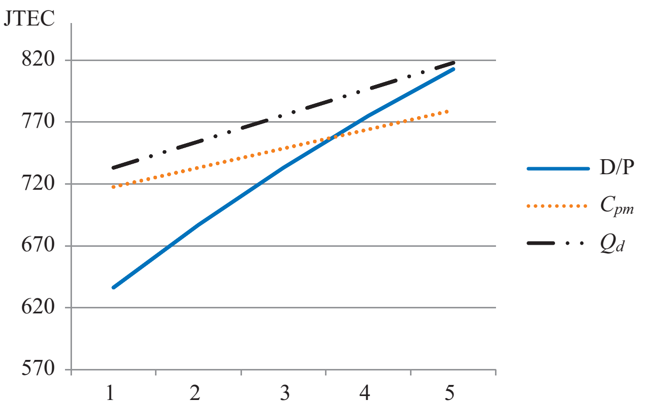

Fig. 11. Sensitive Analysis of Basic Model with Uniform Distribution.

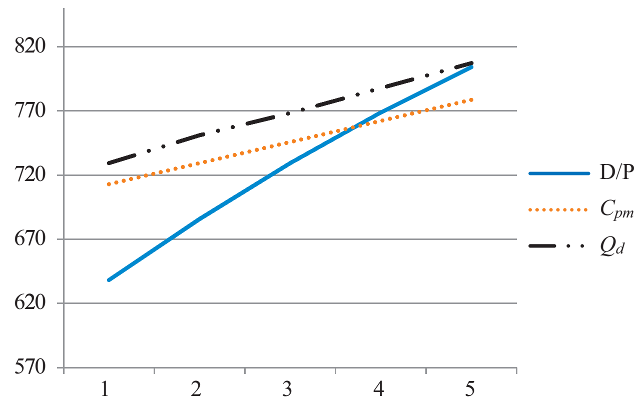

Fig. 13. Sensitive analysis of Weibull model with uniform distribution.

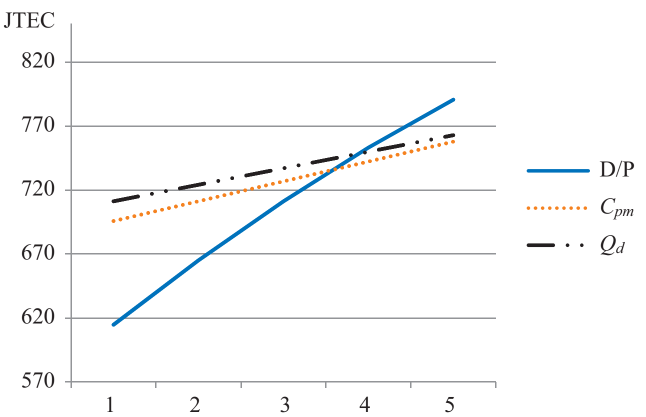

Fig. 12. Sensitive analysis of Geo model with uniform distribution.

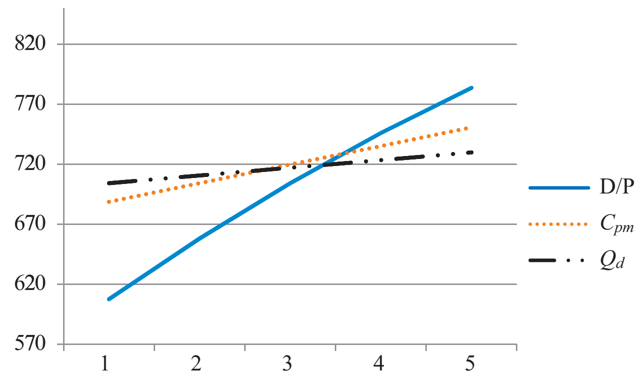

Fig. 14. Sensitive analysis of learning curve model with uniform distribution.

We demonstrated eight different cases (Table 10) and pertaining results are shown on Table 11 and Table 12.

Table 11 shows that the joint total cost increases as demand and production ratio D/P increases because the D/P ratio is influenced by the holding cost and purchase cost. Table 12 shows that increasing the preventive maintenance cost C_{pm} causes an increase in the joint total cost.

When the number of non-re workable defective products Q_d increases, the joint total cost increases. Because the backorder cost is higher than the reduction in holding cost of Q_d, the total joint total cost increases as Q_d increases. Figs. 11-18 exhibit the sensitive analysis results of eight different case and show that, of the three impact factors, the D/P ratio has the most influence in the integrated model.

V. CONCLUSION

An effective supply chain that cooperates with the activities of upstream vendor and downstream buyer will maximize the customer value and minimize the inventory cost. Accordingly,

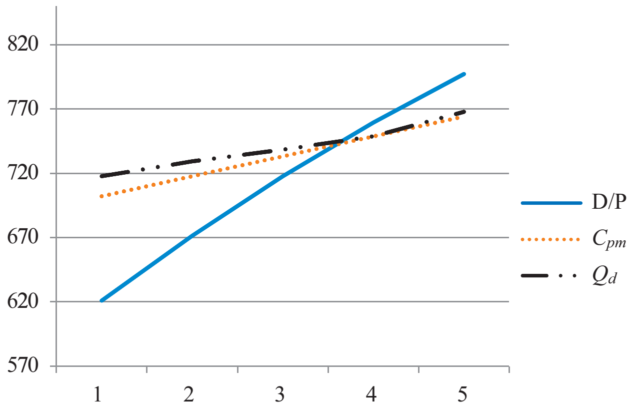

Fig. 15. Sensitive analysis of basic model with gamma distribution.

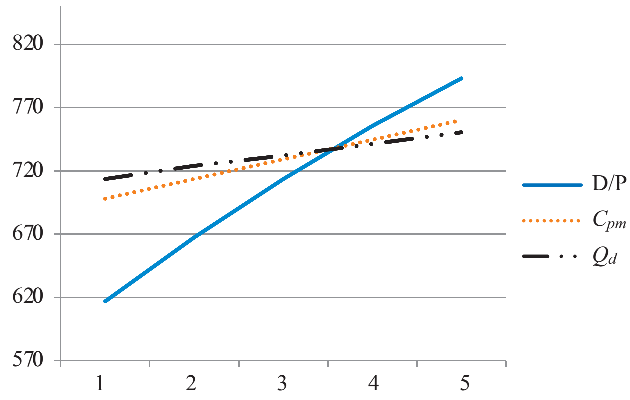

Fig. 17. Sensitive analysis of Weibull model with gamma distribution.

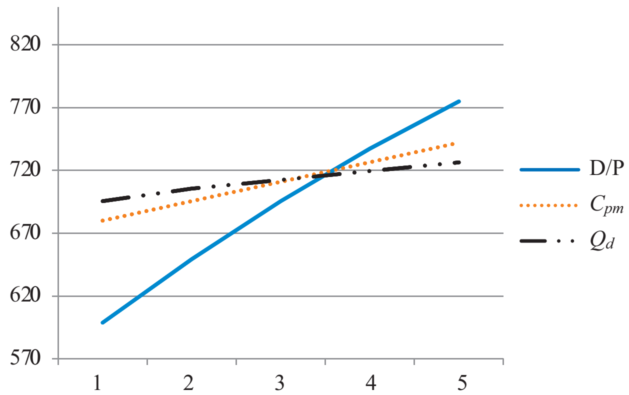

Fig. 16. Sensitive analysis of Geo model with gamma distribution.

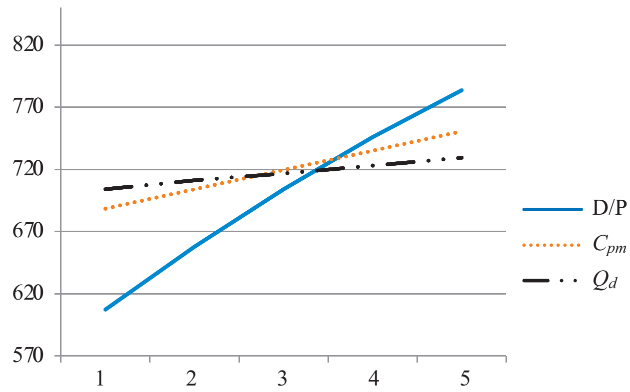

Fig. 18. Sensitive analysis of learning curve model with gamma distribution.

this study developed an integrated model of a production process with one vendor and one buyer based on the concept of EOQ and EPQ. The preventive maintenance strategy of production process fail and minimal repair were also considered to improve the accuracy of the model. The data showed the integrated model can determine the influence of re-workable defective products at each failure and backorder with non-re workable defective products at each failure.

The sensitivity analysis shows that, of the three impact factors, the D/P ratio has the highest influence on the joint total expected cost with integrated model. In addition, four preventive maintenance probability and two shot down probability are using in the proposed model. A numerical example showed that, when $m = 3$, we can find the corresponding value of the decision variables T and m as the optimal solution and the range of joint total expected cost. If the vendors and the buyers need to apply this model in the future, they can choose our model of the probability according to their demand.

To expand the applicability of the proposed model, future studies are suggested to investigate problems involving multiple vendors or multiple buyers. Also, different actual data can be applied to enhance the theory of this model. Finally, researchers are encouraged to expand its applications to other domains such as preventive maintenance time interval, permissible delay in payments and test error.

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APPENDIX

Appendix 1

Proof:

if

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z''T^3}{4} \tag{28}$$

$$Z' = \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r \left(jT \frac{D}{P} \right)$$

and

$$0 \le \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r \left(jT \frac{D}{P} \right) \le 1,$$

$$C_o, C_s, C_{pm}, C_m > Q_d R_b \text{ and } Q_{rw} R_{rw}$$

$$0 \le T \le 1.$$

Hence

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{29}$$

the Eq. (15)>0.

Since Eq. (13) value of T minimizes *JTEC (T, m)*, and *T* can be obtained by solving the Eq. (12).

Appendix 2

Proof:

if

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{30}$$

$$Z' = \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r \left(jT \frac{D}{P} \right)$$

and

$$0 \le \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r \left(jT \frac{D}{P} \right) \le 1,$$

$$C_o, C_s, C_{pm}, C_m > Q_d R_b \text{ and } Q_{rw} R_{rw}$$

$$0 \le T \le 1.$$

Hence

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{31}$$

the Eq. (19) > 0.

Since Eq. (18) value of *T* minimizes *JTEC (T, m)*, and *T* can be obtained by solving the equation Eq. (16).

Appendix 3

Proof:

if

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{32}$$

$$Z' = \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r \left(jT \frac{D}{P} \right)$$

and

$$0 \le \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r \left(jT \frac{D}{P} \right) \le 1,$$

$$C_o, C_s, C_{pm}, C_m > Q_d R_b \text{ and } Q_{rw} R_{rw}$$

$$0 \le T \le 1.$$

Hence

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{33}$$

the Eq. (23) > 0.

Since Eq. (22) value of T which minimizes $JTEC$ (T, m), and T can be obtained by solving the Eq. (20).

Appendix 4

Proof:

if

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{34}$$

$$Z' = \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r\left(jT \frac{D}{P}\right)$$

and

$$0 \le \sum_{j=1}^{\infty} \frac{\overline{P}_{j-1} - \overline{P}_j}{\sum_{j=1}^{\infty} \overline{P}_{j-1}} j \frac{D}{P} r\left(jT \frac{D}{P}\right) \le 1,$$

$$C_o, C_s, C_{pm}, C_m > Q_d R_b \text{ and } Q_{rw} R_{rw}$$

$$0 \le T \le 1.$$

Hence

$$\left(C_o m + C_s + C_{pm}\right) + Z\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right) + \frac{Z'T^2\left(Q_d R_b + C_m + Q_{rw} R_{rw}\right)}{2} > Z'T + \frac{C_h Q_d Z'' T^3}{4} \tag{35}$$

the Eq. (27) > 0.

Since Eq. (26) value of T which minimizes $JTEC$ (T, m), and T can be obtained by solving the Eq. (24).

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