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### Recommended Citation

Abolvafaie, Mahnaz; Koofigar, Hamid Reza; and Malekzadeh, Maryam (2018) "CLASSIFICATION OF HYDRODYNAMIC COEFFICIENTS OF AUTONOMOUS UNDERWATER VEHICLES BASED ON SENSITIVITY ANALYSES IN STANDARD MANEUVERS," *Journal of Marine Science and Technology*. Vol. 26: Iss. 1, Article 1.

DOI: 10.6119/JMST.2018.02\_(1).0001

Available at: <https://jmstt.ntou.edu.tw/journal/vol26/iss1/1>

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# CLASSIFICATION OF HYDRODYNAMIC COEFFICIENTS OF AUTONOMOUS UNDERWATER VEHICLES BASED ON SENSITIVITY ANALYSES IN STANDARD MANEUVERS

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Key words: hydrodynamic coefficients, sensitivity analysis, autonomous underwater vehicle, standard maneuvers.

## ABSTRACT

Due to the important role of hydrodynamic coefficients in the control and guidance of an autonomous underwater vehicle (AUV), sensitivity analysis is proposed here, as a preliminary step to motion control design. Taking the standard maneuvers, including turning circle and horizontal and vertical zigzag, the sensitivity of various hydrodynamic coefficients with respect to velocities and position is determined. Such analyses are then used to classify the model parameters into three categories, as non-sensitive coefficients, coefficients with low influence on the motion and more sensitive coefficients.

## I. INTRODUCTION

A tremendous effort has been devoted to motion control of autonomous underwater vehicles, due to their importance in subsea investigations and marine engineering (Miao, 2013; Wynn et al., 2014; Fang et al., 2015; Jamalzade et al., 2016). Time varying dynamic behavior, uncertainties in hydrodynamic coefficients, and environmental disturbances form a coupled uncertain nonlinear model, which complicates the controller design procedure (Yuh, 2000; Zhang and Chu, 2012; Joe et al., 2014; Wang et al., 2016). The hydrodynamic coefficients may vary by changing the speed, type of maneuvering, and environmental circumstances (Refsnes et al., 2005; Xiao, 2014). Constructing a motion control algorithm by taking all of system parameters as model uncertainties, may be so conservative. On the other hand, neglecting the uncertainties may not lead to a robust performance

(Forces, 2013; Koofigar, 2014). Instead, a sensitivity analysis can be used to assign the most sensitive parameters which affect the vehicle motion (Perrault et al., 2003a; Iwaniec, 2011).

The indirect sensitivity analysis, presented for a ship (Hwang, 1980), is restricted to some specific maneuvers. Later, such method has been modified to be used in more applications to classify the parameters into linear damping, linear inertial force and nonlinear damping coefficients (Kim et al., 2002; Wang et al., 2014). Based on these analyses, the linear damping coefficients have been introduced as the most sensitive parameters (Kim et al., 2002; Yeo and Rhee, 2006). Although the exact mathematical model is not required in indirect method, but the number of simulations is inevitably increased with taking more model coefficients. In fact, the accuracy of the method depends on various simulation studies in different maneuvers (Yeo and Rhee, 2006). Direct analyses are also investigated for ships and submarines. The results may be applied to derive some simplified mathematical models (Yeo and Rhee, 2006; Lin et al., 2008; Wang et al., 2014). Nevertheless, constructing a sensitivity analysis algorithm with simplicity and universality properties for application to AUV dynamics is highly desired in various standard maneuvers.

In this paper, the influence of hydrodynamic coefficients on dynamic behavior of AUV is analyzed in different forward speeds and standard maneuvers. The nonlinear model of the Naval Postgraduate School AUV II (Fossen, 1994) is considered. The sensitivity analysis of hydrodynamic coefficients with respect to velocities and position is used to introduce the parameters as (i) non-sensitive coefficients, which produce no uncertainties, (ii) coefficients with low influence on the motion, which may be estimated by some adaptation mechanisms, and (iii) more sensitive coefficients which cannot be ignored in controller synthesis.

The rest of the paper is organized as follows: In section 2, the six-degree of freedom mathematical model of an AUV is given. The direct sensitivity analysis is introduced in section 3 and applied in some standard maneuvers with different forward speeds and propellers. In section 4, the hydrodynamic coefficients are clas-

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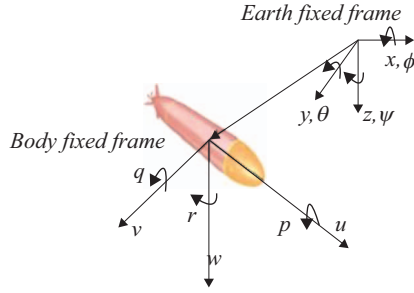


Fig. 1. Body-fixed and earth-fixed reference frames for AUV [Forces, 2013].

sified, based on various illustrative analyses. The conclusion remarks are finally presented in section 5.

## II. MATHEMATICAL MODEL

An AUV with six degrees of freedom is described by defining the position and orientation variables in the earth-fixed frame and linear and angular velocities in the body-fixed coordination, as depicted in Fig. 1.

A general structure for the equations of motion may be given by (Fossen, 1994)

$$M\dot{X} = f \quad (1)$$

where  $M$  is the inertia matrix, including the added mass and rigid-body mass,  $X = [u, v, w, p, q, r]^T$  denotes the vector of linear and angular velocities, and  $f = [f_1, f_2, f_3, f_4, f_5, f_6]^T$  involves the hydrodynamic forces and moments, acting on the AUV, i.e.,

$$\begin{aligned} f_1 = & \frac{\rho L^4}{2} [X'_{pp} p^2 + X'_{qq} q^2 + X'_{rr} r^2 + X'_{pr} pr] + \frac{\rho L^3}{2} [X'_{wq} wq [ + X'_{vp} vp + X'_{vr} vr + uq (X'_{q\delta_s} \delta_s + X'_{q\delta_b} \delta_b X'_{q\delta_{\frac{b}{2}}} \delta_{bs} ) + X'_{r\delta_s} \delta_r ] \\ & + \frac{\rho L^2}{2} [X'_{vv} v^2 + X'_{ww} w^2 + X'_{v\delta_r} uv\delta_r + uw (X'_{w\delta_s} \delta_s + X'_{w\delta_b} \delta_b X'_{w\delta_{\frac{b}{2}}} \delta_{bs} ) + u^2 (X'_{\delta_s \delta_s} \delta_s^2 + X'_{\delta_b \delta_b} \delta_b^2 + X'_{\delta_r \delta_r} \delta_r^2 )] \\ & - (W - B) \sin\theta + \frac{\rho L^3}{2} X'_{q\delta_b} \delta_{sn} \varepsilon(n) + \frac{\rho L^2}{2} [X'_{w\delta_{sn}} uw\delta_{sn} + X'_{\delta_s \delta_{sn}} u^2 \delta_s^2 ] \varepsilon(n) + \frac{\rho L^2}{2} u^2 X'_{prop} + mvr + mx_G (q^2 + r^2) \\ & - my_G pq - mz_G pr \end{aligned} \quad (2)$$

$$\begin{aligned} f_2 = & \frac{\rho L^4}{2} [Y'_{pq} pq + Y'_{qr} qr] + \frac{\rho L^3}{2} [Y'_p up + Y'_r ur + Y'_{vp} vq + Y'_{wp} wp + Y'_{wr} wr] + \frac{\rho L^2}{2} [Y'_v uv + Y'_{vw} vw + Y'_{\delta_s} u^2 \delta_r ] \\ & - \frac{\rho}{2} \int_{x_{tail}}^{x_{nose}} [C_{dy} h(x)(v + xr)^2 + C_{dz} b(x)(w - xq)^2] \frac{(v + xr)}{U_{cf}(x)} dx + (W - B) \cos\theta \sin\phi + mwp - mx_G pq + my_G (p^2 + r^2) - mz_G \end{aligned} \quad (3)$$

$$\begin{aligned} f_3 = & \frac{\rho L^4}{2} [Z'_{pp} p^2 + Z'_{pr} pr + Z'_{rr} r^2] + \frac{\rho L^3}{2} [Z'_q uq + Z'_{vp} vp + Z'_{vr} vr] + \frac{\rho L^2}{2} [Z'_w uw + Z'_{vv} v^2 + u^2 (Z'_{\delta_s} \delta_s + Z'_{\delta_b} \delta_b X'_{\delta_{\frac{b}{2}}} \delta_{bs} ) \\ & + \frac{\rho}{2} \int_{x_{tail}}^{x_{nose}} [C_{dy} h(x)(v + xr)^2 + C_{dz} b(x)(w - xq)^2] \frac{(w + xq)}{U_{cf}(x)} dx] + (W - B) \cos\theta \cos\phi + \frac{\rho L^3}{2} Z'_{qn} uq \varepsilon(n) \\ & + \frac{\rho L^2}{2} [Z'_{wn} uw + Z'_{\delta_s} u^2 \delta_s ] \varepsilon(n) + muq - mvp - mx_G pq - my_G pr + mz_G (p^2 + q^2) \end{aligned} \quad (4)$$

$$\begin{aligned} f_4 = & \frac{\rho L^5}{2} [K'_{pq} pq + K'_{qr} qr] + \frac{\rho L^4}{2} [K'_p up + K'_r ur + K'_{vq} vq + K'_{wp} wp + K'_{wr} wr] + \frac{\rho L^3}{2} [K'_v uv + K'_{vw} vw + u^2 (K'_{\delta_b} \delta_b X'_{\delta_{\frac{b}{2}}} \delta_{bs} ) \\ & + (y_G W - y_B B) \cos\theta \cos\phi - (z_G W - z_B B) \cos\theta \sin\phi + \frac{\rho L^4}{2} K'_{pn} up \varepsilon + \frac{\rho L^3}{2} u^3 K'_{prop} \\ & - (I_z - I_y) qr - I_{xy} pr + I_{yz} (q^2 + r^2) + I_{xz} pr + my_G uq - my_G vq + mz_G (ur - wp) \end{aligned} \quad (5)$$

$$\begin{aligned}
f_5 = & \frac{\rho L^5}{2} \left[ M'_{pp} p^2 + M'_{pr} pr + M'_{rr} r^2 \right] + \frac{\rho L^4}{2} \left[ M'_q uq + M'_{vp} vp + M'_{vr} vr \right] + \frac{\rho L^3}{2} \left[ M'_w uw + M'_{vw} v^2 + u^2 \left( M'_{\delta_s} \delta_s + M'_{\delta_b} \delta_b + K'_{\delta_b} \delta_{bp} \right) \right] \\
& - \frac{\rho}{2} \int_{x_{tail}}^{x_{nose}} \left[ C_{dy} h(x)(v+xr)^2 + C_{dz} b(x)(w-xq)^2 \right] \frac{(w+xq)}{U_{cf}(x)} dx - (x_G W - x_B B) \cos \theta \cos \phi - (z_G W - z_B B) \sin \theta \\
& + \frac{\rho L^4}{2} M'_{qn} up \varepsilon(n) + \frac{\rho L^3}{2} \left[ M'_{wn} uw + M'_{\delta_{sn}} u^2 \delta_s \right] \varepsilon(n) - (I_x - I_z) pr + I_{xy} qr + I_{xz} (p^2 + r^2) + mx_G (vp - uq) + mz_G (-vr - wq)
\end{aligned} \tag{6}$$

$$\begin{aligned}
f_6 = & \frac{\rho L^5}{2} \left[ N'_{pq} pq + N'_{qr} qr \right] + \frac{\rho L^4}{2} \left[ N'_p up + N'_r ur + N'_{vq} vq + N'_{wp} wp + N'_{wr} wr \right] + \frac{\rho L^3}{2} \left[ K'_v uv + K'_{vw} vw + N'_{\delta_r} u^2 \delta_r \right] \\
& - \frac{\rho}{2} \int_{x_{tail}}^{x_{nose}} \left[ C_{dy} h(x)(v+xr)^2 + C_{dz} b(x)(w-xq)^2 \right] \frac{(w+xq)}{U_{cf}(x)} dx + (y_G W - y_B B) \cos \theta \cos \phi \\
& - (z_G W - z_B B) \sin \theta + \frac{\rho L^3}{2} u^3 N'_{prop} - (I_y - I_x) pq - I_{xy} (p^2 + q^2) + I_{yz} pr + I_{xz} qr - mx_G (ur - wp) + my_G (wq - vr)
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
U_{cf}(x) &= \left[ (v+xr)^2 + (w-xq)^2 \right]^{\frac{1}{2}}, \quad X'_{prop} = C_{d0} (\eta|\eta| - 1) \\
\eta &= 0.012 \frac{n}{u}, \quad C_{d0} = 0.00385, \quad C_{dz} = 0.6, \quad C_{t1} = 0.008 \frac{L^2}{2.0} \\
C_t &= 0.008 L^2 \frac{\eta|\eta|}{2.0}, \quad \varepsilon(n) = -1 + \frac{\text{sign}(n) (\sqrt{C_t + 1} - 1)}{\text{sign}(u) (\sqrt{C_{t1} + 1} - 1)}
\end{aligned}$$

In (2)-(7)  $\phi$ ,  $\theta$  are respectively the roll and pitch angles,  $X'$ ,  $Y'$ ,  $Z'$ ,  $K'$ ,  $M'$ ,  $N'$  with any indices are the hydrodynamic coefficients and the rest of parameters are introduced in Table 1.

and angular velocities ( $X_i$ ).

The direct method for sensitivity analysis can be described by rewriting Eq. (1) as

### III. DIRECT SENSITIVITY ANALYSIS

The influence of hydrodynamic coefficients on the system performance maybe calculated by a sensitivity analysis method. Sensitivity is defined as how the system response is affected by changing in hydrodynamic coefficients. The elements of the sensitivity matrix can be approximated by:

$$S_i^j = \frac{\partial X_i}{\partial \Theta_j} = \frac{X_i(\Theta_j - \Delta\Theta_j) - X_i(\Theta_j)}{\Delta\Theta_j} \tag{8}$$

$i = 1, 2, \dots, 6, \quad j = 1, 2, \dots, N_c$

where  $X_i$  may be a linear or angular velocity and  $\Theta_j$ ,  $N_c$  denotes hydrodynamic coefficients and the number of such coefficients, respectively.  $S_i^j$  denotes the sensitivity value of the hydrodynamic coefficient ( $\Theta_j$ ) with respect to the linear

$$M \frac{dX}{dt} = f(X, \Theta) \tag{9}$$

The inertia  $M$  can be considered as an invertible constant matrix for a deeply submerged vehicle (Perrault et al., 2003b). So, multiplying the both sides of (9) by the inverse of inertia matrix  $M$  yields

$$\frac{dX}{dt} = M^{-1} f(X, \Theta) = g(X, \Theta) \tag{10}$$

By differentiating (10) with respect to the hydrodynamic coefficients  $\Theta_j$ , one obtains

$$\frac{\partial}{\partial \Theta} \frac{dX}{dt} = \frac{d}{dt} \frac{\partial X}{\partial \Theta} = \frac{\partial g}{\partial X} \frac{\partial X}{\partial \Theta} + \frac{\partial g}{\partial \Theta} = M^{-1} \frac{\partial f}{\partial X} S + M^{-1} \frac{\partial f}{\partial \Theta}(X, \Theta) \tag{11}$$

**Table 1. The AUV model parameters.**

Parameter	Description
$m$	Mass of the vehicle
$W$	Submerged weight of the vehicle
$B$	Weight of water displaced by the vehicle
$(x_G, y_G, z_G)$	Body fixed coordinates for center of gravity
$I_x$	Moment of inertia about x-axis
$I_y$	Moment of inertia about y-axis
$I_z$	Moment of inertia about z-axis
$I_{xy}$	Products of inertia xy
$I_{xz}$	Products of inertia xz
$I_{yz}$	Products of inertia yz
$(x_B, y_B, z_B)$	Body fixed coordinates for center of buoyancy
$L$	Vehicle length
$\rho$	Density of water
$h(x)$	Width of vehicle at body center along the y-axis
$b(x)$	Height of vehicle at body center along the z-axis
$\delta_r$	Rudder angle
$\delta_s$	Port and starboard stern plane
$\delta_{sn}$	Top and bottom bow plane
$\delta_{bp}$	Port bow plane
$\delta_{bs}$	Starboard bow plane
$n$	Propeller shaft speed

where  $S = \frac{\partial X}{\partial \Theta}$  is the sensitivity matrix. So, the Sensitivity equation is obtained as

$$\dot{S} = M^{-1} \left[ \frac{\partial f}{\partial X} S + \frac{\partial f}{\partial \Theta} \right] \quad (12)$$

Due to different physical units of system variables and hydrodynamic coefficients, comparing the sensitivity values is not reasonable.

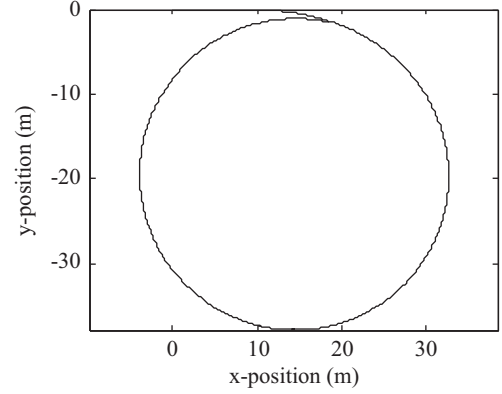
Removing such drawback, the normalized sensitivity matrix can be defined as

$$S_i^j = \frac{\Theta_j}{X_{i,std}} \frac{\partial X_i}{\partial \Theta_j} \quad (13)$$

where

$$X_{i,std} = \left[ 1.0 \frac{m}{s}, 1.0 \frac{m}{s}, 1.0 \frac{m}{s}, 0.01 \frac{rad}{s}, 0.01 \frac{rad}{s}, 0.01 \frac{rad}{s} \right]^T.$$

As the state value  $X_i$  near zero causes undesirable large value of normalized sensitivity, the standard value of state  $X_{i,std}$  is used instead of  $X_i$ .

**Fig. 2. 15° Turning circle maneuver.**

To determine the overall effect of each hydrodynamic coefficient, the sensitivity value of a coefficient with respect to each motion state variable ( $\tilde{S}_i^j$ ) and the total sensitivity value ( $\tilde{S}_i$ ) are respectively defined as

$$\tilde{S}_i^j = \frac{\sum_{k=1}^{N_s} S_i^j(k)}{\sum_{j=1}^{N_s} \sum_{k=1}^{N_s} S_i^j(k)} 100\% \quad (14)$$

and

$$\tilde{S}_i = \frac{1}{6} \sum_{i=1}^6 \tilde{S}_i^j \quad (15)$$

where  $N_s$  is the total number of sampling steps. To exactly determine the sensitive parameters, sensitivity values with respect to the position are obtained as

$$\tilde{S}_{\eta}^j = J \tilde{S}_i^j \quad (16)$$

where  $J$  represents a transformation matrix between the earth-fixed and the body fixed frames. The total sensitivity value with respect to position ( $\tilde{S}_{\eta}$ ) is calculated, similar to (15).

The sensitivity criteria (14), (15) and the total sensitivity value  $\tilde{S}_{\eta}$  are adapted here to classify the hydrodynamic coefficients in the turning circle, horizontal and vertical zigzag maneuvers, as the standard motion trajectories.

#### IV. CLASSIFICATION OF HYDRODYNAMIC COEFFICIENTS

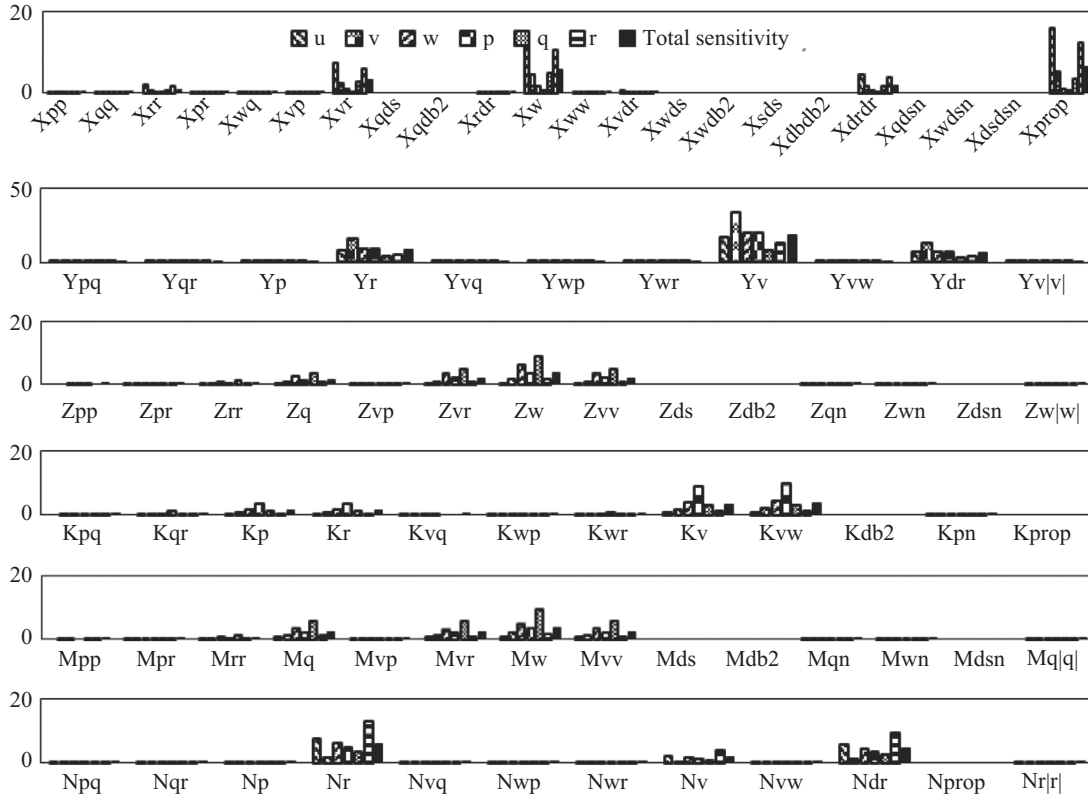
The hydrodynamic coefficients' sensitivity values are calculated here with respect to linear and angular velocities and positions by using different standard maneuvers.

##### 1. Turning Circle Maneuver

By applying the zero yaw rate and commanding the rudder

**Table 2. Classification of coefficients in turning circle maneuver.**

Initial conditions	Direction	Coefficients with low sensitivity	The most sensitive coefficients
u = 2 m/s n = 400 rpm	Surge	-	$X_{rr}, X_{vr}, X_{vv}, X_{\delta_r \delta_r}, X_{prop}$
	Sway	$Y_{ v }$	$Y_r, Y_v, Y_{\delta_r}$
	Heave	$Z_q, Z_{rr}$	$Z_w, Z_{vv}, Z_{vr}$
	Roll	$K_r, K_{qr}, K_p, K_{wr}$	$K_v, K_{vw}$
	Pitch	$M_{rr}, M_{q q }$	$M_q, M_{vr}, M_w, M_{vv}$
	Yaw	-	$N_r, N_v, N_{\delta_r}$
u = 7 m/s n = 750 rpm	Surge	-	$X_{rr}, X_{vr}, X_{vv}, X_{\delta_r \delta_r}, X_{prop}$
	Sway	$Y_{ v }$	$Y_r, Y_v, Y_{\delta_r}$
	Heave	$Z_{vp}, Z_{rr}$	$Z_w, Z_{vv}, Z_{vr}, Z_q$
	Roll	$K_{vq}, K_r, K_{wr}, K_p$	$K_v, K_{vw}, K_{qr}$
	Pitch	$M_{rr}, M_{q q }$	$M_q, M_{vr}, M_w, M_{vv}$
	Yaw	-	$N_r, N_v, N_{\delta_r}$



**Fig. 3. The coefficients' sensitivity values (%) in turning circle maneuver for  $u = 2 \frac{m}{s}$ ,  $n = 400 \text{rpm}$ .**

with a fixed deflection angle, the AUV enters into a circle trajectory. The turning circle maneuvering is demonstrated in Fig. 2. Common values for the rudder angle are limited to  $\delta_r \in [-20, 20]$  (Ernani et al., 2015).

By adopting the forward speed  $u = 2 \text{ m/s}$  and propeller  $n = 400 \text{ rpm}$ , the sensitivity values and the total sensitivity with respect to linear and angular velocities are demonstrated in Fig. 3. Similarly, the sensitivity analysis is presented for  $u = 7 \text{ m/s}$ ,  $n =$

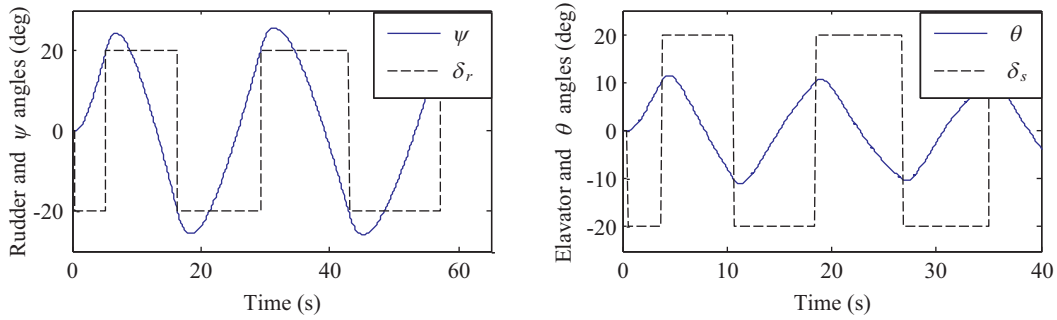


Fig. 4. Rudder and yaw, and elevator and pitch angles in zigzag maneuver.

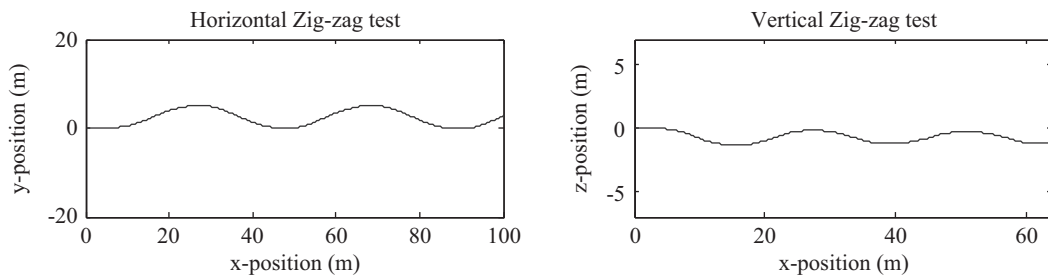


Fig. 5.  $20^\circ/10^\circ$  horizontal zigzag maneuver and  $20^\circ/10^\circ$  vertical zigzag maneuver.

750 rpm (Geridonmez, 2007).

Based on the results of sensitivity analysis with respect to velocities, the parameters can be classified into two categories, as non-sensitive coefficients and sensitive ones. So, the conditions are considered as

If  $\tilde{S}_i < 8.7\%$  (surge),  $\tilde{S}_i < 6.1\%$  (sway),  $\tilde{S}_i < 0.19\%$  (heave),  $\tilde{S}_i < 0.18\%$  (roll),  $\tilde{S}_i < 0.28\%$  (pitch),  $\tilde{S}_i < 1.7\%$  (yaw), the hydrodynamic coefficients are non-sensitive parameters which produce no uncertainties.

The sensitivity analysis with respect to position may be also used to accurately classify the coefficients. Taking  $\tilde{S}_{xy}^j, \tilde{S}_{xz}^j$  as the mean values of sensitivity with respect to x-y and x-z positions, respectively:

If  $\tilde{S}_{xy}^j < 1.7\%$  and  $\tilde{S}_{t_\eta} < 1.32\%$  (surge),  $\tilde{S}_{xy}^j < 7.2\%$  and  $\tilde{S}_{t_\eta} < 5.44\%$  (sway),  $\tilde{S}_{xy}^j < 0.39\%$  and  $\tilde{S}_{t_\eta} < 1.15\%$  (heave),  $\tilde{S}_{xy}^j < 0.38\%$  and  $\tilde{S}_{t_\eta} < 9.2\%$  (roll),  $\tilde{S}_{xy}^j < 0.56\%$  and  $\tilde{S}_{t_\eta} < 1.05\%$  (pitch),  $\tilde{S}_{xy}^j < 1.68\%$  and  $\tilde{S}_{t_\eta} < 2.1\%$  (yaw), then, the hydrodynamic coefficients have a low influence on the motion, and other coefficients have a major influence on the motion. The results of these criteria are given in Table 2. Indeed, to classify the hydrodynamic coefficients into three categories, the above criteria have been selected based on the minimum and maximum sensitivity values of hydrodynamic coefficients for each degree of freedom.

## 2. Zigzag Maneuver

As the start of this maneuvering, the AUV moves with steady speed and zero rudder (elevator) angle. Then, the rudder (elevator) angle is deviated and maintained until the yaw (pitch) angle gets a specified value. In the next stage, the rudder (elevator) angle is deviated to the opposite direction until the yaw (pitch) angle reaches to a fixed value and the cycle is repeated (see Fig. 4).

Common values for the rudder (elevator) angle is  $20^\circ/20^\circ$ ,

$20^\circ/10^\circ, 10^\circ/10^\circ$ , where the first angle refers to the rudder (elevator) angle setting, whereas the second angle denotes how much the yaw (pitch) angle should change before the rudder (elevator) is reversed. The horizontal and vertical zigzag trajectory in x-y and z-x are demonstrated in Fig. 5.

For the standard maneuvers, the sensitivity analysis is applied under  $u = 2$  m/s,  $n = 400$  rpm,  $u = 7$  m/s,  $n = 750$  rpm, and  $u = 12.5$  m/s,  $n = 1500$  rpm (Geridonmez et al., 2007). Considering  $u = 2$  m/s,  $n = 400$  rpm, the sensitivity values and the total sensitivity with respect to linear and angular velocities for horizontal and vertical zigzag are demonstrated in Figs. 6 and 7, respectively.

To make a classification for the parameters, the following criteria are considered:

In horizontal zigzag, if  $\tilde{S}_i < 0.1\%$  (surge),  $\tilde{S}_i < 0.2\%$  (sway),  $\tilde{S}_i < 0.1\%$  (heave),  $\tilde{S}_i < 0.11\%$  (roll),  $\tilde{S}_i < 0.2\%$  (pitch),  $\tilde{S}_i < 0.1\%$  (yaw), and

In vertical zigzag, if  $\tilde{S}_i < 0.013\%$  (surge),  $\tilde{S}_i < 0.017\%$  (sway),  $\tilde{S}_i < 0.003\%$  (heave),  $\tilde{S}_i < 0.6\%$  (roll),  $\tilde{S}_i < 0.0045\%$

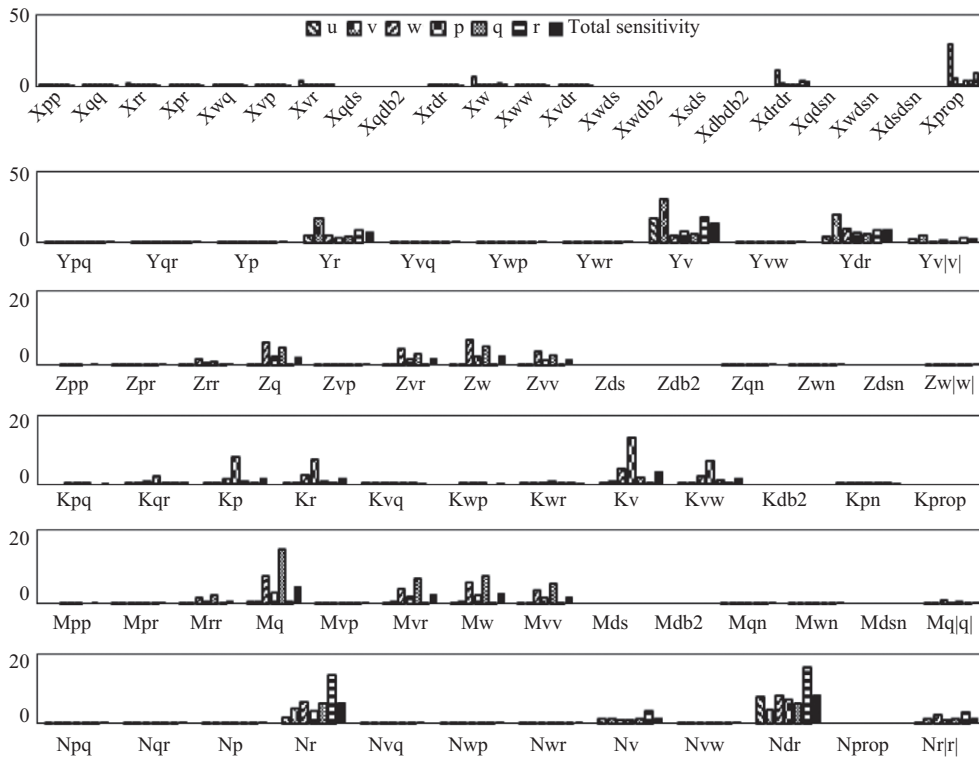


Fig. 6. The coefficients' sensitivity values (%) in horizontal zigzag maneuver for  $u = 2 \frac{m}{s}$ ,  $n = 400 \text{rpm}$ .

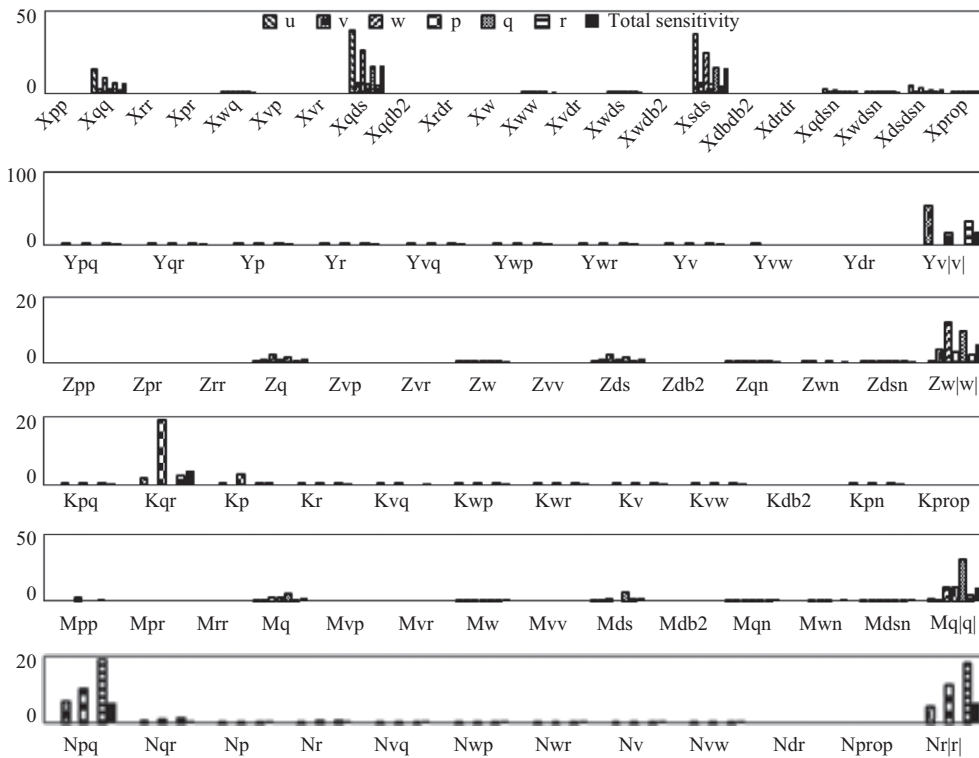


Fig. 7. The coefficients' sensitivity values (%) in vertical zigzag maneuver for  $u = 2 \frac{m}{s}$ ,  $n = 400 \text{rpm}$ .



**Table 3. Classification of coefficients in horizontal zigzag maneuver.**

Initial conditions	Direction	Coefficients with low sensitivity	The most sensitive coefficients
u = 2 m/s n = 400 rpm	Surge	$X_{r\delta_r}$	$X_{rr}, X_{vr}, X_{vv}, X_{\delta_r\delta_r}, X_{prop}$
	Sway	-	$Y_r, Y_v, Y_{\delta_r}, Y_{ v }$
	Heave	$Z_{rr}, Z_{vv}, Z_{vr}, Z_q$	$Z_w$
	Roll	$K_r, K_{qr}, K_p, K_{wr}, K_v, K_{vw}, K_{vq}$	$Z_{pr}, Z_{rr}, Z_{vp}$ -
	Pitch	$M_{rr}, M_w, M_{vv}$	$M_q$
	Yaw	$N_{r r }$	$N_r, N_v, N_{\delta_r}$
u = 12.5 m/s n = 1500 rpm	Surge	$X_{pp}, X_{wq}, X_{r\delta_r}$	$X_{rr}, X_{vr}, X_{vv}, X_{\delta_r\delta_r}, X_{prop}$
	Sway	$Y_p, Y_{wp}$	$Y_r, Y_v, Y_{\delta_r}, Y_{ v }$
	Heave	$Z_{pr}, Z_{rr}, Z_{vp}$	$Z_w, Z_q, Z_{\delta_s}, Z_{\delta_{sn}}, Z_{w v }$
	Roll	$K_r, K_{qr}, K_{wr}, K_{pn}, K_{vq}$	$K_p, K_v, K_{vw}$
	Pitch	$M_{rr}, M_{pr}, M_{q q }$	$M_{vr}, M_q, M_w, M_{vv}$
	Yaw	$N_{pq}, N_p, N_{r r }$	$N_r, N_v, N_{\delta_r}$

**Table 4. Classification of coefficients in vertical zigzag maneuver.**

Initial conditions	Direction	Coefficients with low sensitivity	The most sensitive coefficients
u = 2 m/s n = 400 rpm	Surge	$X_{wq}, X_{q\delta_s}, X_{prop}$	$X_{qq}, X_{w\delta_s}, X_{s\delta_r}, X_{\delta_s\delta_{sn}}, X_{q\delta_{sn}}$
	Sway	$Y_r, Y_{vq}, Y_p, Y_{qr}, Y_{wp}, Y_{ v }, Y_{pq}$	-
	Heave	$Z_{qn}$	$Z_w, Z_q, Z_{\delta_s}, Z_{\delta_{sn}}, Z_{w v }$
	Roll	$K_p, K_{qr}$	-
	Pitch	-	$M_q, M_w, M_{\delta_s}, M_{q q }, M_{qn}, M_{\delta_{sn}}$
	Yaw	$N_{pq}, N_r, N_{r r }, N_{qr}$	-
u = 12.5 m/s n = 1500 rpm	Surge	$X_{wq}, X_{q\delta_s}, X_{prop}$	$X_{qq}, X_{w\delta_s}, X_{s\delta_r}, X_{\delta_s\delta_{sn}}, X_{q\delta_{sn}}$
	Sway	$Y_r, Y_{vq}, Y_p, Y_{qr}, Y_{wp}, Y_{ v }, Y_{pq}$	-
	Heave	$Z_{qn}$	$Z_w, Z_q, Z_{\delta_s}, Z_{\delta_{sn}}, Z_{w v }$
	Roll	$K_p, K_{qr}$	-
	Pitch	-	$M_q, M_w, M_{\delta_s}, M_{q q }, M_{qn}, M_{\delta_{sn}}$
	Yaw	$N_{pq}, N_r, N_{r r }, N_{qr}, N_p, N_{vq}$	-

(pitch),  $\tilde{S}_i < 0.011\%$  (yaw), then, the hydrodynamic coefficients are the non-sensitive parameters which produce no uncertainties.

Next, to accurately classify the coefficients that may cause changes in the output, the following conditions are applied:

In horizontal zigzag, if  $\tilde{S}_{xy}^j < 1.5\%$  and  $\tilde{S}_{t_\eta} < 0.94\%$  (surge),  $\tilde{S}_{xy}^j < 3.2\%$  and  $\tilde{S}_{t_\eta} < 2.5\%$  (sway),  $\tilde{S}_{xy}^j < 0.35\%$  and  $\tilde{S}_{t_\eta} < 0.65\%$  (heave),  $\tilde{S}_{xy}^j < 0.79\%$  and  $\tilde{S}_{t_\eta} < 1.14\%$  (roll),  $\tilde{S}_{xy}^j <$

$0.5\%$  and  $\tilde{S}_{t_\eta} < 1.1\%$  (pitch),  $\tilde{S}_{xy}^j < 1.11\%$  and  $\tilde{S}_{t_\eta} < 1.65\%$  (yaw), and

In vertical zigzag, if  $\tilde{S}_{xz}^j < 2.46\%$  and  $\tilde{S}_{t_\eta} < 1.78\%$  (surge),  $\tilde{S}_{xz}^j < 4.3 \times 10^{-4}\%$  and  $\tilde{S}_{t_\eta} < 2.6\%$  (sway),  $\tilde{S}_{xz}^j < 1.46 \times 10^{-3}\%$  and  $\tilde{S}_{t_\eta} < 0.003\%$  (heave),  $\tilde{S}_{xz}^j < 4 \times 10^{-6}\%$  and  $\tilde{S}_{t_\eta} < 0.4\%$  (roll),  $\tilde{S}_{xz}^j < 1.04 \times 10^{-3}\%$  and  $\tilde{S}_{t_\eta} < 0.006\%$  (pitch),  $\tilde{S}_{xz}^j < 6 \times 10^{-6}\%$  and  $\tilde{S}_{t_\eta} < 2.5\%$  (yaw).

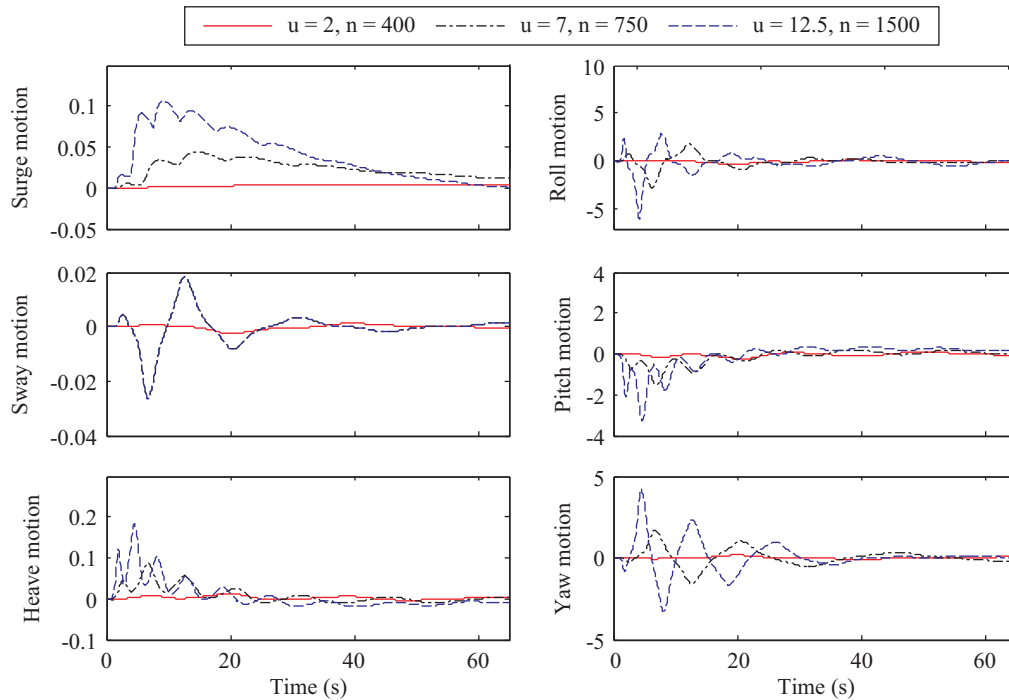


Fig. 8. The normalized sensitive value of hydrodynamic coefficient  $Z_{\alpha}$ , based on different forward speeds and propellers in horizontal zigzag maneuver.

Thus, hydrodynamic coefficients are divided into sensitive and non-sensitive coefficients. The results of these categories has been reported in Tables 3 and 4. Also, the normalized sensitivity value of hydrodynamic coefficients and the number of sensitive parameters are changing with various forward speed and propellers.

As a sample, the normalized sensitivity value of  $Z_{\alpha}$ , is illustrated in Fig 8. Hence, the influence of the hydrodynamic coefficients on the system performance is inevitable in high speed.

## V. CONCLUSION

By performing the sensitivity analysis in different forward speeds and propellers, in standard maneuvers (turning circle, horizontal and vertical zigzag), the influence of hydrodynamic coefficients on dynamic behavior of AUV is investigated. The numerical results are then used to classify the hydrodynamic coefficients as: (i) non-sensitive coefficients which produce no uncertainties, (ii) coefficients with low influence, which may be estimated by some adaptation mechanisms, and (iii) the most sensitive coefficients, which cannot be ignored in controller synthesis. Meanwhile, by increasing forward speed and propellers, the sensitivity of hydrodynamic forces and moments are inevitably increased, especially in horizontal zigzag maneuver.

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