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# NONLINEAR DYNAMICS AND CONTROLLING CHAOS IN A MAGNETIC LEVITATION SYSTEM

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Key words: magnetic levitation, bifurcation, chaos, Lyapunov exponent, state feedback control.

## ABSTRACT

This investigation verifies the chaos motion of a magnetic levitation system with a ferromagnetic ball suspended in a voltage-controlled magnetic field, and explains a system for chaotic control. Then, the detailed dynamic behaviors are numerically investigated by means of Poincaré maps, phase portraits, time responses, and frequency spectra. The results reveal that due to the realistic nonlinear characteristics of magnetic forces, period-doubling bifurcation has been observed to lead to chaos. Chaotic behavior is verified using Lyapunov exponents and Lyapunov dimensions. Finally, we propose a state feedback control technique for the effective control of a chaotic magnetic levitation system. Some simulation results are presented to demonstrate the feasibility of the proposed approach.

## I. INTRODUCTION

Magnetic levitation (maglev) systems are particularly well suited to engineering applications because of their numerous interesting characteristics. They have been widely used in various applications such as high-speed magnetically levitated vehicles, frictionless bearings, vibration isolation of sensitive machinery, and so on. They allow the mechanical components of a system to operate without making contact, hence reducing wear or lubrication. Recently, many studies have addressed the feasibility of electromagnetic levitation in various applications such as magnetic bearings (Samanta and Hirani, 2008; Budig, 2010; Ritonja et al., 2010; Zang et al., 2011; Bachovchin et al., 2012), magnetic levitation suspension (Siyambalapitiya et al., 2012), and high-speed ground transportation (Kong et al., 2011; Yu and Chen, 2011; Min et al., 2012; Yau, 2012). Magnetic levitation systems can also be applied to vibration isolation problems, and

several studies (Nagaya and Ishikawa, 1995; Chang, 2001; Tsuda et al., 2009; Sasaki et al., 2010) have been devoted to controlling the vibrations of magnetic levitation tables.

In practice, machines and devices utilizing such a system exhibit complicated phenomena even though they include control systems. The characteristics of magnetic systems are inherently nonlinear due to the nonlinearities of the electromagnetic fields. Moreover, magnets act as negative springs and cause the system to be unstable, because when the magnetic material is moved closer to the magnet, the magnetic force is increased. To stabilize the system and to eliminate vibrations due to disturbed forces, the negative spring effect must be overcome by controlling the electromagnetic coil current with proper feedback signals such as those from displacement and velocity sensors. However, the highly nonlinear behavior and the inherent instability of such a system complicate the controller design. When this system is subjected to an external disturbance, contact by the levitated object with electromagnets of the system due to large vibration should be avoided.

Therefore, at the design stage, it is necessary to accurately predict the dynamic behaviors of this system over a range of operating conditions. Here, the disturbance may be a shock load of very short duration. In this case, a considerable deviation of the levitated object from the equilibrium state which occurs during machinery start-up or shut-down may cause the system to exhibit very complicated behaviors. In most applications, this situation is not acceptable, so it is important that it be controlled and eliminated. Modern nonlinear theory, which involves bifurcation and chaos, has been widely utilized to study the stability of nonlinear systems (Ghayesh and Amabili, 2013; Jiang and Tao, 2014). Several studies have been carried out to investigate the nonlinear dynamics in a magnetically levitated system (Wang et al., 2007; Zhang et al., 2010).

While studies have focused on the dynamic characteristics of magnetically levitated systems, they do not use the Lyapunov exponent to demonstrate chaos motion. In this work, various numerical analyses, involving a bifurcation diagram, phase portraits, a Poincaré map, frequency spectra, and Lyapunov exponents, are adopted to explicate periodic and chaotic motions of a magnetic ball levitation system with a ferromagnetic ball suspended in a voltage-controlled magnetic field. The algorithms for computing Lyapunov exponents of smooth dynamical systems are well developed (Shimada and Nagashima, 1979; Benettin

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et al., 1980a, b; Wolf et al., 1985) and will be employed in this study to determine whether or not the system exhibits chaotic motion. The method of Lyapunov exponents is applied to verify the occurrence of chaotic motion for the magnetic ball levitation system.

Although chaotic behavior may in some cases be acceptable, it is normally undesirable because it degrades performance and restricts the operating range of numerous electrical and mechanical devices. In many engineering problems involving chaos control, it is important to develop control techniques to drive a chaotic attractor to a periodic orbit. Since the pioneering work of Ott et al. (1990) in controlling chaos, many modified methods and other approaches have subsequently been proposed (Ditto et al., 1990; Hunt, 1991; Cai et al., 2002a; Kecik, 2014). Recently, work regarding the control of a magnetic levitation system has undergone great progress, and several techniques have been proposed (Al-muthairi and Zribi, 2004; Lee et al., 2006; Ahmad and Javaid, 2010; Suster and Jadlovska, 2012). Accordingly, a simple control method based on state feedback properties proposed by Cai et al. (2002b) is used in this paper. This scheme converts chaos into stable motion using feedback combined with the linear state feedback of an available system variable. Chang (2007) also successfully quenched chaotic motion in a steer-by-wire vehicle dynamic system using the state feedback. Chang and Lin (2012) successfully quenched chaotic motion in a permanent magnet synchronous motor for electric vehicles using the linear state feedback property. Finally, numerical simulations have demonstrated the efficiency and feasibility of the proposed approach.

In this study, we investigate the complex dynamic behaviors of a maglev system with suspended ball. In this work, various numerical analyses methods, including a bifurcation diagram, phase portraits, Poincaré maps and frequency spectra, are presented to reveal the complex dynamic behaviors in this system. The Lyapunov exponents and Lyapunov dimensions are used to identify the chaotic motion of the system. The results indicate that the chaotic vibration may be created in the maglev system. Finally, a state feedback control is used to convert chaotic behaviors into periodic motion. In addition, these results will be helpful in stability control for the maglev trains in the future.

## II. DYNAMIC MODEL OF THE MAGNETIC BALL LEVITATION SYSTEM

The magnetic ball levitation system considered in this paper consists of a ferromagnetic ball suspended in a voltage-controlled magnetic field. Fig. 1 shows the schematic diagram of the magnetic ball levitation system. The coil acts as an electromagnetic actuator, while an optoelectronic sensor determines the position of the ferromagnetic ball. By regulating the electric current in the circuit through a controller, the electromagnetic force can be adjusted to be equal to the weight of the steel ball, enabling the ball to levitate in an equilibrium state. Only the vertical motion is considered, and the objective is to keep the ball at a prescribed reference level. The dynamic model of the system

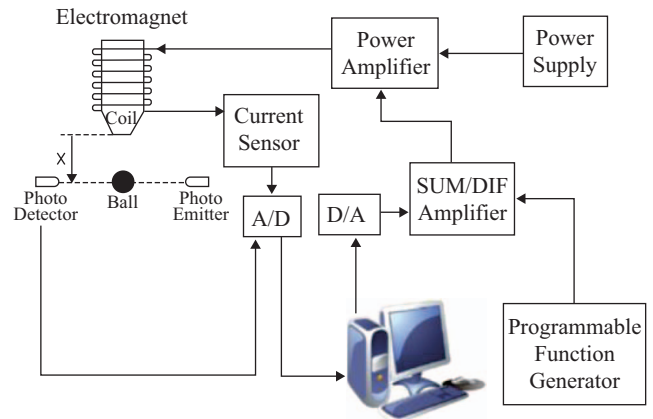


Fig. 1. Schematic diagram of the magnetic ball levitation system.

can be written as (Barie and Chiasson, 1996)

$$\frac{dx}{dt} = v, \quad (1a)$$

$$Ri + \frac{dL(x)i}{dt} = u, \quad (1b)$$

$$m\ddot{x} = mg - C\left(\frac{i}{x}\right)^2, \quad (1c)$$

where  $x$  is the ball position;  $v$  is the ball's velocity;  $i$  is the current in the coil of the electromagnet;  $u$  is the applied voltage ( $u = A_0 \sin \Omega t$ );  $R$  is the coil resistance;  $L$  is the coil inductance;  $g$  is the gravitational acceleration;  $C$  is the magnetic force constant, and  $m$  is the mass of the levitated ball.

The inductance  $L$  is a nonlinear function of the ball's position  $x$ . A typical approximation is to assume that this inductance varies inversely with respect to the ball's position  $x$  (Barie and Chiasson, 1996), which is given by

$$L(x) = L + \frac{L_0 x_0}{x}, \quad (2)$$

where  $L$  is the constant inductance of the coil in the absence of ball,  $L_0$  is the additional inductance contributed by the presence of the ball,  $x_0$  is the equilibrium position. Assuming the suspended ball remains close to its equilibrium position,  $x = x_0$ , and therefore

$$L(x) = L + L_0. \quad (3)$$

Also assuming that  $L \gg L_0$ , Eq. (3) can be simplified as

$$L(x) = L. \quad (4)$$

At equilibrium, the weight of the ball is suspended by the

**Table 1. Physical parameters of a magnetically levitated system.**

Parameter	Unit	Value
$m$	kg	0.05
$g$	m/s <sup>2</sup>	9.8
$R$	$\Omega$	1.0
$L$	H	0.01
$C$		0.0001

electromagnet force that generated by a bias current.

As indicated in Eq. (2), the inductance  $L(x)$  is a nonlinear function of the ball’s position. It is well known that the characteristics of magnetic systems are essentially nonlinear due to the nonlinearities of the electromagnetic fields. Hence, to accurately control or predict the performance of the system, the effects of these nonlinearities should be taken into consideration. Substituting Eq. (2) into Eq. (1b) results in

$$u(t) = Ri + L \frac{di}{dt} - \left( \frac{L_0 x_0 i}{x^2} \right) \frac{dx}{dt}. \tag{5}$$

A conservation of energy argument shows that  $C = L_0 x_0 / 2$  (Barie, 1995). Let the states be chosen such that  $x_1 = x, x_2 = v, x_3 = i$ . Thus, the state-space model of the magnetic levitation system can be written as

$$\frac{dx_1}{dt} = x_2, \tag{6a}$$

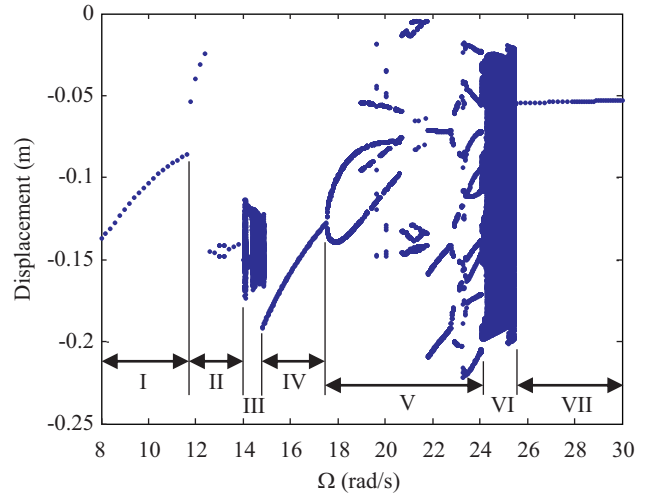
$$\frac{dx_2}{dt} = g - \frac{c}{m} \left( \frac{x_3}{x_1} \right)^2, \tag{6b}$$

$$\frac{dx_3}{dt} = -\frac{R}{L} x_3 + \frac{2C}{L} \left( \frac{x_2 x_3}{x_1^2} \right) + \frac{1}{L} A_0 \sin \Omega t. \tag{6c}$$

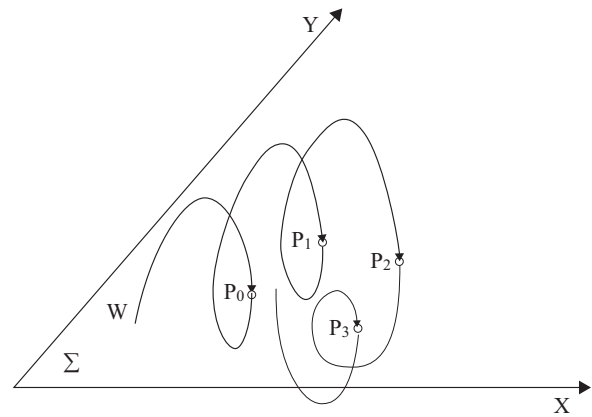
The amplitude of the applied voltage is  $A_0 = 5.0$  Volts. The other parameter values of the above equations are listed in Table 1.

### III. THE OVERALL CHARACTERISTICS OF MAGLEV SYSTEM AND CHAOS ATTITUDE MOTION

A series of numerical simulations based on Eq. (6) were performed to clearly elucidate the characteristics of this system. The dynamic behaviors may be observed more completely over a range of parameter values in the bifurcation diagram. Such diagrams are widely employed to describe transitions from periodic motion to chaotic motion for dynamical systems. The commercial package DIVPRK of IMSL in FORTRAN subroutines for mathematics applications is used to solve these ordinary differential equations (IMSL, 1989). The resulting bifurcation dia-



**Fig. 2. Bifurcation diagram of the system for  $A_0 = 5.0V$ .**



**Fig. 3. Illustration of a Poincaré section.**

gram is shown in Fig. 2. This figure clearly shows that chaotic motions exit in regions III and VI. Period-1 motion appears in regions I, IV, and VII, while a stable equilibrium point is present in regions I, IV, and VII. Herein each response is characterized by a phase portrait, a Poincaré map (velocity vs. phase angle), and a frequency spectrum. To order to detect period or chaos, we select a cross section  $\Sigma$ , where the flow “ $W$ ” must be transverse to  $\Sigma$ . For point  $(X_i, Y_i) \in \Sigma$ , let  $t_0$  be the time of next return to  $\Sigma$ . The map is the Poincaré map, as shown in Fig. 3.

$$P^{t_0}(X_i, Y_i) = (X_{i+1}, Y_{i+1}), \tag{7}$$

Fig. 4 indicates that the equilibrium point of Eq. (6) is stable if the parameter  $\Omega$  exists in region I. When the parameter  $\Omega$  exists in regions II and V, period-doubling bifurcations appear in these regions. Fig. 5 reveals that a cascade of period-doubling bifurcations causes a series of subharmonic components, which show the bifurcations with the new frequency components at  $\Omega/2, 3\Omega/2, 5\Omega/2, \dots$ . As the forcing frequency ( $\Omega$ ) continues to increase into regions III and VI in Fig. 2, a cascade of period-

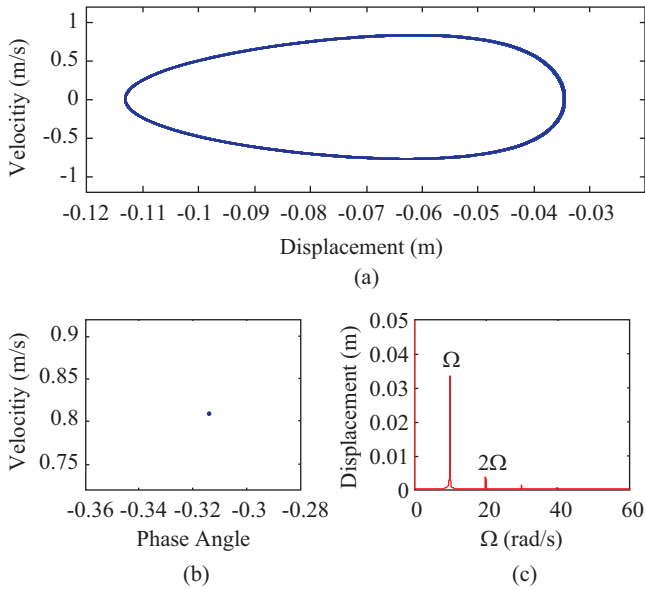


Fig. 4. Period-one orbit for  $\Omega = 10.0$  rad/s: (a) phase portrait; (b) Poincaré map; (c) frequency spectrum.

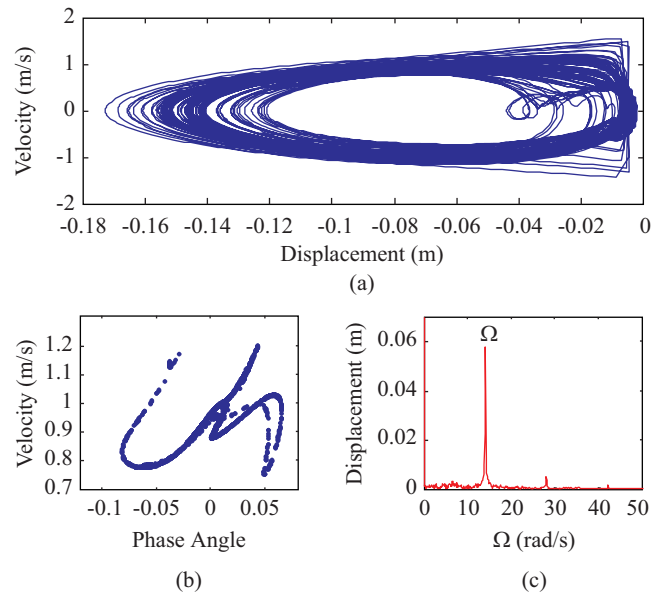


Fig. 6. Chaotic motion for  $\Omega = 14.1$  rad/s: (a) phase portrait; (b) Poincaré map; (c) frequency spectrum.

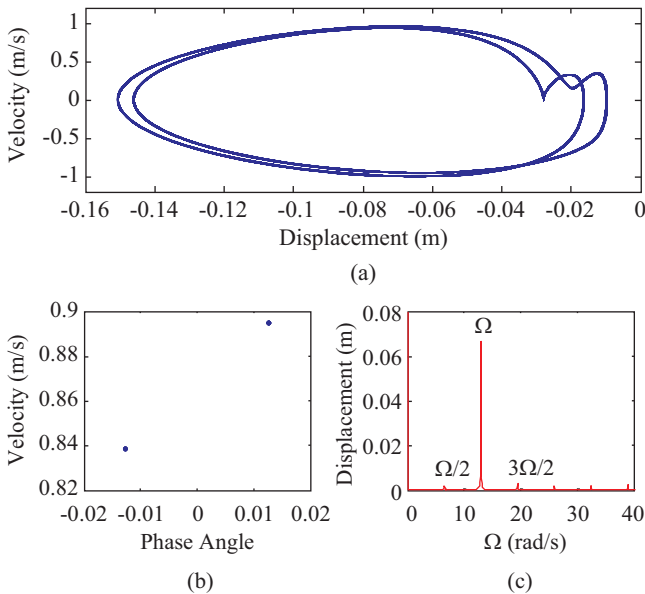


Fig. 5. Period-two orbit for  $\Omega = 13.0$  rad/s: (a) phase portrait; (b) Poincaré map; (c) frequency spectrum.

doubling bifurcations are clearly seen, and lead the system to chaos. Therefore, chatter vibration occurs. Two descriptors, the Poincaré map and the frequency spectrum, characterize the essence of the chaotic behavior. The Poincaré map shows an infinite set of points that are collectively referred to as a strange attractor. The frequency spectrum of the chaotic motion spans a wide range of frequencies. These two features of the strange attractor and the continuous Fourier spectrum are strong indicators of chaos. Fig. 6 clearly shows chaotic behaviors in regions III. Figs. 6(a)-6(c) shows the phase portraits, the Poincaré maps, and the frequency spectra, respectively.

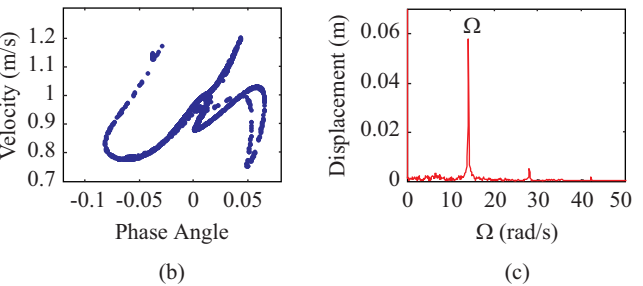


Fig. 7. Evolution of the largest Lyapunov exponent.

#### IV. ANALYSIS OF CHAOTIC PHENOMENA IN MAGLEV BASED ON LYAPUNOV EXPONENT AND LYAPUNOV DIMENSION

The analyses presented in Section III cannot identify likely chaotic motion in the magnetic levitation system, so in this section, the method of Lyapunov exponents is utilized to verify the occurrence of chaotic motion. For every dynamic system, a spectrum of Lyapunov exponents ( $\lambda$ ) (Wolf et al., 1985) indicates the variation of the length, areas, and volumes in the phase space. As a criterion for the existence of chaos, one needs only to calculate the largest exponent, which tells whether nearby trajectories diverge ( $\lambda > 0$ ) or converge ( $\lambda < 0$ ) on average. Any bounded motion in a system with at least one positive Lyapunov exponent is defined as chaotic, while for periodic motion, the Lyapunov

exponents are not positive. Fig. 7 plots the evolution of the largest Lyapunov exponent for the magnetic levitation system, and was computed using the algorithm for calculating the Lyapunov exponents that was presented by Wolf et al. (1985). This figure reveals that the onset of chaotic motion is at about  $\Omega = 14.0$  rad/s and  $24.0$  rad/s, because at these points ( $P_2$  and  $P_4$ ) the sign of the largest Lyapunov exponent changes from negative to positive as the parameter,  $\Omega$ , is slowly increased. At points  $P_1$  and  $P_3$ , the largest Lyapunov exponent is shown to approach zero. At this point, the system may undergo bifurcation.

When  $\Omega$  is larger than  $P_1$ , such as at  $\Omega = 10.0$  rad/s, the Lyapunov exponents given by Eq. (6) are  $\lambda_1 = -1.2699$ ,  $\lambda_2 = -1.2734$  and  $\lambda_3 = -141.7260$ . Their sum is  $\lambda_1 + \lambda_2 + \lambda_3 = -144.2693$ , which is negative, indicating that the motion of the rotor at these values eventually converges to a stable limit cycle. By indicating the Lyapunov exponents of a dynamical system with  $\lambda_1 \geq \dots \geq \lambda_n$ , Kaplan and Yorke (1979) provide an estimation for the Lyapunov dimension  $d_L$  as

$$d_L = j + \frac{1}{|\lambda_{j+1}|} \sum_{i=1}^j \lambda_i, \quad (8)$$

where  $j$  is the largest integer that satisfies  $\sum_{i=1}^j \lambda_i > 0$ . This technique yields a Lyapunov dimension of Eq. (6) for  $\Omega = 10.0$  rad/s, which is  $d_L = 1$ . Because the Lyapunov dimension is an integer, the system has periodic motion. When the parameter  $\Omega$  increases across the bifurcation point, such as at  $\Omega = 25.0$  rad/s, the Lyapunov exponents are  $\lambda_1 = 1.3711$ ,  $\lambda_2 = -5.5069$  and  $\lambda_3 = -140.1338$ , and the Lyapunov dimension is  $d_L = 1.25$ . Because the Lyapunov dimension is not an integer, the system exhibits chaotic motion. Accordingly, for a periodic solution, the Lyapunov dimension equals an integer, but it may not be an integer for a strange attractor.

### V. CONTROLLING CHAOS

To improve the performance of a dynamic system, or to avoid the chaotic behaviors, we need to control a chaotic system to a periodic motion, which is beneficial for working with a particular condition. Therefore, for practicality, it is very important that suitable control methods be developed. Recently, Cai et al. (2002b) suggested a simple and effective control method for converting chaos into periodic motion using the linear state feedback of an available system variable. This approach for the  $n$ -dimensional dynamical system, is explained briefly.

$$\dot{x} = f(x, t), \quad (9)$$

where  $x(t) \in R^n$  is the state vector, and  $f = (f_1, f_2, \dots, f_n, \dots)$ , where  $f_i$  is a linear or a nonlinear function and  $f$  includes at least one nonlinear function. Suppose  $f_i(x, t)$  is the nonlinear function that yields chaotic motion in Eq. (9); then, only one term of the state feedback of the available system variable  $x_m$  is added to the equation that includes  $f_i$ , as follows.

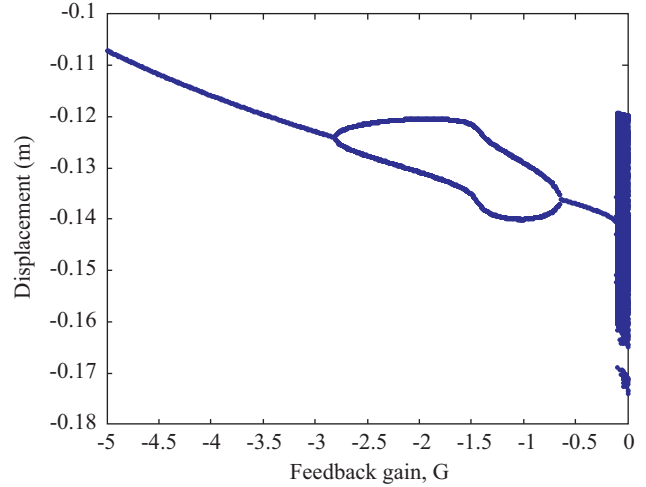


Fig. 8. Bifurcation diagram of system with state feedback control, where  $G$  denotes the feedback gain.

$$\dot{x}_k = f_k(x, t) + G x_m, \quad k, m \in \{1, 2, \dots, n\}, \quad (10)$$

where  $G$  is the feedback gain. Also other functions remain their original forms.

System (6) with state feedback control can be written as follows.

$$\frac{dx_1}{dt} = x_2, \quad (11a)$$

$$\frac{dx_2}{dt} = g - \frac{c}{m} \left( \frac{x_3}{x_1^2} \right)^2 + Gx_2, \quad (11b)$$

$$\frac{dx_3}{dt} = -\frac{R}{L} x_3 + \frac{2C}{L} \left( \frac{x_2 x_3}{x_1^2} \right) + \frac{1}{L} A_0 \sin \Omega t + Gx_3. \quad (11c)$$

In the absence of state feedback control,  $G = 0$ , and system (6) describes chaotic motion for  $\Omega = 14.1$  rad/s, as shown in region III of Fig. 2. Consider the effect of adding state feedback control to the right-hand side of Eq. (6). Fig. 8 displays the resulting bifurcation. This figure clearly reveals that chaotic motion occurs at  $G > -0.1$ , and that the chaotic behavior disappears at about  $G \leq -0.1$ . When the feedback gain,  $G$ , falls below  $-0.1$ , the stable period motion of Eq. (6) appears. By decreasing the feedback gain,  $G$ , between about  $-0.65$  and  $-2.8$ , the period-doubling behavior appears. As the feedback gain,  $G$ , continues to decrease at about  $-2.8$ , the period-one motions take place. The evolution of the largest Lyapunov exponents for the Maglev system with state feedback at  $\Omega = 14.1$  rad/s is displayed in Fig. 9. From this figure, we find that the chaotic motion disappears at  $G \leq T_1$  (about  $T_1 = -0.1$ ), because at this point,  $T_1$ , the largest Lyapunov exponent changes its sign from positive to negative when the feedback gain is slowly decreased. For points,  $T_{2-3}$ ,

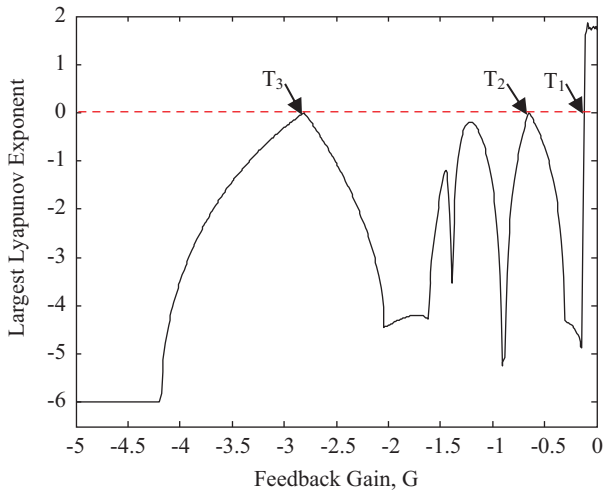


Fig. 9. Largest Lyapunov exponents vs. feedback gains.

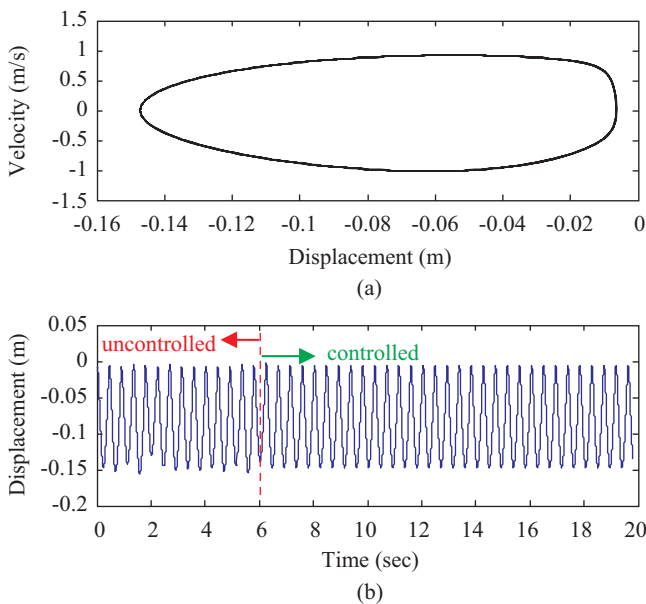


Fig. 10. Transforming chaotic motion to a desired period-one orbit at  $G = -0.5$  and  $\Omega = 14.1$  rad/s: (a) phase portrait of the controlled system (11); (b) time responses of the displacement. The state feedback control signal is introduced after 6 s.

the largest Lyapunov exponents are shown to approach zero. At these points, the system may undergo bifurcations. However, the Lyapunov exponent at such a point provides no means to determine the type of bifurcation; therefore, the bifurcation diagram presented in Fig. 8 must be applied. Therefore, in Fig. 9, the period-doubling bifurcation occurs at  $T_{2,3}$ . In this regard, the state feedback control technique has been proposed so that the Lyapunov exponents of the closed-loop system are negative, and consequently, the chaos is eliminated in the related system. After 6 s, the control signal is applied to the system, as presented in Fig. 10, to show the effectiveness of the designed controller in controlling the chaotic oscillation.

## VI. CONCLUSIONS

This investigation is concerned with the rich dynamic behaviors of a magnetic levitation system, which consists of a ferromagnetic ball that suspended to find an effective method for controlling chaotic vibrations. Dynamic behaviors may be observed over the entire range of parameter values on the bifurcation diagram. This diagram indicates that the magnetic levitation system exhibit period-doubling bifurcation and chaotic motion. The most powerful method for determining whether the system is in chaotic motion involves the use of the Lyapunov exponent. Finally, the state feedback control approach is employed to effectively improve the performance of maglev system or suppress the chaotic motions. Because magnetically levitated vehicles belong to the field of high-speed transportation systems, it is particularly important to investigate their dynamic characteristics from the viewpoint of running stability, safety, and ride quality at high speed. We believe that studying the dynamics and controlling chaotic vibrations of the magnetic ball levitation system will aid in controlling the magnetic levitation trains in the future.

## ACKNOWLEDGMENTS

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