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SUPPLY CHAIN INTEGRATED INVENTORY MODEL CONSIDERING FUZZY DEMAND AND QUANTITY DISCOUNTS WITH UNCERTAIN MANUFACTURING PROCEDURES

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Key words: fuzzy demand, uncertain manufacture procedure, inventory model, quantity discount.

ABSTRACT

This study mainly discusses the best economic production quantity (EPQ) system that can be obtained with the considerations of the possibility of causing defective item production in the uncertain environments. In the current production and manufacturing schedule, the efficiency of a company's operations is often determined by the quality of the supply chain. However, the traditional method for solving the problem pertaining to economic production quantities usually assumes that the defective items and the backorder situations do not exist in the perfect production process. In this study, the system is developed on the basis of the production of the finished good inventory model. Defective products were separated by the system. Then, the nonrepairable defective products were destroyed, and the remaining items were repaired and re-sent to the buyer. Quantity discount was added to this study to represent the influence of the defective product rate on the manufacturing cost. To cooperate with the uncertain manufacturing procedure, fuzzy demand was incorporated into the study to obtain a more realistic and reliable result.

I. INTRODUCTION

The inventory strategy is crucial for firms because it helps in the production and logistics aspects. A comprehensive inventory system can achieve the best level of service and simultaneously reduce the manufacturing and inventory costs to maximize the profits. Based on previous studies, the traditional integrated inventory model usually comprises the perfect production processes that do not involve the production of defective products. However, the problem of defective product production is unavoidable due to human errors, mechanical failures, and other unspecified reasons. Therefore, this study attempts to determine how the defective product rate influences the product costs for buyers and sellers and tries to reduce the losses caused by defective products.

Quantity discounts are often used by suppliers as concessions to attract buyers. However, this study considers that quantity discounts are used to cover the losses caused by defective products, which implies that the buyers pay for the suppliers' losses.

This study employs an integrated supply chain inventory model that includes environmental uncertainties and quantity discounts for minimizing the total cost of the buyer and seller. Moreover, fuzzy demand was applied to the model because of the uncertainties of the buyer's demand and the vendor's defective product rate. Moreover, this study assumes that the production process produces a certain number of defective products. When buyers receive defective products, they return the products to the sellers for repairing. In this case, the vendors offer discounts to the buyers.

The aforementioned points are considered in this study to simulate a realistic situation. The uncertain demand and defective products make the entire manufacture procedure unpredictable. To determine the minimum total cost, the optimal order quantity Q and delivery times per production cycle n should be determined. Then, the first and second-order partial derivative of the expected annual integrated total cost EK (n, Q) with respect to Q and n should be obtained. In this study, the extreme values of n and Q were calculated because the delivery time n is an integer. This study used an interactive method to calculate the optimal solution of n and Q and to compute the minimum total cost.

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After the minimum total cost was obtained, four parameters were included while conducting the sensitivity analysis of EK (n, Q)—screening rate X, annual demand D, percentage of defective products k, and production rate P. The sensitivity analysis results reveal how the influence of these four parameters changed throughout the equation. Subsequently, this study applied the experimental data by using mathematical equations for Q, D, and the EK to assess the three-dimensional map.

Goyal (1976) proposed the integrated inventory model firstly. Then, Goyal (1988) extended the study by analyzing the joint economic-lot-size models proposed by Banerjee (1986) by releasing the lot-for-lot policy. Porteus (1986) was the first researcher to incorporate the influence of defective products into the basic the economic order quantity (EOQ) model. Based on the study, we acknowledged the importance of including the influence of unreliable processes. Schwaller (1988) extended EOQ models to conform to the real-life environment of inventories by adding assumptions of a known proportion of defectives in the incoming lots. Ben-Daya and Hariga (2000) considered the influence of imperfections in the process of a model and assumed that a product has the perfect quality in the initial phase of production. Salameh and Jaber (2000) assumed that the production process and inventory situation, items, or products are not in the perfect quality. Defective and unwanted products can be used in other restrictive procedures, acceptance control production, and inventory situation with the consideration of poor-quality items at the end should be sold out. Goyal and Cardenas-Barron (2002) developed a model to determine the total profit per unit time and purchase products from supply EOQs. Moreover, they proposed a method to determine the best economic production quantity (EPQ) and defective products. Huang (2004) suggested that the use of a model developed under the Just-in time JIT manufacturing environment to determine defective items held by the sellers and the buyers is the best integrated inventory strategy. Huang (2004) also proposed a model that was developed to identify defective products during the continuous consumption of an inventory, and the items that are identified to be defective were reimbursed.

In this study, changes are performed in the inventory quantity discount model and unreliable situations are included. Moreover, the most suitable order quantity from buyers and sellers is determined to achieve a minimized total cost.

In the current highly competitive global markets, many marketing strategies and manufacturers use price discounts to attract consumers. Lal and Staelin (1984) developed a strategy to provide the best price discounts for buyers. Chakravarty and Martin (1988) provided vendors with a method for optimally determining both the discount price and the replenishment interval under a periodic review for the desired joint saving-sharing scheme between the seller and multiple buyers. Munson and Rosenblatt (1988) proposed a third-level quantity discount and a fixed demand rate for a supply chain.

Wang (2005) extended the traditional quantity discount methodology that is solely based on buyers' order sizes by including discount policies that are based on both buyers' order sizes and their annual volume. He revealed that discount policies can be used to achieve nearly optimal system profit and thus provide effective coordination. Yao and Wu (2000) proposed the ranking fuzzy numbers based on decomposition principle and signed distance. Then, Li and Liu (2006) developed a model that explains how to use quantity discount policies to achieve supply chain coordination by considering that only one product is sold after multiple cycles and by considering the probability of the customer's demand in the buyer and seller system. Moreover, they suggested that when the acceptable quantity discount profit is determined mutually by the seller and receiver by using decentralized decision making, the sum of profits of the seller and the receiver increases. Rong and Maiti (2015) analyzed the EOQ model involving fuzzy demand and variable lead time.

In previous studies, fuzzy demand was incorporated into different models to obtain results that are more fitting for real situations. For example, Tu, Lo, and Yang (2010) input the calculation of fuzzy demand into a two-echelon inventory model, and Yang (2014) cooperated fuzzy demand in the Program Evaluation and Review Technique PERT model. Rong and Maiti (2015) investigated the cost minimization inventory model in a fuzzy-stochastic environment by including the decreasing lead time by the crashing cost and the minimum-maximum distribution procedure. Furthermore, Ouyang and Yao (2002) presented a fuzzy continuous-review model with a distribution-free procedure and variable lead time. Based on the different studies, we can conclude that fuzzy demand should be incorporated while developing models to fit the real environment. This study focused on fuzzy demand in the EOQ model. The fuzzy demand was incorporated into different models from many previous studies to obtain results that are more fitting to real situations. For example, Tu, Lo, and Yang (2010) input the fuzzy demand calculation into a two-echelon inventory model. Yang (2014) cooperated fuzzy demand in a PERT model. Rong and Maiti (2015) investigated the cost minimization inventory model in a fuzzy-stochastic environment with lead time crashing cost and a minimum-maximum distribution procedure. Moreover, Ouyang and Yao (2002) presented a fuzzy continuous-review model with the distribution-free procedure and variable lead time. Based on the different studies, we concluded that fuzzy demand should be incorporated while building models to fit the real environment. This study focused on fuzzy demand in the EOQ model.

Yang et al. (2010) established an inventory model for retailers in a supply chain when a supplier offers either a cash discount or a delay payment for an ordered quantity. Lin and Lin (2014) developed a model pertaining to defective products and quantity discounts. The purpose of the model was to determine the optimal pricing and ordering strategy. The analysis was based on the buyer's order quantity. Zhang and Xu (2014) proposed a multiple objective decision making model for a bifuzzy environment and quantity discount policy. Quantity discount was an important factor in their study.

Some related researches were listed below. Pan and Yang (2008) were proposed an integrated inventory models with fuzzy annual demand and fuzzy production rate in a supply chain. Ho and Lin (2011) illustrated the integrated inventory model with quantity discount and price-sensitive demand. Then, Chiu et al., (2014) applied the fuzzy multi objective integrated logistics model to green supply chain problems. Yang et al., (2016) proposed an integrated multi-echelon logistics model with uncertain delivery lead time and quality unreliability. And Hsiao et al., (2017) proposed the research of deteriorating inventory model for Ready-to-eat food under fuzzy environment.

The studies in earlier sections mainly focus on price promotions, discounts, and strategies because these factors can have a direct impact on cost and profit. However, the refreed studies which we listed that ignored the fact that different quantity discount policy may adversely influence the profit. Therefore, the manuscript determined the discounts for various quantities based on the defective product rate. Due to the uncertain environments, fuzzy demand must be incorporated in the study to obtain results that are fit for real conditions.

II. MATERIALS AND METHODS

The following notations and assumptions are discussed throughout this paper to establish the proposed model.

1. Notations

S_v : Set-up cost for the vendor, dollars/time
Q : The number of products transported from the buyer each time, pieces/times
P : Production rate, pieces/year
R : Recovery cost for the vendor; dollars/month
L : Maintenance cost for the vendor; dollars/month
n : Number of deliveries in each production cycle, times
h_v : Holding cost for the vendor, dollars/month
V : Warranty cost for the buyer, dollars/month
Y : The percentage of defective products considered as random variables
$\tilde{Y}$: Triangular fuzzy number; $\tilde{Y} = (Y - \Delta_3, Y, Y + \Delta_4)$, $0 < \Delta_3 < \Delta$, $0 < \Delta_4$. Here, Δ_3 and Δ_4 are determined by the decision maker.
Q_r : Manufacturing cost of the vendor; dollars/month
S_b : Order cost for the buyer, dollars/time
F : Transportation cost per shipment, dollars/trip
$\tilde{D}$: Triangular fuzzy number; $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$ $0 < \Delta_1 < D$, $0 < \Delta_2$. Here, Δ_1 and Δ_2 are determined by the decision maker.
h_B : Holding cost for the buyer, dollars/month
d : Screening cost for the buyer, dollars/month
X : Screening rate, piece/year
σ : Discount rate; $\sigma = m \times \tilde{Y} \times k$. Here, the punishment multiple m is determined by the sellers themselves.

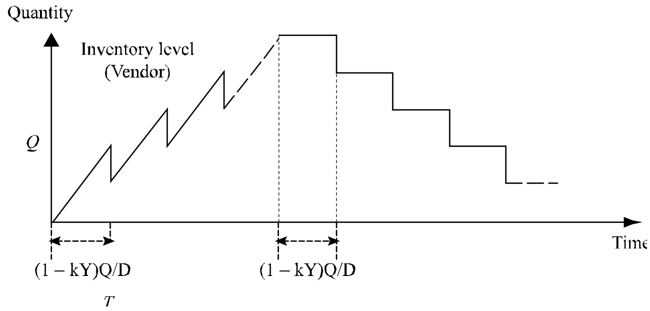

Fig. 1. Schematic of the vendor's cost.

B : Purchase cost for the buyer, dollars/month
T : Successive time interval for each transport
T_c : Cycle time; $T_C = n \times T$
K : Percentage of defective products that cannot be repaired
EK : Expected annual integrated total cost
TrC_V : Vendor transportation cost
HC_V : Vendor holding cost
TC_B : Total cost for the buyer
HC_v : Total holding cost for the vendor
HC_B : Total holding cost for the buyer

Assumptions

(1) This study included a single vendor and a single buyer for a single item.
(2) The production rate is finite.
(3) Shortage is not allowed.
(4) As shortage is not allowed, the production rate of nondefective products should be higher than the buyer's demand.
(5) Quantity discount and defective product rate are directly related.
(6) The returned defective products will be repaired, but not fully.
(7) When a buyer's inventory is Q/2, all products must be inspected. Moreover, defective products must be picked up and sent back to the vendor.
(8) Quantity discount has a restriction because the vendor's cost cannot be more than the buyer's purchasing cost. Otherwise, the vendor will not have any profits.

$$v + \frac{Q_r}{\tilde{D}} + \sigma B < B \Rightarrow \sigma < 1 - \frac{\left(\frac{Q_r}{\tilde{D}} + v\right)}{B}$$

The discount rate is assumed to be $\sigma = m\tilde{Y}k$ (m is a magnification determined by the vendor). The discount rate of the buyer increases with the number of defective products.

2. Vendor's Cost

Vender's cost = setup cost + transportation cost + manufacturing cost + recovery cost + maintenance cost + holding cost

$$TC_V(Q,n) = S_V \times \frac{\tilde{D}}{nQ\left(1-k\tilde{Y}\right)} + F\left(1+2\tilde{Y}-k\tilde{Y}\right)\times\frac{\tilde{D}}{Q\left(1-k\tilde{Y}\right)}$$

$$+Q_r\tilde{D} + Rk\tilde{Y}\tilde{D} + L\tilde{Y}\tilde{D} + h_v Q\left[\frac{n-1}{2} + \frac{\tilde{D}(2-n)}{2p\left(1-k\tilde{Y}\right)}\right]$$

Adjust the former formula by taking $\tilde{D}$ as common.

$$\Rightarrow TC_V(Q,n) = \tilde{D}\left[\frac{S_V}{nQ\left(1-k\tilde{Y}\right)} + \frac{F\left(1+2\tilde{Y}-k\tilde{Y}\right)}{Q\left(1-kY\right)} + Qr + Rk\tilde{Y}\right.$$

$$\left. + L\tilde{Y} + \frac{h_v Q(2-n)}{2p\left(1-k\tilde{Y}\right)}\right] + \frac{h_v Q(n-1)}{2}$$

Definition 1. Kaufmann and Gupta (1991), Zimmermann (1996), and Yao and Wu (2000) suggested that for a fuzzy set $\tilde{B}\in\Omega$ and [0,1], the α cut-off of the fuzzy set $\tilde{B}$ is $\mathrm{B}(\alpha) = \left\{x\in\Omega \mid \mu_B(x)\geq\alpha\right\} = \left[B_L(\alpha), B_U(\alpha)\right]$, where $B_L(\alpha) = a+\alpha(b-d)$ and $B_U(\alpha) = c-\alpha(c-b)$. We can obtain the following equation. The distance between $\tilde{B}$ and $\tilde{0}_1$ is defined as follows:

$$d\left(\tilde{B},\tilde{0}_1\right) = \int_0^t d\left\{\left[B_L(\alpha), B_U(\alpha)\right], \tilde{0}_1\right\} d\alpha$$

$$= \frac{1}{2}\int_0^1 \left[B_L(\alpha) + B_U(\alpha)\right] d\alpha.$$

Thus, this equation becomes

$$d(\tilde{B},\tilde{0}_1) = \frac{1}{2}\int_0^1 \left[B_L(\alpha), B_U(\alpha)\right] d\alpha$$

$$= \frac{1}{2}\int_0^1 \left[B_L(\alpha) + B_U(\alpha)\right] d\alpha = \frac{1}{4}(2b+a+c).$$

A streamlined distance method was used to for the defuzzication of $TC_V(Q, n)$.

$$\tilde{D} = d\left(\tilde{D},\tilde{0}_1\right) = \frac{1}{4}\left[\left(D-\Delta_1\right) + 2D + \left(D+\Delta_2\right)\right]$$

$$= D + \frac{1}{4}\left(\Delta_2 - \Delta_1\right)$$

$$\tilde{Y} = y\left(\tilde{D},\tilde{0}_1\right) = \frac{1}{4}\left[\left(Y-\Delta_3\right) + 2Y + \left(Y+\Delta_4\right)\right]$$

$$= Y + \frac{1}{4}\left(\Delta_4 - \Delta_3\right)$$

$$\Rightarrow TC_V(Q,n) = \left[D + \frac{\left(\Delta_2-\Delta_1\right)}{4}\right]\left[\frac{S_V}{nQ\left(1-k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)}\right.$$

$$+\frac{F\left(1+2(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right)) - k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)}{Q\left(1-k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)} \tag{1}$$

$$+Qr + Rk(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right)) + L(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))$$

$$\left. + \frac{h_v Q(2-n)}{2p\left(1-k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)}\right] + \frac{h_v Q(n-1)}{2}$$

Derivation of vendor transportation cost:

$$TrC_V = F\times\frac{Q+\tilde{Y}Q+(1-k)\tilde{Y}Q}{Q}\times\frac{\tilde{D}}{Q\left(1-k\tilde{Y}\right)}$$

$$= \mathrm{F}\times\frac{Q+2\tilde{Y}Q-k\tilde{Y}Q}{Q}\times\frac{\tilde{D}}{Q\left(1-k\tilde{Y}\right)}$$

$$= \mathrm{F}\left(1+2Y-k\tilde{Y}\right)\times\frac{\tilde{D}}{Q\left(1-k\tilde{Y}\right)}$$

Derivation of vendor holding cost:

$$Hc_V = \frac{h_V\left\{nQ\left[\frac{Q}{P}+T(n-1)\right] - \frac{nQ\left(\frac{nQ}{P}\right)}{2} - T\left[Q+2Q+\ldots(n-1)Q\right]\right\}}{nT}$$

$$= \frac{h_V\left\{nQ\left[\frac{Q+Tnp-Tp}{p}\right] - \frac{n^2Q^2}{2p} - \frac{n^2TQ-nTQ}{2}\right\}}{nT}$$

$$= \frac{h_V\left\{\frac{2nQ^2+2Tn^2QP-2nQTp-n^2Q^2}{2P} - \frac{n^2TpQ-nTpQ}{2p}\right\}}{nT}$$

$$= h_V\left(\frac{2Q^2-nQ^2}{2pT} + \frac{2nQ-2Q-nQ+Q}{2}\right)$$

$$= h_V\left[\frac{(2-n)Q^2}{2pT} + \frac{Q(n-1)}{2}\right]$$

where $T = \dfrac{Q\left(1-k\tilde{Y}\right)}{\tilde{D}}$

$$\Rightarrow HC_V = h_v Q\left[\frac{n-1}{2} + \frac{\tilde{D}(2-n)}{2p\left(1-k\tilde{Y}\right)}\right]$$

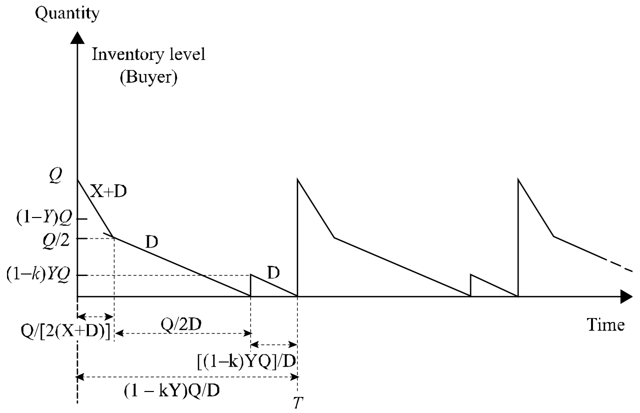

Fig. 2. Schematic of the buyer's cost.

3. Buyer's Cost

Buyers cost = order cost + screening cost + purchase cost + warranty cost + holding cost

$$TC_B(Q,n) = S_B \times \frac{\tilde{D}}{nQ\left(1-k\tilde{Y}\right)} + dX + B\tilde{D}\left(1-\sigma\right) + V\tilde{D}$$

$$+\frac{h_B Q}{4}\left[\frac{\tilde{D}}{\left(X+\tilde{D}\right)\left(1-k\tilde{Y}\right)} + \frac{2\left(1-k\right)^2 Y^2 - \tilde{Y} + 1}{1-k\tilde{Y}}\right]$$

$$\Rightarrow TC_B(Q,n) = \tilde{D}\left[\frac{S_B}{nQ\left(1-k\tilde{Y}\right)} + B\left(1-\sigma\right) + V + \frac{h_B Q}{4\left(X+\tilde{D}\right)\left(1-k\tilde{Y}\right)}\right]$$

$$+dX + \frac{h_B Q\left[2\left(1-k\right)^2 \tilde{Y}^2 - \tilde{Y} + 1\right]}{4\left(1-k\tilde{Y}\right)}$$

The streamlined distance method was used for the defuzzication of $TC_B(Q, n)$.

$$\tilde{D} = d\left(\tilde{D},\tilde{0}_1\right) = \frac{1}{4}\left[\left(D-\Delta_1\right) + 2D + \left(D-\Delta_2\right)\right] = D + \frac{1}{4}\left(\Delta_2 - \Delta_1\right)$$

$$\Rightarrow TC_B(Q,n) = \left[D + \frac{\left(\Delta_2-\Delta_1\right)}{4}\right]\left\{\frac{S_B}{nQ\left(1-k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)} + B\left(1-\sigma\right) + V\right.$$

$$\left. + \frac{h_B Q}{4\left[X + D + \frac{\left(\Delta_2-\Delta_1\right)}{4}\right]\left(1-k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)}\right\} + +dX$$

$$+\frac{h_B Q\left[2\left(1-k\right)^2 (Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))^2 - (Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right)) + 1\right]}{4\left(1-k(Y+\frac{1}{4}\left(\Delta_4-\Delta_3\right))\right)} \quad (2)$$

Derivation of buyer holding cost:

$$Hc_B = \frac{h_B}{T}\left\{\frac{1}{2}\left[\tilde{Y}Q \times \frac{Q}{2\left(X+\tilde{D}\right)}\right] + \frac{Q}{2}\left(1-\tilde{Y}\right)\left[\frac{Q}{2\left(X+\tilde{D}\right)} + \frac{Q}{2\tilde{D}}\right]\right.$$

$$\left. + \frac{\tilde{Y}Q}{2}\left(1-k\right) \times \frac{YQ\left(1-k\right)}{\tilde{D}}\right\}$$

$$= \frac{h_B Q^2}{2T}\left[\frac{\tilde{Y}}{2\left(X+\tilde{D}\right)} + \frac{\left(1-\tilde{Y}\right)}{2\left(X+\tilde{D}\right)} + \frac{\left(1-\tilde{Y}\right)}{2\tilde{D}} + \frac{2\tilde{Y}^2\left(1-k\right)^2}{2\tilde{D}}\right]$$

$$= \frac{h_B Q^2}{2T}\left[\frac{\tilde{Y}+1-\tilde{Y}}{2\left(X+\tilde{D}\right)} + \frac{\left(1-\tilde{Y}\right) + 2\tilde{Y}^2\left(1-k\right)^2}{2\tilde{D}}\right]$$

$$= \frac{h_B Q^2}{4T}\left[\frac{1}{X+\tilde{D}} + \frac{2\tilde{Y}^2\left(1-k\right)^2 - \tilde{Y} + 1}{\tilde{D}}\right]$$

where $T = \frac{Q\left(1-k\tilde{Y}\right)}{\tilde{D}}$

$$\Rightarrow HC_B = \frac{h_B Q}{4}\left[\frac{\tilde{D}}{\left(X+\tilde{D}\right)\left(1-k\tilde{Y}\right)} + \frac{2\left(1-k\right)^2 \tilde{Y}^2 - \tilde{Y} + 1}{1-k\tilde{Y}}\right]$$

4. Solving Procedure

$$EK(Q,n) = TC_V + TC_B$$

$$= S_V \times \frac{\tilde{D}}{nQ\left(1-kY\right)} + F\left(1+2\tilde{Y}-k\tilde{Y}\right) \times \frac{\tilde{D}}{Q\left(1-kY\right)}$$

$$+Q_r\tilde{D} + Rk\tilde{Y}\tilde{D} + L\tilde{Y}\tilde{D} + h_v Q\left[\frac{n-1}{2} + \frac{\tilde{D}\left(2-n\right)}{2p\left(1-k\tilde{Y}\right)}\right] \quad (3)$$

$$+S_B \times \frac{\tilde{D}}{nQ\left(1-k\tilde{Y}\right)} + dX + B\tilde{D}\left(1-\sigma\right) + V\tilde{D}$$

$$+\frac{h_B Q}{4}\left[\frac{\tilde{D}}{\left(X+\tilde{D}\right)\left(1-k\tilde{Y}\right)} + \frac{2\left(1-k\right)^2 \tilde{Y}^2 - \tilde{Y} + 1}{1-k\tilde{Y}}\right]$$

Based on the second-order partial deviation of $EK(Q, n)$, the derivative of $EK(Q, n)$ is computed with respect to Q, which is a convex function in Q for $Q > 0$.

$$\frac{\partial EK(Q,n)}{\partial \mathrm{Q}} = -S_V \times \frac{\tilde{D}}{nQ^2\left(1-k\tilde{Y}\right)} - S_B \times \frac{\tilde{D}}{nQ^2\left(1-k\tilde{Y}\right)}$$

$$-F\left(1+2\tilde{Y}-k\tilde{Y}\right) \times \frac{\tilde{D}}{Q^2\left(1-k\tilde{Y}\right)} + H_v + H_B \quad (4)$$

where $HC_v = h_v\left[\dfrac{n-1}{2}+\dfrac{\tilde{D}(2-n)}{2p(1-k\tilde{Y})}\right]$

$$HC_B = \frac{h_B}{4}\left[\frac{\tilde{D}}{(X+\tilde{D})(1-k\tilde{Y})}+\frac{2(1-k)^2\tilde{Y}^2-\tilde{Y}+1}{1-k\tilde{Y}}\right]$$

$$let\ \frac{\partial EK(Q,n)}{\partial \text{Q}} = 0$$

$$Q^* = \sqrt{\frac{\tilde{D}\left[S_V+S_B+nF\left(1+2\tilde{Y}-k\tilde{Y}\right)\right]}{n\left(1-k\tilde{Y}\right)\left(H_V+H_B\right)}}$$

$$Q^* = \sqrt{\frac{\left[D+\frac{(\Delta_2-\Delta_1)}{4}\right]\left[S_V+S_B+nF(1+2(Y+\frac{1}{4}(\Delta_4-\Delta_3))-k\left(Y+\frac{1}{4}(\Delta_4-\Delta_3)\right)\right]}{n(1-k\left(Y+\frac{1}{4}(\Delta_4-\Delta_3)\right))\left(H_V+H_B\right)}}$$

$$= \sqrt{\frac{(4D+\Delta_2-\Delta_1)\left[S_V+S_B+nF\left(4+8Y-4kY+2(\Delta_4-\Delta_3)\right)-k(\Delta_4-\Delta_3)\right)\right]}{4n\left(4-4kY-k(\Delta_4-\Delta_3)\right)\left(H_V+H_B\right)}} \tag{5}$$

Then, take the derivative of $EK(Q, n)$ with respect to n to understand the effect of n in $EK(Q, n)$.

$$\frac{\partial EK(Q,n)}{\partial \text{n}} = -S_V \times \frac{\tilde{D}}{n^2Q\left(1-k\tilde{Y}\right)} - S_B * \frac{\tilde{D}}{n^2Q\left(1-k\tilde{Y}\right)} + h_v\left[\frac{Q}{2}-\frac{Q\tilde{D}}{2p\left(1-k\tilde{Y}\right)}\right] \tag{6}$$

$$let\ \frac{\partial EK(Q,n)}{\partial \text{n}} = 0$$

$$n^* = \sqrt{\frac{\tilde{D}(S_V+S_B)}{h_vQ\left(1-k\tilde{Y}\right)\left[\frac{Q}{2}-\frac{Q\tilde{D}}{2p\left(1-k\tilde{Y}\right)}\right]}}$$

$$= \sqrt{\frac{2p\tilde{D}\left(1-k\tilde{Y}\right)(S_V+S_B)}{h_vQ^2\left(1-k\tilde{Y}\right)\left[p\left(1-k\tilde{Y}\right)-\tilde{D}\right]}}$$

The streamlined distance method was used for the defuzzication of n^*.

$$\tilde{D} = d\left(\tilde{D},\tilde{0}_1\right) = \frac{1}{4}\left[(D-\Delta_1)+2D+(D-\Delta_2)\right] = D+\frac{1}{4}(\Delta_2-\Delta_1)$$

$$n^* = \sqrt{\frac{2p\left[D+\frac{(\Delta_2-\Delta_1)}{4}\right](1-k\left(Y+\frac{1}{4}(\Delta_4-\Delta_3)\right))(S_V+S_B)}{h_vQ^2(1-k\left(Y+\frac{1}{4}(\Delta_4-\Delta_3)\right))\left\{p\left(1-k(Y+\frac{1}{4}(\Delta_4-\Delta_3))\right)-\left[D+\frac{(\Delta_2-\Delta_1)}{4}\right]\right\}}} \tag{7}$$

Then, the the second-order partial derivative of $EK(Q, n)$ was taken with respect to n.

$$\frac{\partial EK(Q,n)}{\partial \text{n}^2} = (S_V+S_B)\times\frac{2\tilde{D}}{Qn^3\left(1-k\tilde{Y}\right)} > 0 \tag{8}$$

The result of the equation proves that the solution has a minimum. The expected total cost increases when the defective product rate increases. Moreover, the quantity of the manufactured products must be increased to complete the original order. Therefore, a well-designed quantity discount policy is crucial.

5. Algorithm

(1) Set n = 1.
(2) Substitute n = 1 into equation (5) to evaluate Q_1.
(3) Substitute Q_1 into equation (4) to evaluate EK.
(4) Set n = n + 1 and substitute n = n + 1 into equation (5) to evaluate Q_2. Then, repeat step 2 and step 3 to obtain EK.
(5) Substitute Q_2 into equation (4) to evaluate EK.
6) If $EK' < EK$, return to step 4; otherwise, EK is the optimal solution.

III. RESULTS

This study presents a detailed numerical example to illustrate the results of the proposed models:

D = 5000 pieces/year, S_V = 3000 \$/setup, S_b = 300 \$/cycle, X = 1000 pieces/year, H_V = 1 \$/piece, H_b = 4 \$/piece, P = 8000 pieces/year, V = 1.5 \$/piece, d = 0.5 \$/piece, Y = 0.01, Q_r = 10 \$ piece, B = 25 \$/piece, K = 0.3, F = 800, s = mYk = 0.3, R = 2 \$/piece, M = 100

The minimum cost solution required multiples of (Δ_1, Δ_2). Consider that (Δ_1, Δ_2) is determined by the decision maker to handle uncertain problems. All the results are provided in the tables below.

(1) When ($\Delta_1 < \Delta_2$), then $d(\tilde{D},\tilde{0}_1) > D$. Thus, $V_Q > 0$, and $V_W > 0$. When the value of ($\Delta_2 - \Delta_1$) decreases, both V_Q and V_W decrease. The smaller the value of ($\Delta_2 + \Delta_1$) in this fuzzy model, the more similar to is the fuzzy model to the traditional model.
(2) When $\Delta_1 > \Delta_2$ then $d(\tilde{D},\tilde{0}_1) < D$. Thus, $V_Q < 0$, and $V_W < 0$. When the value of ($\Delta_2 - \Delta_1$) increases, both V_Q and V_W increase.
(3) When $\Delta_1 = \Delta_2$ = **2500** and $\Delta_3 = \Delta_4$ = **0.006**, then $\boldsymbol{d(\tilde{D},\tilde{0}_1) = D = 5000}$. In this case, this fuzzy model is exactly the same as the traditional models, and both $\boldsymbol{V_Q}$ and $\boldsymbol{V_W}$ are equal to zero.
(4) The mathematical relationship diagram of **EK** and ($\Delta_2 - \Delta_1$) is displayed in Fig. 3.
(5) Fig. 4 illustrates the comparison between $\boldsymbol{V_Q}$ and $\boldsymbol{V_W}$ with

($\Delta_2 - \Delta_1$). The figure reveals that the slope of V_W is larger than that of V_Q, which represents the variation in V_W is higher than that in V_Q.

(6) Based on the results displayed in Table 1, the quantity of manufactured products increases to fulfill the demand when the defective product rate increases.

(7) Table 2 presents that when the defective product rate increases, the expected total cost increases. To minimize the cost, the defective product rate should be reduced.

TABLE I NUMERICAL EXAMPLE RESULTS

Table 1. Result of Q^* when $n^* = 2$.

$(D-\Delta_1, D, D+\Delta_2)$ \ $(Y-\Delta_3, Y, Y+\Delta_4)$	0.012	0.011	0.01	0.009	0.008
(4750,5000,5500)	4638.34	4635.94	4633.53	4631.13	4628.72
(4500,5000,6000)	4665.20	4662.78	4660.36	4657.94	4655.52
(4250,5000,6500)	4691.89	4689.46	4687.03	4684.60	4682.16
(4000,5000,7000)	4718.43	4715.99	4713.54	4711.10	4708.65
(3750,5000,7500)	4744.82	4742.36	4739.90	4737.45	4734.99
(2500,5000,7500)	4611.33	4608.94	4606.55	4604.15	4601.76
(2500,5000,6250)	4473.74	4471.42	4469.09	4466.76	4464.44
(3000,5000,6000)	4501.61	4499.27	4496.93	4494.59	4492.25
(3500,5000,5750)	4529.29	4526.94	4524.59	4522.24	4519.88
(4000,5000,5500)	4556.81	4554.44	4552.08	4549.71	4547.34
(4500,5000,5250)	4584.15	4581.77	4579.39	4577.01	4574.63

Table 2. Result of EK when $n^* = 2$.

$(D-\Delta_1, D, D+\Delta_2)$ \ $(Y-\Delta_3, Y, Y+\Delta_4)$	0.012	0.011	0.01	0.009	0.008
(4750,5000,5500)	160917.6	160905.0	160892.4	160879.8	160867.2
(4500,5000,6000)	162819.4	162806.6	162793.9	162781.1	162768.4
(4250,5000,6500)	164720.6	164707.7	164694.8	164681.9	164669.1
(4000,5000,7000)	166621.2	166608.2	166595.2	166582.2	166569.2
(3750,5000,7500)	168521.4	168508.2	168495.1	168482.0	168468.8
(2500,5000,7500)	159015.3	159002.8	158990.3	158977.9	158965.4
(2500,5000,6250)	149495.0	149483.2	149471.4	149459.6	149447.8
(3000,5000,6000)	151400.3	151388.3	151376.4	151364.4	151352.5
(3500,5000,5750)	153304.9	153292.8	153280.7	153268.7	153256.6
(4000,5000,5500)	155209.0	155196.7	155184.5	155172.3	155160.1
(4500,5000,5250)	157112.4	157100.1	157087.7	157075.4	157063.1

Table 3. Fuzzy defective product rates of the example.

$\tilde{Y}$	0.012	0.011	0.01	0.009	0.008
Δ_3	0.002	0.004	0.006	0.008	0.01
Δ_4	0.01	0.008	0.006	0.004	0.002
Y	0.01	0.01	0.01	0.01	0.01

This study computes the defuzzified values Y by using the streamlined distance method, which is conducted using the following equation:

$$\tilde{Y} = y\left(\tilde{D},\tilde{0}_1\right) = \frac{1}{4}\left[\left(Y-\Delta_3\right)+2Y+\left(Y-\Delta_4\right)\right] = Y + \frac{1}{4}\left(\Delta_4 - \Delta_3\right)$$

Table 4. Result of V_Q when n^* = 2.

$(D-\Delta_1, D, D+\Delta_2)$ \ $(Y-\Delta_3, Y, Y+\Delta_4)$	0.012	0.011	0.01	0.009	0.008
(4750,5000,5500)	0.6903	0.6381	0.5858	0.5336	0.4814
(4500,5000,6000)	1.2732	1.2207	1.1682	1.1157	1.0631
(4250,5000,6500)	1.8527	1.7999	1.7471	1.6943	1.6415
(4000,5000,7000)	2.4288	2.3758	2.3227	2.2696	2.2165
(3750,5000,7500)	3.0017	2.9483	2.8950	2.8416	2.7882
(2500,5000,7500)	0.1039	0.0519	0	−0.0520	−0.1039
(2500,5000,6250)	−2.8830	−2.9334	−2.9839	−3.0344	−3.0849
(3000,5000,6000)	−2.2781	−2.3288	−2.3796	−2.4304	−2.4812
(3500,5000,5750)	−1.6770	−1.7281	−1.7791	−1.8302	−1.8813
(4000,5000,5500)	−1.0797	−1.1311	−1.1824	−1.2338	−1.2852
(4500,5000,5250)	−0.4861	−0.5378	−0.5894	−0.6411	−0.6928

Table 5. Result of V_W when n^* = 2.

$(D-\Delta_1, D, D+\Delta_2)$ \ $(Y-\Delta_3, Y, Y+\Delta_4)$	0.012	0.011	0.01	0.009	0.008
(4750,5000,5500)	1.2122	1.2043	1.1963	1.1884	1.1805
(4500,5000,6000)	2.4083	2.4003	2.3923	2.3843	2.3763
(4250,5000,6500)	3.6041	3.5960	3.5879	3.5798	3.5718
(4000,5000,7000)	4.7996	4.7914	4.7832	4.7751	4.7669
(3750,5000,7500)	5.9947	5.9865	5.9782	5.9699	5.9617
(2500,5000,7500)	0.0157	0.0078	0	−0.0078	−0.0157
(2500,5000,6250)	−5.9723	−5.9797	−5.9871	−5.9946	−6.0020
(3000,5000,6000)	−4.7739	−4.7814	−4.7890	−4.7965	−4.8040
(3500,5000,5750)	−3.5760	−3.5836	−3.5912	−3.5987	−3.6063
(4000,5000,5500)	−2.3784	−2.3861	−2.3937	−2.4014	−2.4091
(4500,5000,5250)	−1.1812	−1.1889	−1.1967	−1.2044	−1.2122

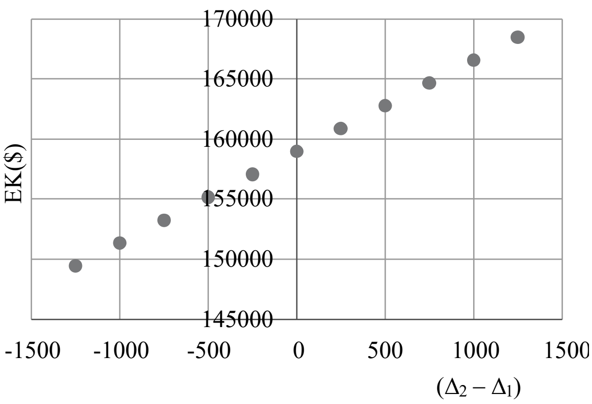

Fig. 3. Mathematical relationship diagram of EK and ($\Delta_2 - \Delta_1$).

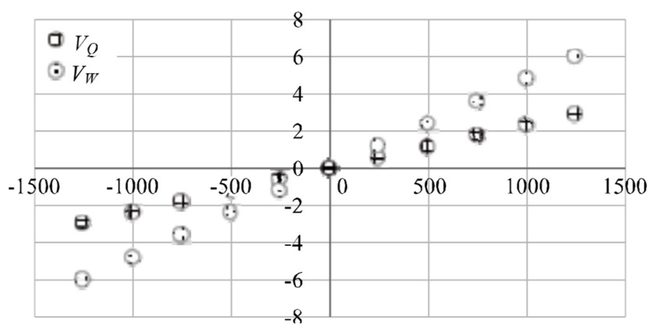

Fig. 4. Comparison of V_Q and V_W with ($\Delta_2 - \Delta_1$).

IV. CONCLUSION

The global market has a highly competitive environment. The pricing strategy and quality often affect the purchase orientation of customers. Suppliers often lower their prices to compete in the market. However, but this is not a good marketing strategy because it might increase the rate of manufacturing defective products. To solve the problem of the uncertain manufacturing procedure that occurs due to the unpredictable demand and rate of manufacturing defective products, this study employs many mathematical programs for processing. The results of the study indicate that the expected total cost increases when the rate of manufacturing defective products increases. Moreover, the quantity of the manufactured products should increase to complete the original order. Therefore, a well-designed quantity discount policy is crucial. This study incorporated quantity discount, fuzzy demand, and an uncertain manufacture procedure into the integrated inventory model. The sensitive analysis conducted in this study indicates that if $(\Delta_2 - \Delta_1)$ increases, both V_Q and V_W increase simultaneously. Moreover, the smaller the values of $(\Delta_2 + \Delta_1)$ and $(\Delta_3 + \Delta_4)$ in the fuzzy model, the more similar is the fuzzy model to the traditional model.

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REFERENCES

Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. Decision Science 17(3), 292-311.

Ben-Daya, M. and M. Hariga (2000). Economic lot scheduling problem with imperfect production process. Journal of the Operational Research Society, 875-881.

Chakravarty, A. K. and G. E. Martin (1988). An optimal joint buyer-seller discount pricing model. Journal of Computers and Operations Research, 271-281.

Chiu, C. Y., Y. Lin and M. F. Yang (2014). Applying fuzzy multi objective integrated logistics model to green supply chain problems. Journal of Applied Mathematics 2014, 767095.

Goyal, S. K. (1976). An integrated inventory model for a single supplier-single customer problem. International Journal of Production Research 15(1), 107-111.

Goyal, S. K. (1988). A joint economic-lot-size model for purchaser and vendor: a comment. Decision Sciences 19(1), 236-241.

Goyal, S. K. and L. E. Cardenas-Barron (2002). Note on: Economic production quantity model for items with imperfect quality-a practical approach. International Journal of Production Economics 77(1), 85-87.

Hsiao, H. I., M. Tu, M. F. Yang and W. C. Tseng (2017). Deteriorating inventory model for Ready-to-eat food under fuzzy environment. International Journal of Logistics Research and Applications 20(6), 560-580.

Huang, C. K. (2004). An optimal policy for a single vendor single-buyer integrated production-inventory problem with process unreliability consideration. International Journal of Production Economics 91(1), 91-98.

Kaufmann, A. and M. M. Gupta (1991). Introduction to Fuzzy Arithmetic: Theory and Applications, Van Nostrand Reinhold, New York.

Lal, R. and R. Staelin (1984). An approach for developing an optimal discount pricing policy. Management Science 30(2), 1524-1539.

Li, J. and L. Liu (2006). Supply chain coordination with quantity discount policy. International Journal of Production Economics 101(1), 89-98.

Lin, Y. J. and C. H. Ho (2011). Integrated inventory model with quantity discount and price-sensitive demand. An Official Journal of the Spanish Society of Statistics and Operations Research 19(1), 177-188.

Ouyang, L.Y. and J. S. Yao (2002). A minimax distribution free procedure for mixed inventory model involving variable lead time with fuzzy demand. Computers & Operations Research 29, 471-487.

Pan, C. H. and M.F. Yang (2008). Integrated inventory models with fuzzy annual demand and fuzzy production rate in a supply chain. International Journal of Production Research 46(3), 753-770.

Porteus, E.L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. Annals of Operations Research 34(1), 137-144.

Rong, M. and M. Maiti, (2015), On an EOQ model with service level constraint under fuzzy-stochastic demand and variable lead-time. Applied Mathematical Modelling 39(17), 5230-5240.

Salameh, M. K. and M.Y. Jaber (2000). Economic production quantity model for items with imperfect quality. International Journal of Production Economics 64(1-3), 59-64.

Schwaller, R.L. (1988). EOQ under inspection costs. Product. Journal of Inventory Management 29(3), 22-24.

Tu, H. J., M. C. Lo and M. F. Yang (2010). A two-echelon inventory model for fuzzy demand with mutual beneficial pricing approach in a supply chain. African Journal of Business Management 5(14), 58-70.

Wang, Q. (2005). Discount pricing policies and the coordination of decentralized distribution systems. Decision Sciences Journal 36(4), 627-646.

Yang, M. F., T. S. Huang and Y. Lin (2010). Variable lead time fuzzy inventory system under backorder rate with allowed shortage. International Journal of Innovative Computing Information and Control 6(11), 5015-5034.

Yang, M. F., Y. Lin, W. F. Kao and L. H. Ho (2016). An integrated multi-echelon logistics model with uncertain delivery lead time and quality unreliability. Mathematical Problems in Engineering 2016, 8494268.

Yao, J. S and K. Wu (2000). Ranking fuzzy numbers based on decomposition principle and signed distance. Fuzzy Sets and Systems 116(2), 275-288.

Zhang, Z. and J. Xu (2014). Applying nonlinear MODM model to supply chain management with quantity discount policy under complex fuzzy environment. Journal of Industrial Engineering and Management 7(3), 660-680.

Zimmermann, H. J. (1996). Fuzzy Set Theory and its Application, 3rd edition, Kluwer Academic Publishers, Dordrecht.