Journal of Marine Science and Technology

Volume 28 | Issue 5 Article 4

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Recommended Citation
Chang, Wen-Jer; Ku, Cheung-Chieh; Yen, Chih-Yu; and Chen, Guan-Wei (2020) "GAIN-SCHEDULED CONTROL OF DISCRETIZED SHIP AUTOPILOT SYSTEM SUBJECT TO POLEASSIGNMENT AND PASSIVITY CONSTRAINTS," *Journal of Marine Science and Technology*: Vol. 28: Iss. 5, Article 4.
DOI: 10.6119/JMST.202010_28(5).0004
Available at: https://jmstt.ntou.edu.tw/journal/vol28/iss5/4

Journal of Marine Science and Technology, Vol. 28, No. 5, pp. 349-356 (*2020*)
DOI: 10.6119/JMST.202010_28(5).0004

GAIN-SCHEDULED CONTROL OF DISCRETIZED SHIP AUTOPILOT SYSTEM SUBJECT TO POLE-ASSIGNMENT AND PASSIVITY CONSTRAINTS

Wen-Jer Chang, Cheung-Chieh Ku, Chih-Yu Yen and Guan-Wei Chen

Key words: pole-assignment constraint, ship autopilot system, passivity theory, gain-scheduled control.

ABSTRACT

This paper addresses a control performance problem for discretized ship autopilot system with uncertainty. To completely express the uncertainty, Linear Parameter Varying (LPV) modelling technology is employed such that the ship autopilot system is described via several linear systems and weighting function. Furthermore, gain-scheduled scheme is applied to design a controller to achieve the passivity and pole-assignment constraints. Moreover, a Parameter-Dependent Lyapunov Function (PDLF) is used to derive some sufficient conditions. Through the proposed design method, the attenuation performance and stability of the system can be guaranteed. Besides, the transient responses are furtherly improved such that the ship autopilot system possesses short settling time for abominable operation. To demonstrate the proposed design method, some numerical simulations are finally provided.

I. INTRODUCTION

Lim and Forsythe have firstly proposed a mathematical model to describe the dynamics of ship autopilot systems. Through the model (Lim and Forsythe, 1983), many works (Song et al., 2005; Zhang and Ren, 2005) have been done for their own controller design method. Unfortunately, only few results were efforted to investigate the robust issue of ship autopilot systems. It is well known that the modelling error and time-varying parameter named as uncertainty cause instability and poor performance. Generally, the uncertainty is appeared during some transformation procedures, such as discretization and linearization. Therefore, variant robust control problems (Belov and Andrianova, 2019; Esfahani and Pieper, 2019; Lee and Lee, 1999) have been received much attention for uncertain systems. According to the maturity of electronic elements, the practical stabilization problems are usually regarded as the discrete control problems. Referring to (Song et al., 2005), the ship autopilot system was thus directly discretized from continuous-time dynamics. However, the discretization often causes a strong uncertainty. To express the uncertainty, many literature (Belov and Andrianova, 2019) utilize a bounded norm with some regular functions such as sine or cosine function. To avoid the limitation of bounded norm, the Linear Parameter Varying (LPV) system (Daafouz and Bernussou, 2001; Prempain et al., 2002; Lee, 2006; Qin and Wang, 2007; Caignu et al., 2010; White et al., 2013; Zhang et al., 2015) was provided to complete description for the complex uncertainties.

Referring to the literature (Zhang et al., 2015), the extreme values of carrying region are used to model several linear systems and weighting function to establish a LPV model for time-varying systems. Due to the weighting function, the time-varying property is characterized via scheduling the linear systems to simulate the uncertain systems. Besides, a gain-scheduled scheme (Caignu et al., 2010) was widely applied to amplify the significant advances of the LPV system for robustness. Through the LPV system and gain-scheduled scheme, the solutions in (Prempain et al., 2002; Wing and Wang, 2007; White et al., 2013) were proposed to analyze and synthesize the robust stability of practical systems. For the stability issues, Lyapunov function is usually chosen to discuss the energy change of systems. Referring to (Daafouz and Bernussou, 2001; Ku and Chen, 2015), a Parameter-Dependent Lyapunov Function (PDLF) provides some relaxed results in discussing the stability analysis and controller synthesis. Based on the PDLF, the state-feedback scheme is always adopted to establish a gain-scheduled controller for the required control performance (Daafouz and Bernussou, 2001; Ku and Chen, 2015). According to the inertia of ship, the transient response of the autopilot system is required to improve the control performance, furtherly. For the transient response, pole-assignment constraint (Chilali and Gahinet, 1996; Chilali et al., 1996; Hernrion et al., 2003; Hong and Nam, 2003; Plalcios and Titli, 2005) is thus provided to force the closed-loop poles into the assigned region.

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Considering the pole-assignment constraint, a D-stability technology (Chilali and Gahinet, 1996; Chilali et al., 1996; Hernrion et al., 2003) was proposed to assign the region which determines the transient response of the closed-loop system. Moreover, the region can be assigned as sector, circle or plane cases for the different requirements. Furthermore, a design method has been proposed to achieve multiple pole-assignment constraints in (Plalcios and Titli, 2005). Based on the results, the D-stability technology can be directly applied to constrain the closed-loop poles of the polynomial systems such as fuzzy system (Chang et al., 2018; Hong and Nam, 2003; Plalcios and Titli, 2005) and LPV system (Ramezanifar et al., 2012). Nevertheless, the pole-assignment constraint for polynomial systems is hardly achieved by satisfying several conditions at the same time. Thus, a controller design method to guarantee the closed-loop poles of polynomial system in the assigned region is an interesting issue. In addition to the pole-assignment constraint, the attenuation performance is also concerned to attenuate the effect of wave on the heading of ship autopilot system. Referring to (Deng and Bu, 2012; Ku, 2016; Lozano et al., 2000; Xie et al., 1998), the passivity theory possesses a general attenuation performance including control scheme, positive real theory and several passive types via giving power supply function. Through the pole-assignment and passivity constraints, the transient and steady-state responses of ship autopilot system can be improved in the finite time.

According to the above motivations, the robust asymptotical stability and stabilization issue of discretized ship autopilot system is investigated subject to the pole-assignment and passivity constraints in this paper. Considering the modelling errors caused by the discretization, the LPV system is employed to describe the ship autopilot system with uncertainties. Based on the D-stability approach, the closed-loop poles are assigned to achieve excellent transient response. For the attenuation performance, the passivity theory is applied to constrain the wave affecting operation of ship. Using PDLF, some Linear Matrix Inequality (LMI) (Boyd et al., 1994) conditions are derived to guarantee the pole-assignment and passivity constraints under the robust asymptotical stability. Via the simulation results, the ship autopilot system driven by the designed controller is asymptotically stable subject to pole-assignment and passivity constraints.

This paper is organized as follows: In Section II, the discretized ship autopilot system is described. For the system, a gain-scheduled controller design method is proposed in Section III. In Section IV, the simulations of the discretized ship autopilot system driven by the designed gain-scheduled controller are provided. Some conclusions are stated in Section V.

II. SYSTEMS DESCRIPTION AND PROBLEM FORMULATION

Referring to (Song et al., 2005), the following discretized

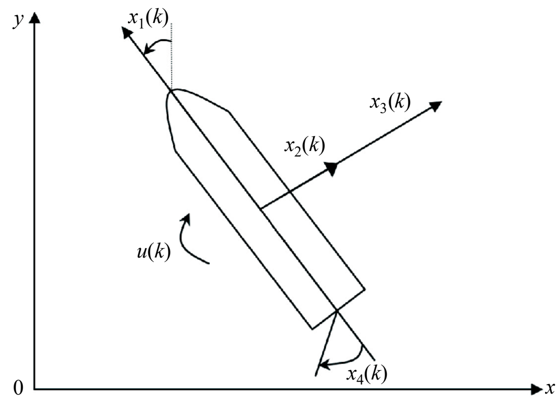

Fig. 1 Coordinate of Ship Autopilot System

equations for the ship motion can be obtained. To consider the modelling error, the time constant T is assumed as the time varying parameter regarding as uncertainty. Moreover, the disturbed term is added for the possible external disturbance.

$$x_1(k+1) = x_1(k) + 0.4x_2(k) \tag{1a}$$

$$x_2(k+1) = x_2(k) + 0.4x_3(k) + 0.002v(k) \tag{1b}$$

$$x_3(k+1) = \frac{-0.04564}{8.54\times T(k)}x_2(k) + \left(1 - \frac{0.4\left(T(k)+8.54\right)}{8.54\times T(k)}\right)x_3(k) - \frac{6.044x_4(k) - 7.044u(k)}{21.35\times T(k)} \tag{1c}$$

$$x_4(k+1) = x_4(k) - 0.16\left(x_4(k) - u(k)\right) \tag{1d}$$

$$y(k) = x_1(k) + v(k) \tag{1e}$$

where $x_1(k)$ represents the difference of heading angle and desired heading angle of ship, $x_2(k)$ represents the navigational angle velocity, $x_3(k)$ represents the navigational angle acceleration; $x_4(k)$ represents the actual rudder angle of ship; $u(k)$ represents the steering angle, and $v(k)$ is chosen as zero-mean white noise with unit variance. The dynamics of (1) can be referred to Fig. 1. For the possible value of $T(k)$ in the system, the time-varying range is determined as follows:

$$T(k) \in [36.25 \quad 108.75] \tag{2}$$

Using the LPV modelling approach (Zhang et al., 2015), the system (1) is described as follows:

$$\begin{aligned} x(k+1) &= \mathbf{A}(\vartheta(k)) + \mathbf{B}(\vartheta(k))u(k) + \mathbf{E}v(k) \\ &= \sum_{i=1}^{2}\vartheta_i(k)\left(\mathbf{A}_i x(k) + \mathbf{B}_i u(k) + \mathbf{E}v(k)\right) \end{aligned} \tag{3a}$$

$$y(k) = \mathbf{C}x(k) + \mathbf{D}v(k) \tag{3b}$$

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0.4 & 0 & 0 \\ 0 & 1 & 0.4 & 0 \\ 0 & -0.000064 & 0.9508 & -0.000296 \\ 0 & 0 & 0 & 0.84 \end{bmatrix}$$

where

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0.4 & 0 & 0 \\ 0 & 1 & 0.4 & 0 \\ 0 & -0.00013 & 0.9438 & -0.00088 \\ 0 & 0 & 0 & 0.84 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 0.0003 \\ 0.1363 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0.0010 \\ 0.1363 \end{bmatrix},$$

$$\mathbf{E}^T = \begin{bmatrix} 0 & 0.0002 & 0 & 0 \end{bmatrix}^T, \ \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{D} = 1 \quad \vartheta_1(k) = |\sin(k)|, \quad \vartheta_2(k) = 1 - |\sin(k)|$$
$$\vartheta(k) = \begin{bmatrix} \vartheta_1(k) & \vartheta_2(k) \end{bmatrix} \text{ and } ,$$
$$x^T(k) = \begin{bmatrix} x_1^T(k) & x_2^T(k) & x_3^T(k) & x_4^T(k) \end{bmatrix}^T$$

For simplifying the notation, $\vartheta(k) \triangleq \vartheta$ is used in the following context.

The following gain-scheduled controller is designed for (3).

$$u(k) = -\mathbf{F}(\vartheta)\mathbf{G}(\vartheta)x(k) = -\left(\sum_{j=1}^{2} \vartheta_j \mathbf{F}_j\right)\left(\sum_{j=1}^{2} \vartheta_j \mathbf{G}_j\right)x(k) \tag{4}$$

Based on (4), the following closed-loop system is built.

$$\begin{aligned} x(k+1) &= \mathbf{A}(\vartheta) - \mathbf{B}(\vartheta)\mathbf{F}(\vartheta)\mathbf{G}(\vartheta) + \mathbf{E}v(k) \\ &= \mathbf{X}(\vartheta)x(k) + \mathbf{E}v(k) \\ &= \sum_{i=1}^{2}\sum_{j=1}^{2} \vartheta_i \vartheta_j \left(\left(\mathbf{A}_i - \mathbf{B}_i \mathbf{F}_j \mathbf{G}_j \right) x(k) + \mathbf{E}v(k) \right) \\ &= \sum_{i=1}^{2}\sum_{j=1}^{2} \vartheta_i \vartheta_j \left(\mathbf{X}_{ij} x(k) + \mathbf{E}v(k) \right) \end{aligned} \tag{5}$$

where $\mathbf{X}_{ij} = \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{F}_j \mathbf{G}_j \right)$.

In this paper, the following definitions are applied for attenuation performance and pole-assignment constraint.

Definition 1 (Lozano et al., 2000)

The closed-loop system (5) is called passive if there exists the matrices $\mathbf{S}_1$, $\mathbf{S}_3$ and $\mathbf{S}_2 \geq 0$ such that

$$2\sum_{k=0}^{k_p} y^T(k)\mathbf{S}_1 v(k) > \sum_{k=0}^{k_p} y^T(k)\mathbf{S}_2 y(k) + \sum_{k=0}^{k_p} v^T(k)\mathbf{S}_3 v(k) \tag{6}$$

where $k_p > 0$ #

Definition 2 (Palacios and Titli, 2005)

The region D is defined as follows:

$$D = \left\{ x + jy \in C : (x-q)^2 + y^2 = r^2 \right\} \tag{7}$$

If the poles of the closed-loop system (5) without external disturbance lie in the circle C, it includes the centered point $(q, 0)$ and radius $1 > r > 0$ where $0 < |q + r| < 1$. #

According to (Palacios and Titli, 2005), the considered system (5) subject to pole-assignment can be noted as follows:

$$\begin{aligned} x(k+1) &= \left(\mathbf{X}(\vartheta) - q\mathbf{I} \right) / r\, x(k) \\ &= \left\{ \sum_{i=1}^{2} \vartheta_i \left(\mathbf{X}_{ii} - q\mathbf{I} \right) / r + \sum_{i=1}^{2}\sum_{i<j} \vartheta_i \vartheta_j \left(\mathbf{X}_{ij} \right) \right\} x(k) \end{aligned} \tag{8}$$

In the following context, the system (8) is substituted for the original system (5) to design the gain-scheduled controller (4). And then, the designed controller is applied to uncertain ship autopilot system (1) to demonstrate the applicability and usefulness of the proposed design method.

Remark 1

Generally, the pole assignment performance is fundamentally related to $i = j$ but $i < j$ and to locate the poles of the dominant terms in the prescribed region. #

Besides, the following lemma is applied to analyze the D-stability of the non-disturbed system.

Lemma 1 (Hong and Nam, 2003)

If there exists a symmetric matrix $\mathbf{P}$ satisfying the following inequality, the system is asymptotically D-stable.

$$\left(\mathbf{A} - q\mathbf{I} \right)^T \mathbf{P} \left(\mathbf{A} - q\mathbf{I} \right) - r^2 \mathbf{P} < 0 \tag{9}$$

where q and r satisfy Definition 1. #.

III. CONTROLLER DESIGN METHOD

In this section, some sufficient conditions are derived for discussing the stability and attenuating performance of (8). Furthermore, the conditions are converted into LMI form for applying convex optimization algorithm.

Theorem

Given scalars q and r satisfying $0 < |q + r| < 1$, and matrices $\mathbf{S}_1$, $\mathbf{S}_2 \geq 0$ and $\mathbf{S}_3$, if there exists the positive definite matrices $\mathbf{P}_i$, any matrices $\mathbf{G}_j$ and feedback gains $\mathbf{F}_j$ to satisfy the following condition, then the closed-loop system (8) is asymptotically stable subject to pole-assignment and passivity constraints.

$$\begin{bmatrix} \mathbf{U}_{ii} & * & * & * \\ -\mathbf{S}_1^T\mathbf{C}\mathbf{G}_i & \mathbf{S}_3 - \mathbf{D}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D} & * & * \\ (\mathbf{M}_{ii} - q\mathbf{G}_i)/r & \mathbf{E}_1 & -\mathbf{Q}_l & * \\ (\mathbf{S}_2)^{0.5}\mathbf{C}\mathbf{G}_i & (\mathbf{S}_2)^{0.5}\mathbf{D}^T & 0 & -\mathbf{I} \end{bmatrix} < 0 \tag{10a}$$

for $i = 1, 2$ and $l = 1, 2$

$$\begin{bmatrix} (\mathbf{U}_{ii} + \mathbf{U}_{jj})/2 & * & * & * \\ -\mathbf{S}_1^T\mathbf{C}\mathbf{G}_i & \mathbf{S}_3 - \mathbf{D}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D} & * & * \\ (\mathbf{M}_{ij} + \mathbf{M}_{ji})/2 & \mathbf{E} & -\mathbf{Q}_l & * \\ (\mathbf{S}_2)^{0.5}\mathbf{C}\mathbf{G}_i & \mathbf{D}^T & 0 & -\mathbf{I} \end{bmatrix} < 0 \tag{10b}$$

for $i = 1$, $j = 2$ and $l = 1, 2$

where $\mathbf{Q}_i = \mathbf{P}_i^{-1}$, $\mathbf{R}_i = \mathbf{G}_i^{-1}$, $\mathbf{U}_{ii} = \mathbf{Q}_i - \mathbf{R}_i^T - \mathbf{R}_i$ and $\mathbf{M}_{ij} = \mathbf{A}_i\mathbf{R}_j - \mathbf{B}_i\mathbf{F}_j$.

Proof:

Let us choose the following PDLF.

$$V(x(k)) = x^T(k)\mathbf{P}(\vartheta)x(k) \tag{11}$$

Taking the difference of (11), the following equation can be directly inferred along the trajectories of (8).

$$\begin{aligned} &\Delta V(x(k)) = (\mathbf{X}(\vartheta)x(k) + \mathbf{E}v(k))^T \mathbf{P}(\vartheta(k+1)) \\ &\times(\mathbf{X}(\vartheta)x(k) + \mathbf{E}v(k)) - x^T(k)\mathbf{P}(\vartheta)x(k) \\ &\leq \sum_{i=1}^{2}\vartheta_i\Big\{\big(((\mathbf{X}_i - q\mathbf{I})/r)x(k) + \mathbf{E}v(k)\big)^T \mathbf{P}(\vartheta(k+1)) \\ &\times\big(((\mathbf{X}_i - q\mathbf{I})/r)x(k) + \mathbf{E}v(k)\big) - x^T(k)\mathbf{P}_i x(k)\Big\} \\ &+\sum_{i=1}^{2}\sum_{i>j}\vartheta_i\vartheta_j\Big\{\big(((\mathbf{X}_{ij} + \mathbf{X}_{ji})/2)x(k) + \mathbf{E}v(k)\big)^T \mathbf{P}(\vartheta(k+1)) \\ &\times\big(((\mathbf{X}_{ij} + \mathbf{X}_{ji})/2)x(k) + \mathbf{E}v(k)\big) - x^T(k)\mathbf{P}_i x(k)\Big\} \end{aligned} \tag{12}$$

Defining $\mathbf{P}(\vartheta(k+1)) = \mathbf{P}(\varepsilon(k)) = \sum_{l=1}^{2}\varepsilon_l(k)\mathbf{P}_l$, (12) can be thus rewritten as follows:

$$\begin{aligned} &\Delta V(x(k)) \leq \sum_{i=1}^{2}\sum_{l=1}^{2}\vartheta_i\varepsilon_l(k)\Big\{\big(((\mathbf{X}_i - q\mathbf{I})/r)x(k) + \mathbf{E}v(k)\big)^T \\ &\times\mathbf{P}_l\big(((\mathbf{X}_i - q\mathbf{I})/r)x(k) + \mathbf{E}v(k)\big) - x^T(k)\mathbf{P}_i x(k)\Big\} \\ &+\sum_{i=1}^{2}\sum_{i>j}\sum_{l=1}^{2}\vartheta_i\vartheta_j\varepsilon_l(k)\Big\{\big(((\mathbf{X}_{ij} + \mathbf{X}_{ji})/2)x(k) + \mathbf{E}v(k)\big)^T \mathbf{P}_l \\ &\times\big(((\mathbf{X}_{ij} + \mathbf{X}_{ji})/2)x(k) + \mathbf{E}v(k)\big) - x^T(k)\mathbf{P}_i x(k)\Big\} \end{aligned} \tag{13}$$

For all $v(k) \neq 0$ and $x(0) = 0$, we define the following cost function.

$$\begin{aligned} &\boldsymbol{\Gamma}(x,v,k) = \sum_{k=0}^{k_p}\big(y^T(k)\mathbf{S}_2 y(k) + v^T(k)\mathbf{S}_3 v(k) - 2y^T(k)\mathbf{S}_1 v(k)\big) \\ &\leq \sum_{k=0}^{k_p}\big(y^T(k)\mathbf{S}_2 y(k) + v^T(k)\mathbf{S}_3 v(k) - 2y^T(k)\mathbf{S}_1 v(k) + \Delta V(x(k))\big) \\ &< \sum_{k=0}^{k_p}\begin{bmatrix} x^T(k) \\ v^T(k) \end{bmatrix}\left\{\left(\sum_{i=1}^{2}\sum_{l=1}^{2}\vartheta_i\varepsilon_l(k)\boldsymbol{\Psi}_{iil}\right) + \sum_{i=1}^{2}\sum_{i>j}\sum_{l=1}^{2}\vartheta_i\vartheta_j\varepsilon_l(k)\boldsymbol{\Psi}_{12l}\right\}\begin{bmatrix} x^T(k) \\ v^T(k) \end{bmatrix}^T \end{aligned} \tag{14}$$

where

$$\boldsymbol{\Psi}_{iil} = \begin{bmatrix} (\mathbf{X}_{ii} - q\mathbf{I})^T\mathbf{P}_l(\mathbf{X}_{ii} - q\mathbf{I})/r^2 - \mathbf{P}_i + \mathbf{C}^T\mathbf{S}_2\mathbf{C} & * \\ \mathbf{E}^T\mathbf{P}_l(\mathbf{X}_{ii} - q\mathbf{I})/r - \mathbf{S}_1^T\mathbf{C} + \mathbf{D}_i^T\mathbf{S}_2\mathbf{C} & \mathbf{E}^T\mathbf{P}_l\mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D} - \mathbf{D}^T\mathbf{S}_2\mathbf{D} \end{bmatrix}$$

and

$$\boldsymbol{\Psi}_{12l} = \begin{bmatrix} \left(\dfrac{(\mathbf{X}_{ij} + \mathbf{X}_{ji})}{2}\right)^T\mathbf{P}_l\left(\dfrac{(\mathbf{X}_{ij} + \mathbf{X}_{ji})}{2}\right) - \dfrac{(\mathbf{P}_i + \mathbf{P}_j)}{2} + \mathbf{C}^T\mathbf{S}_2\mathbf{C} & * \\ \mathbf{E}^T\mathbf{P}_l\left(\dfrac{(\mathbf{X}_{ij} + \mathbf{X}_{ji})}{2}\right) - \mathbf{S}_1^T\mathbf{C} + \mathbf{D}^T\mathbf{S}_2\mathbf{C} & \mathbf{E}^T\mathbf{P}_l\mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D} - \mathbf{D}^T\mathbf{S}_2\mathbf{D} \end{bmatrix}$$

Applying Schur complement (Boyd et al., 1994) to (10), one can obtain the following inequalities.

$$\begin{bmatrix} (\mathbf{M}_{ii} - q\mathbf{R}_i)^T\mathbf{P}_l(\mathbf{M}_{ii} - q\mathbf{R}_i)/r^2 + \mathbf{U}_{ii} + \mathbf{R}_i^T\mathbf{C}^T\mathbf{S}_2\mathbf{C}\mathbf{R}_i & * \\ \mathbf{E}^T\mathbf{P}_l(\mathbf{M}_{ii} - q\mathbf{R}_i)\mathbf{P}_l/r - \mathbf{S}_1^T\mathbf{C}\mathbf{R}_i + \mathbf{D}_i^T\mathbf{S}_2\mathbf{C}\mathbf{R}_i & \mathbf{E}^T\mathbf{P}_l\mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D} - \mathbf{D}^T\mathbf{S}_2\mathbf{D} \end{bmatrix} < 0 \tag{15a}$$

and

$$\begin{bmatrix} \left(\frac{\left(\mathbf{M}_{ij}+\mathbf{M}_{ji}\right)}{2}\right)^T \mathbf{P}_l \left(\frac{\left(\mathbf{M}_{ij}+\mathbf{M}_{ji}\right)}{2}\right) + \frac{\left(\mathbf{U}_{ii}+\mathbf{U}_{jj}\right)}{2} + \mathbf{R}_i^T \mathbf{C}^T \mathbf{S}_2 \mathbf{C} \mathbf{R}_i & * \\ \mathbf{E}^T \mathbf{P}_l \left(\frac{\left(\mathbf{M}_{ij}+\mathbf{M}_{ji}\right)}{2}\right) - \mathbf{S}_1^T \mathbf{C} \mathbf{R}_i + \mathbf{D}_i^T \mathbf{S}_2 \mathbf{C} \mathbf{R}_i & \mathbf{E}^T \mathbf{P}_l \mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T \mathbf{S}_1 - \mathbf{S}_1 \mathbf{D} - \mathbf{D}^T \mathbf{S}_2 \mathbf{D} \end{bmatrix} < 0 \tag{15b}$$

Based on $\mathbf{Q}_i - \mathbf{R}_i^T - \mathbf{R}_i \geq -\mathbf{R}_i^T \mathbf{P}_i \mathbf{R}_i$, the following inequalities can be obtained.

$$\begin{bmatrix} \left(\mathbf{M}_{ii} - q\mathbf{R}_i\right)^T \mathbf{P}_l \left(\mathbf{M}_{ii} - q\mathbf{R}_i\right) / r^2 - \mathbf{R}_i^T \mathbf{P}_i \mathbf{R}_i + \mathbf{R}_i^T \mathbf{C}^T \mathbf{S}_2 \mathbf{C} \mathbf{R}_i & * \\ \mathbf{E}^T \mathbf{P}_l \left(\mathbf{M}_{ii} - q\mathbf{R}_i\right) \mathbf{P}_l / r - \mathbf{S}_1^T \mathbf{C} \mathbf{R}_i + \mathbf{D}_i^T \mathbf{S}_2 \mathbf{C} \mathbf{R}_i & \mathbf{E}^T \mathbf{P}_l \mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T \mathbf{S}_1 - \mathbf{S}_1 \mathbf{D} - \mathbf{D}^T \mathbf{S}_2 \mathbf{D} \end{bmatrix} < 0 \tag{16a}$$

and

$$\left[\begin{array}{c|c} \left(\frac{\left(\mathbf{M}_{ij}+\mathbf{M}_{ji}\right)}{2}\right)^T \mathbf{P}_l \left(\frac{\left(\mathbf{M}_{ij}+\mathbf{M}_{ji}\right)}{2}\right) - \frac{\left(\mathbf{R}_i^T \mathbf{P}_i \mathbf{R}_i + \mathbf{R}_j^T \mathbf{P}_j \mathbf{R}_j\right)}{2} + \mathbf{R}_i^T \mathbf{C}^T \mathbf{S}_2 \mathbf{C} \mathbf{R}_i & * \\ \hline \mathbf{E}^T \mathbf{P}_l \left(\frac{\left(\mathbf{M}_{ij}+\mathbf{M}_{ji}\right)}{2}\right) - \mathbf{S}_1^T \mathbf{C} \mathbf{R}_i + \mathbf{D}_i^T \mathbf{S}_2 \mathbf{C} \mathbf{R}_i & \mathbf{E}^T \mathbf{P}_l \mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T \mathbf{S}_1 - \mathbf{S}_1 \mathbf{D} - \mathbf{D}^T \mathbf{S}_2 \mathbf{D} \end{array}\right] < 0 \tag{16b}$$

Pre- and post-multiplying (16) by $diag\left\{\mathbf{G}_j^T, \mathbf{I}\right\}$ and $diag\left\{\mathbf{G}_j, \mathbf{I}\right\}$, one has

$$\begin{bmatrix} \left(\mathbf{X}_{ii} - q\mathbf{I}\right)^T \mathbf{P}_l \left(\mathbf{X}_{ii} - q\mathbf{I}\right) / r^2 - \mathbf{P}_i + \mathbf{C}^T \mathbf{S}_2 \mathbf{C} & * \\ \mathbf{E}^T \mathbf{P}_l \left(\mathbf{X}_{ii} - q\mathbf{I}\right) / r - \mathbf{S}_1^T \mathbf{C} + \mathbf{D}_i^T \mathbf{S}_2 \mathbf{C} & \mathbf{E}^T \mathbf{P}_l \mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T \mathbf{S}_1 - \mathbf{S}_1 \mathbf{D} - \mathbf{D}^T \mathbf{S}_2 \mathbf{D} \end{bmatrix} < 0 \tag{17a}$$

and

$$\begin{bmatrix} \left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right)^T \mathbf{P}_l \left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right) - \left(\mathbf{P}_i + \mathbf{P}_j\right)/2 + \mathbf{C}^T \mathbf{S}_2 \mathbf{C} & * \\ \mathbf{E}^T \mathbf{P}_l \left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right) - \mathbf{S}_1^T \mathbf{C} + \mathbf{D}_i^T \mathbf{S}_2 \mathbf{C} & \mathbf{E}^T \mathbf{P}_l \mathbf{E} + \mathbf{S}_3 - \mathbf{D}^T \mathbf{S}_1 - \mathbf{S}_1 \mathbf{D} - \mathbf{D}^T \mathbf{S}_2 \mathbf{D} \end{bmatrix} < 0 \tag{17b}$$

Obviously, the inequalities in (17) can be applied to ensure $\mathbf{\Psi}_{iil} < 0$ and $\mathbf{\Psi}_{12l} < 0$ if the conditions in (10) are held. Since $\mathbf{\Psi}_{iil} < 0$ and $\mathbf{\Psi}_{12l} < 0$, $\mathbf{\Gamma}\left(x, v, k\right) < 0$ can also be inferred from (14) to guarantee the following inequality.

$$\sum_{k=0}^{k_p} 2 y^T(k) \mathbf{S}_1 v(k) > \sum_{k=0}^{k_p} \left(y^T(k) \mathbf{S}_2 y(k) + v^T(k) \mathbf{S}_3 v(k) \right) \tag{18}$$

Based on (18), one can easily find that the closed-loop system achieves passivity performance defined by Definition 1. Additionally, assuming $v(k) = 0$, the following inequalities are obtained from (17).

$$\left(\mathbf{X}_{ii} - q\mathbf{I}\right)^T \mathbf{P}_l \left(\mathbf{X}_{ii} - q\mathbf{I}\right) / r^2 - \mathbf{P}_i < -\mathbf{C}^T \mathbf{S}_2 \mathbf{C} \tag{19a}$$

and

$$\begin{aligned} &\left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right)^T \mathbf{P}_l \left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right) \\ &-\left(\mathbf{P}_i + \mathbf{P}_j\right)/2 < -\mathbf{C}^T \mathbf{S}_2 \mathbf{C} \end{aligned} \tag{19b}$$

Since $\mathbf{S}_2 \geq 0$, the following inequalities are easily obtained.

$$\left(\mathbf{X}_{ii} - q\mathbf{I}\right)^T \mathbf{P}_l \left(\mathbf{X}_{ii} - q\mathbf{I}\right) / r^2 - \mathbf{P}_i < 0 \tag{20a}$$

and

$$\left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right)^T \mathbf{P}_l \left(\left(\mathbf{X}_{ij} + \mathbf{X}_{ji}\right)/2\right) - \left(\mathbf{P}_i + \mathbf{P}_j\right)/2 < 0 \tag{20b}$$

Based on Lemma 1 and (20a), if the condition (10a) holds then the closed-loop poles for each linear system in (8) can be forced into the *D*-region defined by Definition 2. Furthermore, based on (12) and (20), $\Delta V\left(x(k)\right) < 0$ can be found for guaranteeing the asymptotical stability of the closed-loop system in (8). Thus, if the conditions in Theorem hold, the uncertain ship autopilot system is asymptotically stable subject to pole-assignment and passivity constraints. The proof is completed. #

It is obviously found that the conditions in (10) are LMI form which can be directly solved via convex optimization algorithm (Boyd et al., 1994). Based on the proposed design method, the simulation results of uncertain ship autopilot system (1) are provided in the next section.

IV. SIMULATION RESULTS

Applying the convex optimization algorithm, the following feasible solutions can be obtained via solving Theorem with the given $q = 0$, $r = 0.4$, $\mathbf{S}_1 = 0$, $\mathbf{S}_2 = \mathbf{I}$ and $\mathbf{S}_3 = -\mathbf{I}$.

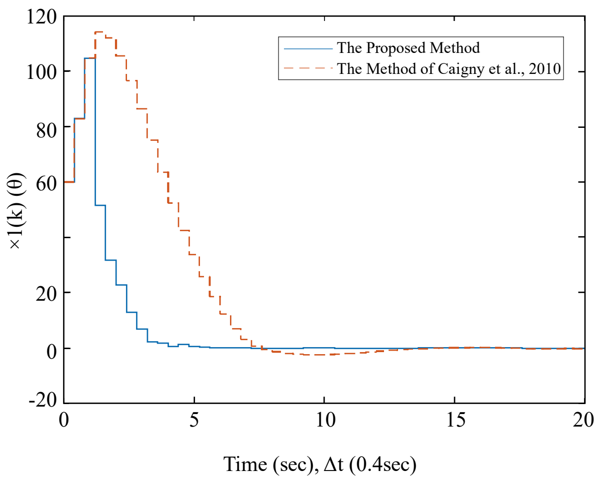

Fig. 2 Response of $x_1(k)$

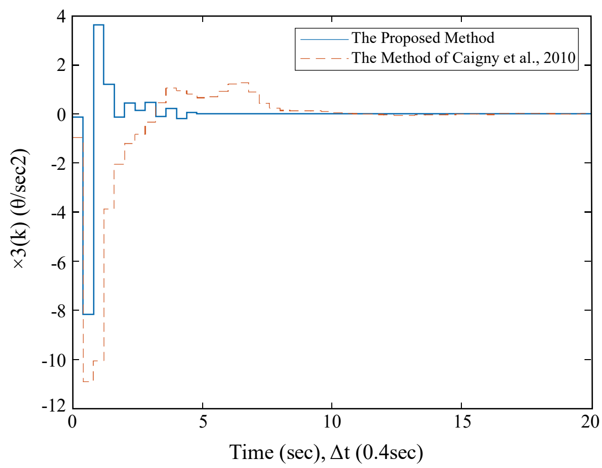

Fig. 4 Response of $x_3(k)$

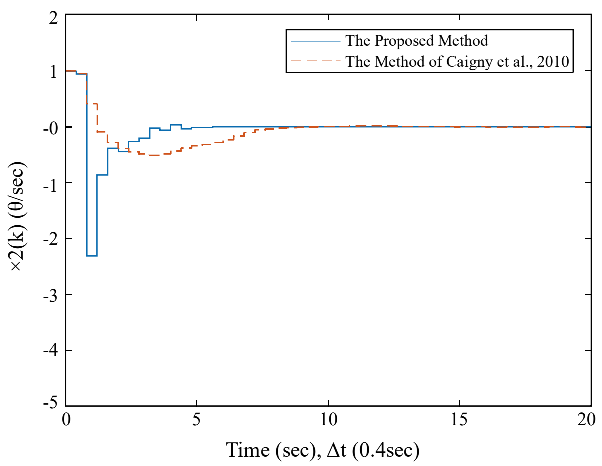

Fig. 3 Response of $x_2(k)$

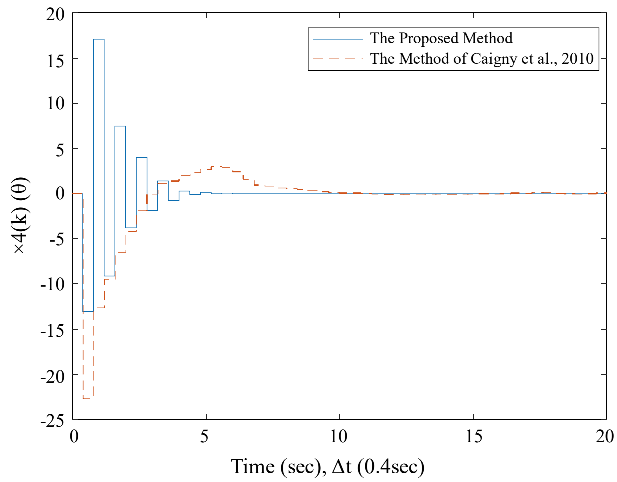

Fig. 5 Response of $x_4(k)$

$$\mathbf{P}_2 = 10^{-3} \times \begin{bmatrix} 0.1299 & 0.1621 & 0.0416 & 0.0022 \\ 0.1621 & 0.2467 & 0.0681 & 0.0040 \\ 0.0416 & 0.0681 & 0.0196 & 0.0011 \\ 0.0022 & 0.0040 & 0.0011 & 0.0005 \end{bmatrix},$$

$$\mathbf{G}_1 = 10^{-4} \times \begin{bmatrix} 0.3169 & 0.0151 & -0.0456 & -0.0025 \\ 0.1272 & 0.0233 & -0.0245 & 0.0016 \\ 0.0039 & 0.0065 & 0.0019 & 0.0001 \\ -0.0022 & 0.0004 & 0.0002 & 0.0041 \end{bmatrix},$$

$$\mathbf{G}_2 = 10^{-4} \times \begin{bmatrix} 0.3061 & 0.1202 & 0.0049 & -0.023 \\ 0.0191 & 0.0286 & 0.0079 & 0.0005 \\ -0.0477 & -0.0265 & 0.0023 & -0.0006 \\ -0.0049 & -0.0016 & 0.0001 & 0.0041 \end{bmatrix}, \quad (21)$$

$\mathbf{F}_1 = 10^8 \times [-0.5093 \quad 0.5979 \quad 0.2483 \quad -4.2772]$ and

$\mathbf{F}_2 = 10^8 \times [-0.4731 \quad 0.5589 \quad 0.2499 \quad -4.2390]$

According to the gains in (21), the following gain-scheduled controller (4) can be established.

$$u(k) = -\sum_{i=1}^{2} \vartheta_i \mathbf{F}_i \left(\sum_{i=1}^{2} \vartheta_i \mathbf{G}_i \right) x(k) \quad (22)$$

With (22), the responses of (1) are stated in Figs.2-5 with $x(0) = [\pi \quad 1 \quad -0.12 \quad 0]^T$. From Figs. 2-5, one can find that the states of system (1) are converged to zero via (22). Besides, according to Fig. 6, the closed-loop poles are obviously located at the circle region specified by $(q,r) = (0,0.4)$. Since $\mathbf{S}_1 = 0$, $\mathbf{S}_2 = \mathbf{I}$ and $\mathbf{S}_3 = -\mathbf{I}$, it means that a consideration of H_∞ performance is a special case of passivity theory. Thus, the following ratio value is got via substituting the simulation data and is obviously smaller than the given values.

$$\sum_{0}^{k_{p=20}} y^T(k)\mathbf{S}_2 y(k) \Big/ \sum_{0}^{k_{p=20}} v^T(k)\mathbf{S}_3 v(k) = 0.7942 \quad (23)$$

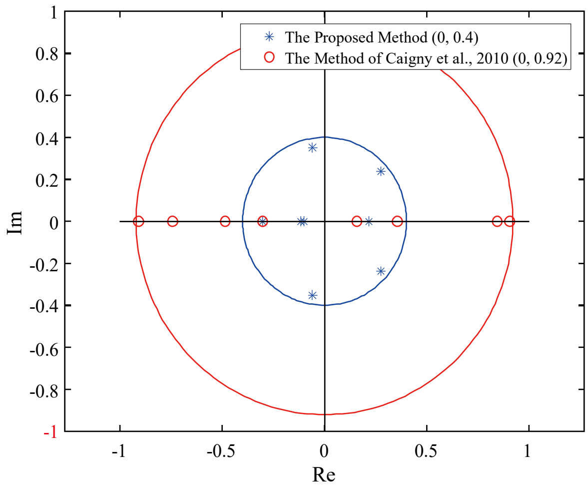

Fig. 6 Location of Closed-Loop Poles

According to (23), the passivity performance is obviously achieved via the controller (22). Concluding the above results, the passivity performance and asymptotical *D*-stability of the proposed method can be verified.

In order to show the advantage of this paper, the LPV-based control method (Caigny et al., 2010) is applied to the same system (1). Applying the method (Caigny et al., 2010), the following feasible solutions can be obtained.

$$
\mathbf{P}_1 = \begin{bmatrix} 0.1923 & 0.3014 & 0.1181 & 0.0683 \\ 0.3014 & 0.9222 & 0.3553 & 0.1910 \\ 0.1181 & 0.3553 & 0.2420 & 0.0583 \\ 0.0683 & 0.1910 & 0.0583 & 0.1273 \end{bmatrix},
$$

$$
\mathbf{P}_2 = \begin{bmatrix} 0.1669 & 0.2148 & 0.0849 & 0.0451 \\ 0.2148 & 0.6408 & 0.2377 & 0.1182 \\ 0.0849 & 0.2377 & 0.1942 & 0.0265 \\ 0.0451 & 0.1182 & 0.0265 & 0.1198 \end{bmatrix},
$$

$$
\mathbf{G}_1 = \begin{bmatrix} 10.4242 & -2.9921 & -0.4268 & -1.3324 \\ -3.1346 & 3.8838 & -3.1605 & -1.6542 \\ -0.4177 & -3.5754 & 8.7682 & 1.2887 \\ -0.5167 & -2.9770 & 1.3699 & 11.1102 \end{bmatrix}, \qquad (24)
$$

$\mathbf{F}_1 = 10^3 \times \begin{bmatrix} -0.2957 & -3.2741 & 1.0569 & 6.4535 \end{bmatrix}$ and

$\mathbf{F}_2 = 10^3 \times \begin{bmatrix} -0.4840 & -2.1611 & -0.8905 & 3.7588 \end{bmatrix}$

$$
\mathbf{G}_2 = \begin{bmatrix} 10.0875 & -2.7593 & -0.9619 & -0..0053 \\ -2.8575 & 3.9821 & -1.8782 & -1.4495 \\ -0.6167 & -3.4977 & 9.3373 & 2.3509 \\ -1.2703 & -1.1217 & -0.6773 & 9.8154 \end{bmatrix}
$$

Based on the gains in (24), the responses of (1) driven by the designed controller are also shown in Figs. 2-5 with the same initial condition. Referring to Figs. 2-5, the proposed design method provides a shorter settling time than the method of (Caigny et al., 2010). Referring to Fig. 6, the region of constraining the closed-loop poles assigned by the proposed design method is smaller region than one assigned by the method of (Caigny et al., 2010). Moreover, one can find that the following ratio value is bigger than the value in (23). That means the stronger H_∞ performance of the designed controller of this paper is guaranteed.

$$
\sum_{0}^{k_{p=20}} y^T(k)\mathbf{S}_2 y(k) \Big/ \sum_{0}^{k_{p=20}} v^T(k)\mathbf{S}_3 v(k) = 0.9908 \qquad (25)
$$

Based on the above results, it can be concluded that the proposed design method provides some improvements to the method of (Caigny et al., 2010) in controlling the uncertain ship autopilot system (1).

V. CONCLUSION

This paper proposed a gain-scheduled controller design method for the discretized ship autopilot system subject to the pole assignment and passivity constraints. To completely simulate the uncertainty, the LPV system and gain-scheduled scheme were used to guarantee the robustness. For the wave effect of ship, the external disturbance was also considered and attenuated via the passivity performance. Besides, the closed-loop poles were assigned in the specific region to improve the transient responses of the ship autopilot system. For the considerations, some sufficient conditions were derived via PDLF to reduce the conservatism caused by pole-assignment constraint. Moreover, the conditions were converted into LMI form which can directly apply convex optimal algorithm. At last, the simulation results were provided to show the application of the proposed method.

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