Journal of Marine Science and Technology

Volume 28 | Issue 5 Article 2

A SURVEY OF ADVANCED CONTROL METHODS FOR PERMANENT MAGNET STEPPER MOTORS

Yong Woo Jeong
Department of Electrical Engineering, Hanyang University, Seoul, South Korea

Youngwoo Lee
Department of Electrical Engineering, Chonnam National University, Gwangju, South Korea

Chung Choo Chung
Division of Electrical and Biomedical Engineering, Hanyang University, Seoul, South Korea., cchung@hanyang.ac.kr

Recommended Citation

Jeong, Yong Woo; Lee, Youngwoo; and Chung, Chung Choo (2020) "A SURVEY OF ADVANCED CONTROL METHODS FOR PERMANENT MAGNET STEPPER MOTORS," *Journal of Marine Science and Technology*: Vol. 28: Iss. 5, Article 2.
DOI: 10.6119/JMST.202010_28(5).0002
Available at: https://jmstt.ntou.edu.tw/journal/vol28/iss5/2

Journal of Marine Science and Technology, Vol. 28, No. 5, pp. 331-342 (2020)
DOI: 10.6119/JMST.202010_28(5).0002

A SURVEY OF ADVANCED CONTROL METHODS FOR PERMANENT MAGNET STEPPER MOTORS

Yong Woo Jeong[1], Youngwoo Lee[2], and Chung Choo Chung[3]

Key words: permanent magnet stepper motor, microstepping, nonlinear control, position control.

ABSTRACT

Permanent Magnet (PM) stepper motors have been widely used in industry due to their low material costs and robustness against the environment. Furthermore, it is relatively easy to implement a control system compared to other type motors. It has been recently reported that advanced control methods show improved tracking performance over the conventional microstepping control. In this paper, we make a survey of advanced control methods for PM stepper motors. First, we introduce basic principles of open-loop control of PM stepper motors, including microstepping. Then we explain various advanced feedback control techniques based on Lyapunov stability. Second, we briefly summarize how PM stepper motors can be modelled as a linear parameter varying system and its tracking performance optimization based on H_2 sense is made with nonlinear torque modulation. Third, we show the equivalency of field orient control and field weakening control to microstepping with nonlinear torque modulation. Then we introduce Proximate In-Phase Current Estimator for PM stepper motors and Phase-Compensated Microstepping for PM stepper motors. Their performances are illustrated by experiments on PM stepper motors.

I. INTRODUCTION

Permanent Magnet (PM) stepper motors have been widely used in positioning applications due to their durability, high efficiency, and power density, as well as their high torque to inertia ratio and absence of rotor winding. Another merit is that PM stepper motors can operate in open-loop control, i.e., full stepping or half-stepping (Kuo, 1979; Acarnley, 2002; Chiasson, 2005). However, full/half-stepping for PM stepper motors has limitation in achieving precision motion control due to its step size and oscillatory motions between steps. Standard PM stepper motors have relatively large step sizes, usually 1/200 of a revolution or 1.8 degrees. Such large step sizes may cause motor-shaft oscillations at low speeds. To solve this problem, microstepping was invented in 1974 by Durkos (Yeadon and Yeadon, 2001). For example, if a PM stepper motor is two-phase winding, microstepping for PM stepper motors is defined as a control method in which two sinusoidal inputs shifted 90 degrees are given to PM stepper motors for position tracking. This novel technique allows the PM stepper motor to stop and hold a position between the step positions if its stall torque is enough. Microstepping largely eliminates the jerky character of low speed operations as well as noises at intermediate speeds.

Although microstepping has been widely used in industry, the performance of the position control at high speed is degraded for many reasons, such as the back-electromagnetic force (back-emf), external disturbance, and system uncertainties. With the increase in power and the decrease in the cost of embedded processors in recent years, drives and control systems for PM stepper motors have become increasingly sophisticated. The availability of low cost embedded processors and significant advances in power electronics have motivated the design of complex control algorithms for PM stepper motors. Thus, for positioning applications, PM stepper motors can be substituted for expensive servo motors such as PM synchronous motors as a cheaper replacement in closed-loop operations (Clarkson and Acarnley, 1988; Bodson et al., 1993; Le et al., 2016). Plenty of journal papers have been published on PM stepper motor control and on the nonlinear control of PM stepper motors. An adaptive backstepping method using full-state feedback was developed in (Xu et al., 1998). Sliding-mode controllers using position and current feedback were proposed in (Nollet et al., 2008; Seshagiri, 2009; Defoort et al., 2009). Most of these papers regard the PM stepper as a complex nonlinear system with direct quadrature (DQ) transformation and the design of control algorithms for PM stepper motors was very complex for several reasons. It is a multivariable control problem since there are two independent control inputs (Shin et al., 2011). s

Paper submitted 01/24/20; accepted 03/05/20. Corresponding Author: Chung Choo Chung(e-mail: cchung@hanyang.ac.kr)
[1]Department of Electrical Engineering, Hanyang University, Seoul, South Korea.
[2]Department of Electrical Engineering, Chonnam National University, Gwangju, South Korea.
[3]Division of Electrical and Biomedical Engineering, Hanyang University, Seoul, South Korea.

Table 1. Input Voltages for Full/Microstepping

Phase	Full stepping				Micro Stepping
	Step 1	Step 2	Step 3	Step 4	
B	V_{max}	$-V_{max}$	$-V_{max}$	V_{max}	$V_{max}\cos\left(N_r\theta^d\right)$
A	V_{max}	V_{max}	$-V_{max}$	$-V_{max}$	$V_{max}\sin\left(N_r\theta^d\right)$
$\bar{A}$	$-V_{max}$	V_{max}	V_{max}	$-Vmax$	$-V_{max}\cos\left(N_r\theta^d\right)$
$\bar{B}$	$-V_{max}$	$-V_{max}$	V_{max}	V_{max}	$-V_{max}\sin\left(N_r\theta^d\right)$

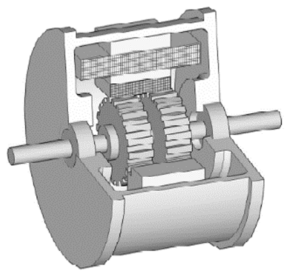

Fig. 1. Cut-off figure of Hybrid Stepper Motor.

Various control methods based on vector control with DQ transformation were developed to eliminate nonlinear terms. By extension, many kinds of research have implemented the nonlinear control theory in order to improve the position (or velocity) tracking performance (Zribi and Chiasson, 1991; Bodson et al., 1993; Marino et al., 1995; Sira-Ramirez, 2000; Nollet et al., 2008; Tomei and Verrilli, 2011). Although using DQ transformation gives us the benefit of easily interpreting the PM stepper motors as a direct current (DC) motor, it additionally needs DQ transformation which requires position-loop feedback. Moreover, there are no methods for designing and analyzing it, based on microstepping, that have been widely used for improved resolution and significantly increased motion stability in PM stepper motors.

PM stepper motor dynamics without DQ transformation were recently introduced over the last decade (Le and Jeon, 2007; Le and Jeon, 2009a; Le and Jeon, 2009b; Nguyen et al., 2017; Kim and Chung, 2011; Kim et al., 2011; Kim et al., 2012a; Kim et al., 2012b; Kim et al., 2012c; Shin et al., 2013; Lee et al., 2016; Kim and Chung, 2016a; Kim et al., 2016b; Shin et al., 2016a; Shin et al., 2016b; Kim et al., 2017; Lee et al., 2017; Kim et al., 2018; Lee et al., 2019). These research results show enhanced tracking performances of PM stepper motors even without DQ transformation. To our knowledge, there is no survey paper overviewing such advanced control methods without DQ transformation. To help the readers grasp an overview of PM stepper motor control, this survey paper presents and summarizes the recent advanced control methods for PM stepper motors control and state estimation techniques without DQ transformations. Although the previous results are based on 2-phase PM stepper motors, since the dynamics of PM stepper motors are almost the same as the dynamics of the PM synchronous motor that has been widely used in various industries, this kind of review paper is also useful for those trying to control other kinds of 3-phase permanent magnet synchronous motors (Lee et al., 2018).

The paper is organized as follows. In Section II, we introduce basic principles of open-loop control of PM stepper motors including microstepping. Then a brief summary of various nonlinear feedback control techniques based on Lyapunov stability follows in Section III. In Section IV, experimental results on the various control methods are presented to support their effectiveness on real PM stepper motors. Conclusions follow in Section V.

II. PRINCIPLES OF PM STEPPER MOTOR OPERATIONS

The Permanent Magnet (PM) Stepper Motor operates with electromagnetic force which comes from the interaction between the rotor and stators magnetic field. Figure 1 shows the cut-off figure of a hybrid stepper motor. Several open-loop control methods for PM stepper motors exist. In this section, we will briefly explain the three basic conventional PM stepper motors control methods, i.e., full stepping, half stepping and microstepping. With these three methods, we will discuss their fundamental limitations in achieving high precision position and/or velocity tracking performance.

1. Full/Half Stepping

Full/half stepping are intuitive and straightforward positioning control methods for PM stepper motors. As we can guess the method from the name, in full step operation, the rotor moves one step angle for specific voltage signal sequences as listed in Table. 1. The amount of step angle is different according to the stepper motor type. For example, a 1.8 degrees step motor takes 200 steps per motor revolution with 50 teeth. For example, in the case of full stepping, there are two types of full step excitation modes (Kue, 1979; Acarnley, 2002; Chiasson, 2005). In the single-phase mode, also known as one-phase on full step excitation, the motor

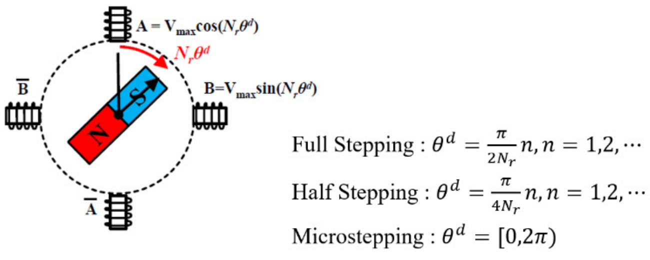

Fig. 2. Rotor Position for Full/Half/Micro stepping.

operates with only one phase (a group of windings) energized at a time. This mode requires the least amount of power from the driver of any of the excitation modes. In dual-phase mode, also known as two-phase on full step excitation, the motor is operated with both phases energized at the same time. This mode provides improved torque and speed performance. Dual-phase excitation provides about 30% to 40% more torque than single-phase excitation but does require twice as much power from the driver. Half step excitation is alternating single and dual-phase operation resulting in steps that are half the basic step angle. Due to the smaller step angle, this mode provides twice the resolution and smoother operation. Half stepping produces roughly 15% less torque than dual phase full stepping. In industry, there are many commercial stepping motor drivers available for full/half stepping control such as L297/L298 (STMicroelectronics, 2000; STMicroelectronics, 2001). These controllers command the switching sequence for on/off of the H-bridge gating signal depending on the direction and step method. They use current feedback with a simple comparator to regulate the current flowing to each leg of the H-Bridge.

2. Microstepping

Figure 3. shows standard hybrid type steppers which have relatively large step sizes, usually 1.8 degrees for 50 teeth. Such large step sizes may cause motor-shaft oscillations at low speeds (Bodson et al., 2006). Due to pulling torque and pull out torque, there are limitations in the design velocity profile (Kue, 1979; Acarnley, 2002; Chiasson, 2005). Microstepping was invented in 1974 by Durkos for improved resolution and significantly increased motion stability reducing vibration. Microstepping for two-phase PM steppers is a control method in which two sinusoidal inputs shifted 90 degrees are given to a PM stepper for position tracking. Microstepping allows a PM stepper to stop and hold a position between the full and half step positions, thereby largely eliminating the jerky character of low-speed operations and noises at intermediate speeds. See Fig. 2 and Table.1. Low resolution microstepping, up to 1/16 step, can be easily implemented using a commercially available PWM power driver such as DRV84x2 and microprocessor (Texas Instruments, 2009a; Texas Instruments, 2009b). Similar to full/half stepping control, conventional microstepping uses closed-feedback for current feedback without

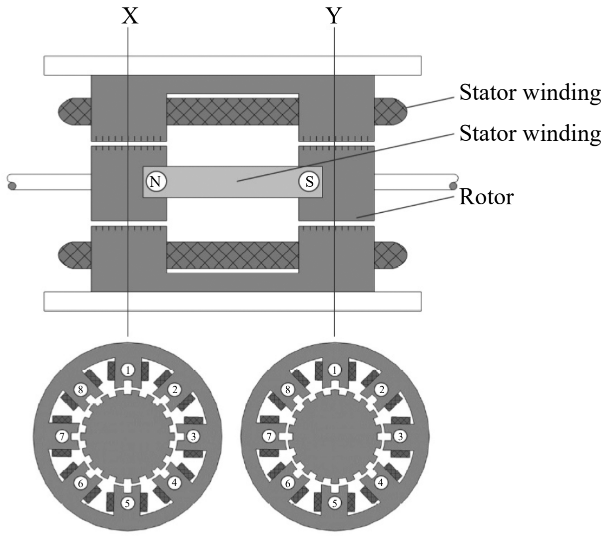

Fig 3. Two Phase Stepper Motor.

measurement of either position or velocity. Although this type of stepper motor control method shows an enhanced control performance compared to the full/half stepping, it cannot always provide a precise positioning tracking performance when internal/external disturbances, such as load torque or back-emf of current loops, are injected. The amount of tracking error is well described in (Kim et al., 2012a). To deal with this kind of problem, feedback control methods of PM stepper motors have been researched based on the mathematical model. Section III describes the currents and/or position feedback control methods of PM stepper motors. Before describing the nonlinear feedback control and estimation method, let us briefly explain the mathematical model of a PM stepper motor.

3. Electro-Mechanical Dynamics of PM Stepper Motor

The electro-mechanical dynamics of PM stepper motors consist of two parts, the mechanical and the electrical dynamics. The mechanical dynamics are given by Newton's laws relating torque to acceleration. The electrical part is represented by Kirchoff's laws and can be derived using an equivalent circuit model. The mechanical and electrical subsystems are coupled through torque which depends on currents and inductances that depend on the position. The mechanical dynamics of 2-phase PM stepper motors are given by (Khorrami et al., 2003):

$$
\begin{aligned}
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{1}{J}\left[-K_m i_\alpha \sin\left(N_r\theta\right) + K_m i_\beta \cos\left(N_r\theta\right) - B\omega - \tau_l\right] \\
\dot{i}_\alpha &= \frac{1}{L}\left[v_\alpha - Ri_\alpha + K_m\omega\sin\left(N_r\theta\right)\right] \\
\dot{i}_\beta &= \frac{1}{L}\left[v_\beta - Ri_\beta - K_m\omega\cos\left(N_r\theta\right)\right]
\end{aligned}
\tag{1}
$$

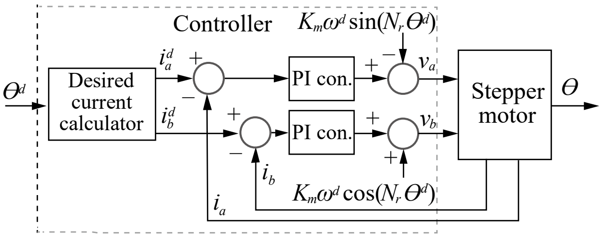

Fig. 4. Block diagram of PI controller in the current-loop

where v_α, v_β and i_α, i_β are the voltages [V] and currents [A] in phases A and B, respectively. θ is the rotor (angular) position [rad], ω is the rotor (angular) velocity [rad/s], B is the viscous friction coefficient [N · m · s/rad], J is the inertia of the motor [Kg · m2], K_m is the motor torque constant [N · m/A], R is the resistance of the phase winding [W], L is the inductance of the phase winding [H], and N_r is the number of rotor teeth. The load torque perturbation is τ_l. The electro-mechanical dynamic equation (1) could be represented after Direct Quadrature (DQ) transformation in the form of:

$$\begin{aligned}
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{1}{J}\left[-K_m i_q - B\omega - \tau_l\right] \\
\dot{i}_d &= \frac{1}{L}\left[v_d - Ri_d + N_r L\omega i_q\right] \\
\dot{i}_q &= \frac{1}{L}\left[v_q - Ri_q - N_r L\omega i_d - K_m\omega\right]
\end{aligned} \tag{2}$$

where, i_d, i_q are the direct and quadrature current and v_d, v_q are direct and quadrature voltage inputs. We see that motor torque related dynamics becomes similar to a DC motor if the nonlinear term $N_r L\omega i_d$ is canceled. Thus equation (2) is mostly preferred for motion control by control engineers.

III. ADVANCED FEEDBACK CONTROL

In this section, we will describe various advanced feedback control methods based on this mathematical model of PM stepper motors. First, microstepping with only currents feedback is introduced. Secondly, microstepping with only position feedback is introduced. Finally, we will present control methods using both current and position measurements. Microstepping using torque modulation and Proximate In-Phase Current Estimator will also be introduced.

1. Microstepping With Only Currents Feedback

When we implement the microstepping method for positioning control of PM stepper motors, simple current feedback control is utilized. For motion control using microstepping, we generate a sequence of desired current references (i_α^d, i_β^d) in the form of sinusoidal signals based on the positioning profiles (θ^d) such as:

$$\begin{aligned}
i_\alpha^d &= \frac{V^d}{R}\cos\left(N_r\theta^d\right) \\
i_\beta^d &= \frac{V^d}{R}\sin\left(N_r\theta^d\right).
\end{aligned} \tag{3}$$

However, it is not sufficient for precision tracking performance in high-speed motion since the back-emf effect is not negligible in that region. The back-emf effect increases as the velocity increases so it distorts the motor torque and degrades the tracking performance of position control. One method for computing the phase voltages (v_α, v_β) is using the proportional and integral current feedback control which cancels out the back-emf by feedforward compensation with the desired position/velocity profile such as:

$$\begin{aligned}
v_\alpha &= K_p\left(i_\alpha^d - i_\alpha\right) + K_I\int_0^t\left(i_\alpha^d - i_\alpha\right)d\tau - K_m\omega^d\sin\left(N_r\theta^d\right) \\
v_\beta &= K_p\left(i_\beta^d - i_\beta\right) + K_I\int_0^t\left(i_\beta^d - i_\beta\right)d\tau + K_m\omega^d\cos\left(N_r\theta^d\right)
\end{aligned} \tag{4}$$

where K_p, K_I are proportional, integral gains. Fig. 4. shows the structure of the PI control with compensation of the back-emf. Notice that the control law (4) requires current measurement, the desired velocity and position. This kind of control structure has a benefit that it partially compensates for the back-emf effect by calculating it based on the desired velocity profile. Although there are differences between the desired and real velocities, and also a difference between the desired and real position, such differences can be partially compensated for by the integral action. It cannot, however, achieve high precision since it does not use position and velocity information at all.

The Phase-Compensated Microstepping technique has been developed to enhance the performance of tracking errors using only currents feedback. Refer to (Shin et al., 2013) for details. Under the presence of uncertain external load torque, the method does not effectively compensate for only using the current feedback. A measurement or estimation of mechanical states should be added to cope with the external disturbance while maintaining precision position/velocity tracking.

2. Stepper Motor Control With Only Position Feedback

To enhance the angular position control performance of PM stepper motors, feedback control is necessary to compensate for the external disturbances. In this subsection, we will describe the advanced position feedback control techniques without DQ transformation. In subsection 2.1, we will explain the passivity based current estimation to substitute current measurement sensor and microstepping with Lyapunov based current control. In subsection 2.2, we will describe the performance optimization position control for PM stepper motors.

2.1. Lyapunov Based Currents Control with Passivity based Currents Estimation with only position measurement.

In this subsection, we will show the Lyapunov based currents tracking control using the passivity phase current observer. This control method implements the precision positioning control with position measurement only. If we define the state by:

$$x=\begin{bmatrix}\theta & \omega & i_\alpha & i_\beta\end{bmatrix}^T,$$

it is shown that the nonlinear passivity state observer can be designed to estimate the current states without measurement such as:

$$\dot{\hat{x}} = A\hat{x} + f_0\left(\theta,\hat{\omega},\hat{i}_\alpha,\hat{i}_\beta\right) + Bu + L\left(\theta-\hat{\theta}\right) \tag{5}$$

where $\hat{x}=\begin{bmatrix}\hat{\theta} & \hat{\omega} & \hat{i}_\alpha & \hat{i}_\beta\end{bmatrix}^T, L=\begin{bmatrix}l_1 & l_2 & l_3 & l_4\end{bmatrix}^T, C=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}$,

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\frac{F}{J} & 0 & 0\\ 0 & 0 & -\frac{R}{L} & 0\\ 0 & 0 & 0 & -\frac{R}{L}\end{bmatrix}, B=\begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix},$$

$$f_0=\begin{bmatrix}0\\ \frac{1}{J}\left(-K_m i_\alpha \sin(N_r\theta)+K_m i_\beta \cos(N_r\theta)\right)\\ \frac{K_m}{L}\omega\sin(N_r\theta)\\ -\frac{K_m}{L}\omega\cos(N_r\theta)\end{bmatrix}.$$

$\hat{x}$ is the estimation of x, and L is a vector of observer gains. Note that the measured position is used in $\sin\left(N_r\theta\right)$ and $\cos\left(N_r\theta\right)$ of the nonlinear observer (Kim and Chung, 2011). With this observer, the system does not need to add the additional shunt resistance to measure the currents and Analogue-digital Converter (ADC) to an embedded system. Also, we can generate the desired current references for each phase using a similar method to microstepping. With these desired currents and estimated currents, we can design the current controller guaranteeing the exponential convergence of current tracking errors by only using position measurements. With the observer, the following control law is used to ensure the motion stability. We have shown that we can achieve the Lyapunov-based controller using the estimated states in (5).

Theorem 1 (Kim et al., 2012a)**:** If the Lyapunov-based controller is given by:

$$\begin{aligned} v_\alpha &= \left(Ri_\alpha - K_m\omega\sin\left(N_r\theta\right)\right) + L\left(\dot{i}_\alpha^d + \rho e_\alpha\right)\\ v_\beta &= \left(Ri_\beta + K_m\omega\cos\left(N_r\theta\right)\right) + L\left(\dot{i}_\beta^d + \rho e_\beta\right) \end{aligned} \tag{6}$$

where $i_\alpha^d = \frac{V_{\max}}{R}\cos\left(N_r\theta^d\right), i_\beta^d = \frac{V_{\max}}{R}\sin\left(N_r\theta^d\right)$, θ^d is the desired angle from a position profile, and the control gain is a positive constant, then the tracking error is exponentially converged. Although this method shows improved positioning control compared to the conventional microstepping control method, it is only focused on the current tracking performances so it does not solve the problem of microstepping control such as degrading performance when the external load torque or other mechanical disturbances exist. Refer to (Kim et al., 2012a) for details.

2.2. Linear Parameter Varying H_2 Control

Traditionally, it is common to use nonlinear control methods for torque control of PM stepper motors. For a long time, there has been research for optimization techniques for nonlinear systems (Van der Shaft, 1991; Van der Shaft, 1992; Chung and Hauser; 1997). However, it is still a difficult issue to evaluate closed-loop performance. To solve this problem, we have provided a new optimization method using linear parameter varying (LPV) synthesis for PM stepper motor dynamics. From (1), let us define nonlinear varying parameters:

$$\delta_1 = k_m\sin\left(N_r\theta\right), \delta_2 = k_m\cos\left(N_r\theta\right).$$

Then since the nonlinear varying parameters are bounded, the nonlinear varying parameters can be represented as follows:

$$\begin{aligned} \underline{\delta_1} &= -k_m \le \delta_1\left(\theta\right) \le \overline{\delta_1} = k_m\\ \underline{\delta_2} &= -k_m \le \delta_2\left(\theta\right) \le \overline{\delta_2} = k_m \end{aligned}. \tag{7}$$

Therefore, the tracking error dynamics is in the form of :

$$\begin{aligned} \dot{e} &= A_e\left(\delta\right)e + B_e u\\ y &= C_e e \end{aligned} \tag{8}$$

where $e=\begin{bmatrix}\theta^d-\theta & \omega^d-\omega & i_\alpha^d - i_\alpha & i_\beta^d - i_\beta\end{bmatrix}^T$

$$A_e\left(\delta\right)=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\frac{B}{J} & -\frac{\delta_1\left(\theta\right)}{J} & \frac{\delta_2\left(\theta\right)}{J}\\ 0 & \frac{\delta_1\left(\theta\right)}{J} & -\frac{R}{L} & 0\\ 0 & \frac{\delta_2\left(\theta\right)}{J} & 0 & \frac{R}{J}\end{bmatrix}$$

$$B_e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, C_e = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \delta(\theta) = \begin{bmatrix} \delta_1(\theta) \\ \delta_2(\theta) \end{bmatrix}.$$

We are going to design the optimal controller to guarantee closed-loop stability and its tracking performance by assuming that the load torque can be compensated by proper estimation techniques. Therefore, the LPV system can be expressed in the state-space form as follows:

$$\begin{aligned} \dot{e} &= A_e(\delta)e + B_e u \\ z &= C_z e + D_z u \\ y &= C_e e \end{aligned} \quad . \tag{9}$$

Here, $z \in \mathrm{R}^4$ are an performance states. $C_z \in \mathrm{R}^{4\times4}$ and $D_z \in \mathrm{R}^{4\times2}$ are control-input weighting matrices, respectively. The auxiliary control law is:

$$\begin{aligned} u &= K(\delta)\hat{e} \\ &= K(\delta)(e-\tilde{x}) \\ &= v(\delta)+w(\delta) \end{aligned} \tag{10}$$

where $v(\delta) = K(\delta)e \in \mathrm{R}^{2\times1}$ is the virtual control input and $w(\delta) = -K(\delta)\tilde{x} \in \mathrm{R}^{2\times1}$ is the bounded exogenous disturbance signal. Therefore, from (9) and (10) the LPV system can be represented in the state-space form as follows:

$$\begin{aligned} \dot{e} &= A_e(\delta)e + B_e w(\delta) + B_e v(\delta) \\ z &= C_z e + D_z v(\delta) \\ y &= C_e e \end{aligned} \quad . \tag{11}$$

To verify the stability of the closed-loop system consisting of the observer in (Lee et al., 2017) and tracking error dynamics (11), we can derive H2 optimal solution from Theorem 2 below.

Theorem 2 (Lee et al., 2017)**:** Consider the closed-loop LPV system (11). If there are two symmetric matrices X_2, $Z > 0$ and $Y^{(i)} > 0$ for a given $\gamma > 0$ such that:

$$\begin{bmatrix} A^{(i)}X_2 + (A^{(i)}X_2)^T + B_2Y^{(i)} + (B_2Y^{(i)})^T & B_1 \\ B_1^T & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} Z & (C_1X_2 + D_{12}Y^{(i)}) \\ (C_1X_2 + D_{12}Y^{(i)})^T & X_2 \end{bmatrix} > 0$$

$$trace(Z) < \gamma^2$$

where $X_2 = X^{-1}$. Then, the closed-loop system (9) is parametrically dependent quadratically stabilizable by the state feedback $K(\delta) = \sum_{i=1}^{4} \xi_i K^{(i)}$ with $K^{(i)} = Y^{(i)}X_2^{-1}$. Moreover, $K(\delta)$ is also guaranteed for H2 performance, $\| T_{zw}(\delta) \|_2 < \gamma$.

3. Stepper Motor Control With Both Current and Position Feedback

In this section, we describe the advanced control for precision positioning control with position and current measurements. Firstly, we will explain the Field Oriented Control (FOC) and Field Weakening Control (FWC) without DQ transformation. After that, we will describe the backstepping positioning control. Secondly, we will explain the advanced method for improving the positioning performance by introducing the Proximate In-Phase Current Estimator (PIpCE) to improve the phase delay of the conventional low-pass filter.

3.1 Microstepping with Nonlinear Torque Modulation Technique

It is shown that microstepping with the torque modulation technique improves the position tracking performance of PM stepper motors. They utilize a novel commutating schemes for generating the desired current of each phase using the following equations (Kim et al., 2012b). Suppose that we need the desired torque, τ^d for a certain motion, the corresponding desired phase currents should be given by:

$$i_\alpha^d = \frac{\tau^d}{K_m}\cos\left(N_r\theta^*\right),\ i_\beta^d = \frac{\tau^d}{K_m}\sin\left(N_r\theta^*\right) \tag{12}$$

either to hold its position or to drive the motor at a specific point θ^*. Notice that the θ^* is the measured angular position of PM stepper motors and it differs from the θ^d as we mentioned in the microstepping above. Based on Equation (12), they establish the FOC and FWC methods without DQ transformation as follows:

1) Field Oriented Control (FOC) without DQ transformation

In order to maximize torque, direct current should be maintained at zero, i.e., $i_d^d = 0$. This condition indicate that the alpha-beta current references should be as follows:

$$\begin{aligned} i_\alpha^d &= I^d\cos\left(N_r\theta + 0.5\pi\right) = -\frac{\tau^d}{K_m}\sin\left(N_r\theta\right) \\ i_\beta^d &= I^d\sin\left(N_r\theta + 0.5\pi\right) = \frac{\tau^d}{K_m}\cos\left(N_r\theta\right) \end{aligned} \quad . \tag{13}$$

2) Field Weakening Control (FWC) without DQ transformation

In Equation (2), $-K_m\omega$ is the back-emf term. At high velocity, the control input for FOC can be saturated due to the cancellation of back-emf. However, control input saturation can be avoided if the negative i_d^d is maintained to help cancel back-emf. This is called as Field Weakening Control (FWC) and the desired currents (i_α^d, i_β^d) for FWC is as follows:

$$
\begin{aligned}
i_\alpha^d &= I^d \cos\left(N_r\theta + 0.5\pi + \theta_l\right) = -\frac{\tau^d}{K_m \cos\left(\theta_l\right)} \sin\left(N_r\theta + \theta_l\right) \\
i_\beta^d &= I^d \sin\left(N_r\theta + 0.5\pi + \theta_l\right) = \frac{\tau^d}{K_m \cos\left(\theta_l\right)} \cos\left(N_r\theta + \theta_l\right)
\end{aligned} \tag{14}
$$

where $\theta_l = \mathrm{atan}\left(N_r L K_m^2 \omega^2 / (R^2 + (N_r \omega L)^2)\tau^d\right)$.

The nonlinear torque modulation was proposed to achieve the desired torque to be robust against the external disturbances such as load torque. The mechanical dynamics tracking errors can be defined as:

$$
\begin{aligned}
e_\theta &= \theta^d - \theta, \\
e_\omega &= \omega^* - \omega
\end{aligned} \tag{15}
$$

where θ^d is the desired dynamic position and ω^* will be defined in the following lemma. The tracking error dynamics of the mechanical dynamics is given by:

$$
\begin{aligned}
\dot{e}_\theta &= \omega^d - \omega \\
\dot{e}_\omega &= \dot{\omega}^* - \frac{1}{J}\left[\tau^d - B\omega - \tau_L\right]
\end{aligned}. \tag{16}
$$

Lemma 1 (Kim et al., 2012b): Consider the tracking error dynamics (10). If the nonlinear torque modulation is designed by:

$$
\begin{aligned}
\omega^* &= \omega^d + k_1\left(\theta^d - \theta\right) \\
\tau^d &= k_2\left(\omega^* - \omega\right) + \left(\theta^d - \theta + B\omega + J\dot{\omega}^* + \tau_L\right)
\end{aligned} \tag{17}
$$

where k_1, k_2 are positive constants and $\omega^d = \dot{\theta}^d$ is the desired velocity, then the origin of the tracking error dynamics (16) is exponentially stable.

3.2. Proximate In-Phase Current Estimator (PIpCE)

In PM stepper motor control, because the currents can be easily measured by monitoring the resistors of the motor driver, the use of current sensors is preferred for industrial applications. However, there is a limitation in increasing the closed-loop bandwidth of the current loop due to measurement noises in the high frequency range. Therefore, low-pass filters (LPFs) have been employed to reduce high-frequency noise. LPFs cause a phase lag in the current measurement, and can lead to torque ripples (Bodson et el., 2006) In addition, this phase lag may result in step-out or speed reversal at high speed operation. In this subsection, we propose a microstepping method using a proximate in-phase current estimator (PIpCE) to reduce torque ripple and achieve precise position control even in high speed operation using only current sensors. If we use the current estimator defined by:

$$
\begin{aligned}
\dot{\hat{i}}_a &= k(i_a - \hat{i}_a) + \dot{i}_a^d \\
\dot{\hat{i}}_b &= k(i_b - \hat{i}_b) + \dot{i}_b^d
\end{aligned} \tag{18}
$$

where k is the filtering gain, $\hat{i}_a$ and $\hat{i}_b$ are the estimated currents, and the current tracking errors e_a and e_b are defined as:

$$
\begin{aligned}
e_a &= i_a^d - i_a \\
e_b &= i_b^d - i_b
\end{aligned} \tag{19}
$$

where i_a^d and i_b^d are the desired phase currents, it is shown that the current estimator becomes PIpCE. Let us define the estimated current tracking errors $\hat{e}_a$ and $\hat{e}_b$, and the estimated errors $\tilde{i}_a$ and $\tilde{i}_b$ as:

$$
\begin{aligned}
\hat{e}_a &= i_a^d - \hat{i}_a \\
\hat{e}_b &= i_b^d - \hat{i}_b \\
\tilde{i}_a &= i_a - \hat{i}_a \\
\tilde{i}_b &= i_b - \hat{i}_b
\end{aligned}. \tag{20}
$$

The current tracking errors (19) should be transformed to the estimated current tracking errors $\hat{e}_a$ and $\hat{e}_b$ because the estimated currents $\hat{i}_a$ and $\hat{i}_b$ are required instead of the measured currents i_a and i_b.

Theorem 3 *(Lee et al., 2016)***:** Given the phase currents dynamics of (1), suppose that the PIpCE and proportional-integral feed-forward(PIFF) controllers are designed as:

$$
\begin{aligned}
\dot{\hat{i}}_a &= k\tilde{i}_a + \dot{i}_a^d \\
\dot{\hat{i}}_b &= k\tilde{i}_b + \dot{i}_b^d \\
\hat{e}_{az} &= \int_0^t \hat{e}_a \, d\tau \\
\hat{e}_{bz} &= \int_0^t \hat{e}_b \, d\tau \\
v_a &= R\hat{i}_a + L\dot{i}_a^d + k_P\hat{e}_a + k_I\hat{e}_{az} \\
v_b &= R\hat{i}_b + L\dot{i}_b^d + k_P\hat{e}_b + k_I\hat{e}_{bz}
\end{aligned} \tag{21}
$$

where $k_P > L - R$ and $k_I > L - k_P$ are the controller gains, and $\hat{e}_{az}$ and $\hat{e}_{bz}$ are integral terms of the current tracking errors. Then, e_a and e_b are uniformly ultimately bound.

Proof: refer to (Lee et al., 2016).

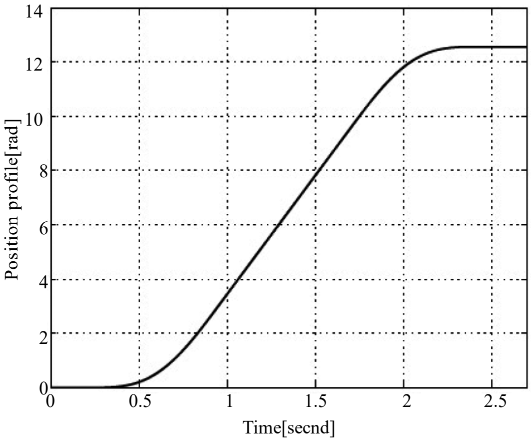

Fig. 5. Reference Position Profile. Adapted from (Kim et el., 2012b)

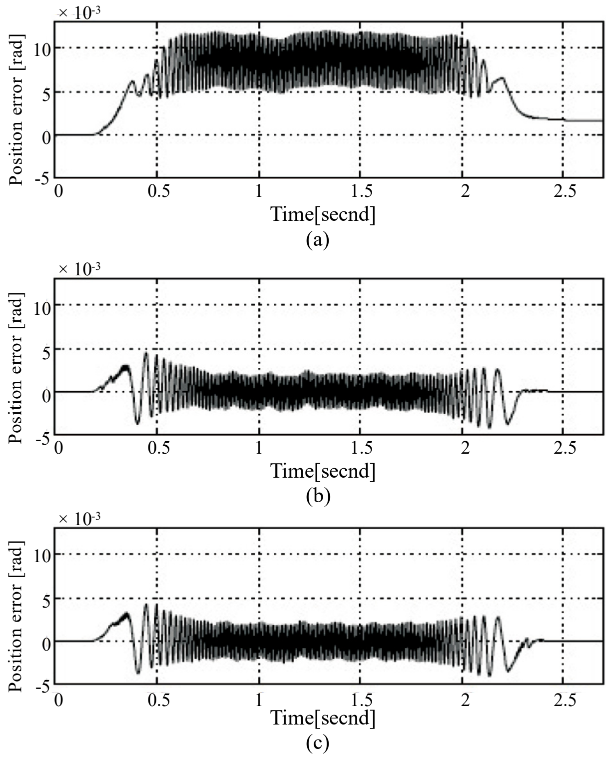

Fig. 6. Position tracking errors e_θ of Lyapunov-based control, FOC, and FWC. (a) Lyapunov-based control. (b) FOC. (c) FWC. Adapted from (Kim et el., 2012b)

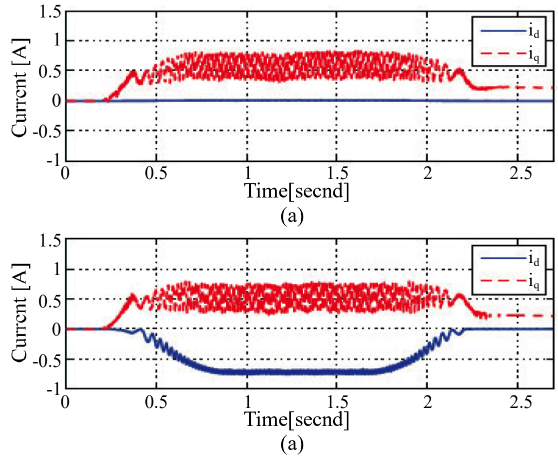

Fig. 7. i_d, i_a for FOC and FWC. Adapted from (Kim et el., 2012b)

IV. EXPERIMENTAL RESULTS

To validate the performance of the advanced control technique for PM stepper motors, we show the experiment result and discuss the characteristics of each advanced control. The detail experimental setup is well described in each related paper.

1. Microstepping With Nonlinear Torque Modulation Technique

Experiments were performed for three cases to verify the performance of the microstepping with nonlinear torque modulation and Lyapunov current controller with passivity state observer. Figure. 5 is the position profile. The PM stepper motor is commanded to move from 0[rad] to 4π [rad]. To conduct a comparative study, we perform the experiment with three cases.

1) Case 1 (Lyapunov-based Control): Lyapunov based currents control with passivity based currents estimation.
2) Case 2 (FOC): Field-oriented control (FOC) with nonlinear torque modulation.
3) Case 3 (FWC): Field-weakening control (FWC) with nonlinear torque modulation.

The position tracking error of the Lyapunov based control, FOC and FWC are plotted in Fig. 6. We observed that the steady-state response was enhanced with the nonlinear torque modulation technique with FOC and FWC compared to conventional microstepping. In the steady-state period, the inevitable position ripples are observed due to the resonance frequency of the system. Additionally, to validate the FOC and FWC techniques mentioned above, we compare the direct and quadrature currents. The results are illustrated in Fig. 7. Overall, the FOC/FWC methods (cases 2 and 3) showed a little more accurate transient behaviors than the Lyapunov based control method (case 1). The tracking errors of all three methods were nearly zero during the steady state period.

2. Linear Parameter Varying H_2 Control

To verify the performance of the LPV H2 controller, experiments were performed for three cases. For the detail experiment environment, please refer to (Lee et al., 2017).

1) Case 1 (LPV 1): Nonlinear H_2 controller with low state (tracking error) weighting and high-input weighting.
2) Case 2 (LPV 2): Nonlinear H_2 controller with high state (tracking error) weighting and low-input weighting.
3) Case 3 (FOC): Field-oriented control (FOC)

The position tracking error of both the FOC and the proposed

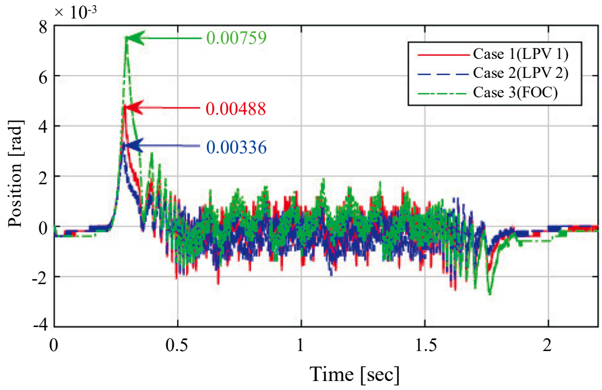

Fig.8. Position tracking errors of the three methods with seventh-order profile. Adapted from (Lee et al., 2017).

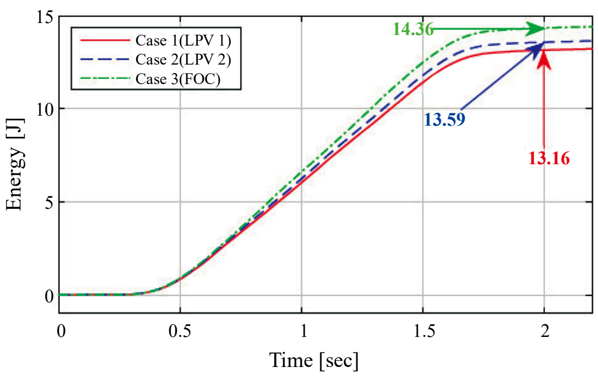

Fig. 9. Energy consumption of the three methods with seventh-order profile. Adapted from (Lee et al., 2017).

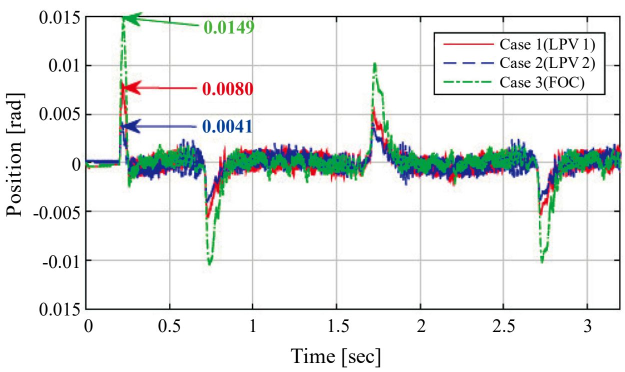

Fig. 10. Position tracking errors of the three methods with a sinusoidal profile. Adapted from (Lee et al., 2017)

method is plotted in Fig. 8. We observed that the transient response was degraded in the FOC method for current tracking. On the other hand, the proposed methods (cases 1 and 2) reduced the peak phenomenon in the transient because the control gain was scheduled. When the acceleration period started, the large peaks of tracking error were observed due to static friction regardless of the control method. In the steady-state period, the inevitable position ripples are observed due to the resonance and frequency of the system. In Fig. 9, we can analyze that there is a trade-off between maximizing tracking performance and minimizing energy consumption since the cost function takes both state errors and inputs into account. The position tracking of the three methods is compared in Fig. 10. At time 0.2 [s], a peak-phenomenon in the tracking errors was observed for all methods because of static friction. Overall, the proposed methods (cases 1 and 2) showed more accurate transient behaviors than the FOC (case 3).

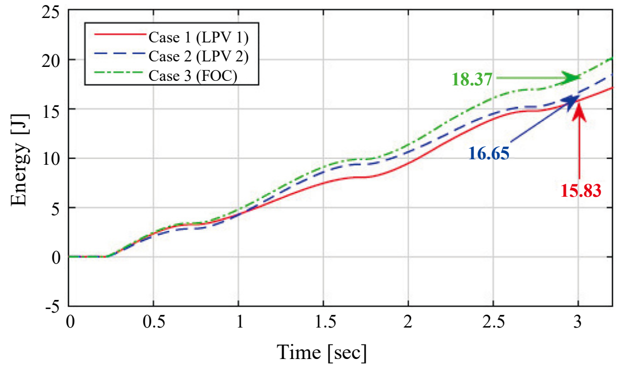

Fig. 11. Energy consumption of the three methods with a sinusoidal profile. Adapted from (Lee et al., 2017).

The tracking errors of the three methods were all nearly zero during the steady state period. Fig. 11. shows the energy consumption of the three methods. The proposed methods (cases 1 and 2) were more energy efficient than the FOC method. By incorporating greater input weighting, energy consumption was minimized in case 1.

3. Proximate In-Phase Current Estimator (PIpCE)

To evaluate the performance of the proposed method, we performed experiments for two cases as follows:

1) Case 1 (PIFF with conventional LPFs)
2) Case 2 (PIFF with proposed PIpCE).

The current tracking performances for both cases are illustrated in Fig. 12. In case 1, we see that the conventional LPFs result in a greater phase lag than the proposed PIpCE.

Both methods use the same control structure, which implement the feedforward and feedback control law. Furthermore, the proposed method provides better current tracking performance in terms of the magnitudes of currents. It is worth noticing that the tracking error was not removed, which is a result of the non-constant back-emf effect. The position tracking errors for both cases are shown in Fig. 13. The ripple width of case 1 was more than double that of case 2. A slightly improved transient response was also observed in the position tracking of case 2. During the steady-state period, a low-frequency ripple (approximately 2–3 Hz) resulted from a flexible coupler between the motor and the sensor. We performed another experiment to analyze the resonance mode under high-speed operation with a small step angle. In the constant velocity period, the maximum velocity changed from 11.8 to

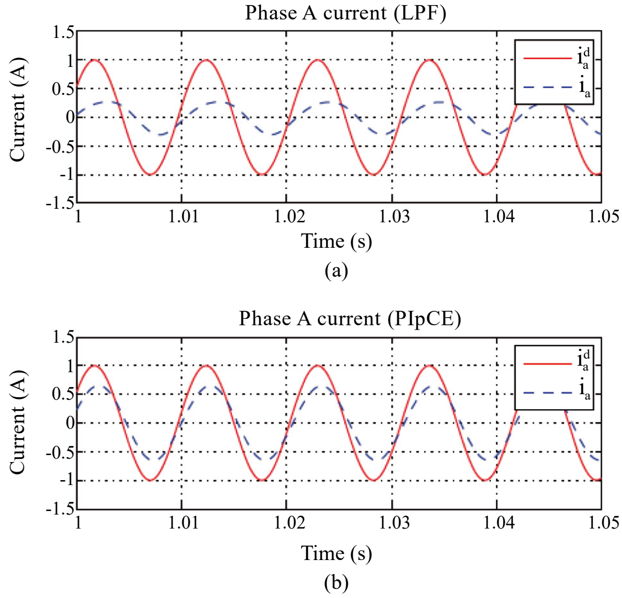

Fig. 12. Current tracking performance of both methods during acceleration velocity period (ω_{max} =11.8 rad/s), (a) Case 1: PIFF controller and conventional LPF, (b) Case 2: PIFF controller and proposed PIpCE. Adapted from (Lee et al., 2016).

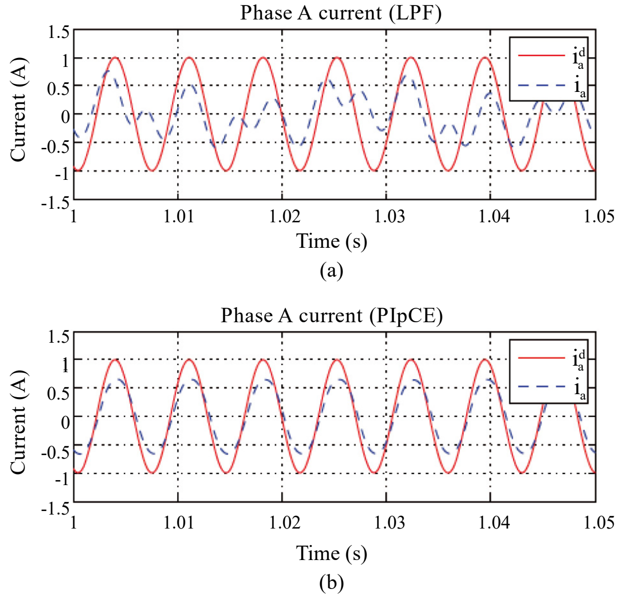

Fig. 14. Current tracking performance of both methods during acceleration velocity period (ω_{max} =17.7 rad/s), (a) Case 1: PIFF controller and conventional LPF, (b) Case 2: PIFF controller and proposed PIpCE. Adapted from (Lee et al., 2016)

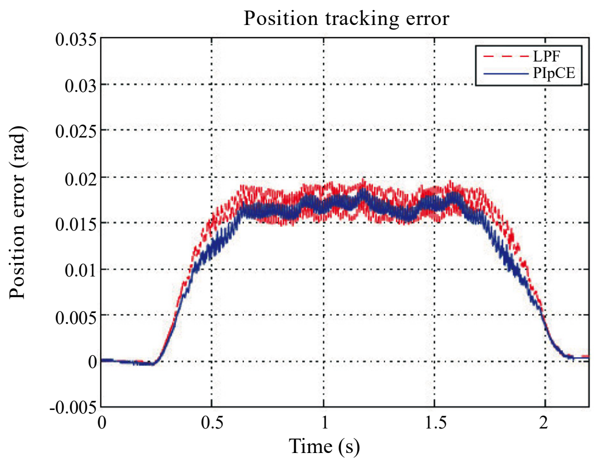

Fig. 13. Position tracking performance of both methods (ω_{max} =11.8 rad/s). Adapted from (Lee et al., 2016)

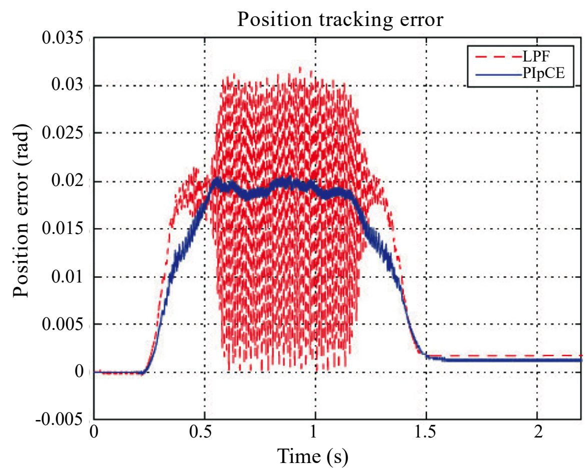

Fig. 15. Position tracking performance of both methods (ω_{max} =17.7 rad/s). Adapted from (Lee et al., 2016).

17.7 rad/s. The current tracking performances of steady-state periods are shown in Fig. 14. The proposed PIpCE still had better current tracking performances than the conventional method, which provided poor current tracking performance at high speed including noticeable harmonics. On the other hand, harmonics were not evident in the current tracking of the proposed PIpCE. The position tracking performance of both methods is illustrated in Fig. 15. The position tracking performance in case 2 was uniform even at high speed, whereas the results in case 1 were due to the system being on the verge of step-out.

V. CONCLUSION

In this survey paper, we explain the advanced control methods for Permanent Magnet (PM) stepper motors to enhance position tracking performances. First, we introduce basic principles of open-loop control of PM stepper motors including microstepping, then explain various advanced feedback control techniques based on Lyapunov stability. Second, we briefly summarize how a PM stepper motors can be modelled as a linear parameter varying system and its tracking performance optimization based on sense is made with the nonlinear torque modulation. Third, we show equivalency of field orient control and field weakening control to microstepping with nonlinear torque modulation. Then we introduce Proximate In-Phase Current Estimator (PIpCE) for PM stepper motors and a phase compensated Phase-Compensated Microstepping for PM stepper motors. Their performances are illustrated by experiments on PM stepper motors.

REFERENCES

Acarnley, P. P. (2002). Stepping motors: a guide to theory and practice (No. 63). Iet.

Bodson, M., J. N. Chiasson, R. T. Novotnak and, R. B. Rekowski (1993). High-performance nonlinear feedback control of a permanent magnet stepper motor. IEEE Transactions on Control Systems Technology 1(1), 5-14.

Bodson, M., J. S. Sato and S. R. Silver (2006). Spontaneous speed reversals in stepper motors. IEEE Transactions on Control Systems Technology 14(2), 369-373.

Chiasson, J. (2005). Modeling and high performance control of electric machines (Vol. 26). John Wiley & Sons.

Chung, C. C. and J. Hauser (1997). Nonlinear $H\infty$ control around periodic orbits. Systems & control letters 30(2-3), 127-137.

Clarkson, P. J. and P. P. Acarnley (1988). Closed-loop control of stepping motor systems. IEEE transactions on industry applications 24(4), 685-691.

Defoort, M., F. Nollet, T. Floquet and W. Perruquetti (2009). A third-order sliding-mode controller for a stepper motor. IEEE Transactions on Industrial Electronics 56(9), 3337-3346.

Khorrami, F., P. Krishnamurthy and H. Melkote (2003). Modeling and adaptive nonlinear control of electric motors. Springer Science & Business Media.

Kim, W. and C. C. Chung (2011). Novel position detection method for permanent magnet stepper motors using only current feedback. IEEE Transactions on Magnetics 47(10), 3590-3593.

Kim, W., C. Yang and C. C. Chung (2011). Design and implementation of simple field-oriented control for permanent magnet stepper motors without DQ transformation. IEEE Transactions on Magnetics 47(10), 4231-4234.

Kim, W., D. Shin and C. C. Chung (2012a). The Lyapunov-based controller with a passive nonlinear observer to improve position tracking performance of microstepping in permanent magnet stepper motors. Automatica 48(12), 3064-3074.

Kim, W., D. Shin and C. C. Chung (2012b). Microstepping with nonlinear torque modulation for permanent magnet stepper motors. IEEE Transactions on Control Systems Technology 21(5), 1971-1979.

Kim, W., D. Shin and C. C. Chung (2012c). Microstepping using a disturbance observer and a variable structure controller for permanent-magnet stepper motors. IEEE Transactions on Industrial Electronics 60(7), 2689-2699.

Kim, W., D. Shin, Y. Lee and C. C. Chung (2016). Simplified torque modulated microstepping for position control of permanent magnet stepper motors. Mechatronics 35, 162-172.

Kim, W. and C. C. Chung (2016). Robust output feedback control for unknown non-linear systems with external disturbance. IET Control Theory & Applications 10(2), 173-182.

Kim, W., Y. Lee, D. Shin and C. C. Chung (2017). Compensative Microstepping Based Position Control with Passive Nonlinear Adaptive Observer for Permanent Magnet Stepper Motors. Journal of Electrical Engineering & Technology 12(5), 1991-2000.

Kim, W., X. Chen, Y. Lee, C. C. Chung and M. Tomizuka (2018). Discrete-time nonlinear damping backstepping control with observers for rejection of low and high frequency disturbances. Mechanical Systems and Signal Processing 104, 436-448.

Kuo, B. C. (1979). Step motors and control systems (Vol. 2). SRL Pub Co.

Le, N. Q. and J. W. Jeon (2007, October). An open-loop stepper motor driver based on FPGA. In 2007 International Conference on Control, Automation and Systems (pp. 1322-1326). IEEE.

Le, Q. N. and J. W. Jeon (2009). Neural-network-based low-speed- damping controller for stepper motor with an FPGA. IEEE Transactions on Industrial Electronics 57(9), 3167-3180.

Le, Q. N. and J. W. Jeon (2009, February). An improved method of speed damping for a stepper motor with a smooth speed estimation. In 2008 IEEE International Conference on Robotics and Biomimetics (pp. 1438-1443). IEEE.

Le, K. M., H. Van Hoang and J. W. Jeon (2016). An advanced closed-loop control to improve the performance of hybrid stepper motors. IEEE Transactions on Power Electronics 32(9), 7244-7255.

Lee, Y., D. Shin, W. Kim and C. C. Chung (2016). Proximate in-phase current estimator to reduce torque ripple in permanent-magnet stepping motor. IEEE Transactions on Industrial Electronics 63(3), 1707-1716.

Lee, Y., D. Shin, W. Kim and C. C. Chung (2017). Nonlinear H_2 Control for a Nonlinear System With Bounded Vary ing Parameters: Application to PM Stepper Motors. IEEE/ASME Transactions on Mechatronics 22(3), 1349-1359.

Lee, Y., S. H. Lee and C. C. Chung (2018). LPV $H\infty$ Control with Disturbance Estimation for Permanent Magnet Synchronous Motors. IEEE Transactions on Industrial Electronics 65(1), 488-497.

Lee, Y., L. Sun, J. Moon, C. C. Chung and M. Tomizuka (2019). Reference Modulation for Performance Enhancement of Motion Control Systems With Nonlinear Parameter Variations. IEEE/ASME Transactions on Mechatronics 24(5), 2040-2051.

Marino, R., S. Peresada and P. Tomei (1995). Nonlinear adaptive control of permanent magnet step motors. Automatica 31(11), 1595-1604.

Nguyen, V. Q., N. V. P. Tran, H. N. Tran, K. M. Le and J. W. Jeon (2017). A closed-loop stepper motor drive based on EtherCAT. In IECON 2017-43rd Annual Conference of the IEEE Industrial Electronics Society (pp. 3361-3365). IEEE.

Nollet, F., T. Floquet and W. Perruquetti (2008). Observer-based second order sliding mode control laws for stepper motors. Control engineering practice 16(4), 429-443.

Rahman, M. F. and A. N. Poo (1988). An application oriented test procedure for designing microstepping step motor controllers. IEEE Transactions on Industrial Electronics 35(4), 542-546.

Seshagiri, S. (2009). Position control of permanent magnet stepper motors using conditional servo compensators. IET control theory & applications 3(9), 1196-1208.

Shin, D., W. Kim, Y. Lee and C. C. Chung (2013). Phase - compensated microstepping for permanent-magnet stepper motors. IEEE transactions on industrial electronics 60(12), 5773-5780.

Shin, D., W. Kim and C. C. Chung (2011, July). Position control of a permanent magnet stepper motor by MISO backstepping in semi-strict feedback form. In 2011 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM) (pp. 808-813). IEEE.

Shin, D., W. Kim, Y. Lee and C. C. Chung (2016). Enhanced nonlinear damping for a class of singularly perturbed interconnected nonlinear systems. Automatica 65, 36-42.

Shin, D., W. Kim, Y. Lee and C. C. Chung (2016). Nonlinear control with state-dependent reset integrator for a class of singularly perturbed interconnected nonlinear systems. IEEE Transactions on Control Systems Technology 25(4), 1193-1203.

Sira-Ramírez, H. (2000). A passivity plus flatness controller for the permanent magnet stepper motor. Asian Journal of Control 2(1), 1-9.

STMicroelectronics. (2000). DUAL FULL-BRIDGE DRIVER. L298 datasheet, Jan.

STMicroelectronics. (2001). STEPPER MOTOR CONTROLLERS. L297 datasheet, Dec.

Texas Instruments. (2009). TMS320F2803x Piccolo™ Microcontrollers. TMS320F2803x datasheet, Apr.

Texas Instruments. (2009). DRV84x2 Dual Full-Bridge PWM Motor Driver. DRV84x2 datasheet, Dec.

Tomei, P. and C. M. Verrelli (2011). Observer-based speed tracking control for sensorless permanent magnet synchronous motors with unknown load torque. IEEE Transactions on Automatic Control 56(6), 1484-1488.

Van der Schaft, A. J. (1991). On a state space approach to nonlinear $H\infty$

control. Systems & Control Letters 16(1), 1-8.

Van Der Schaft, A. J. (1992). L 2-gain analysis of nonlinear systems and nonlinear state feedback H∞ control. IEEE transactions on automatic control 37(6), 770-784.

Xu, J. X., T. H. Lee, Q. W. Jia and M. Wang (1998). On adaptive robust backstepping control schemes suitable for PM synchronous motors. International Journal of control 70(6), 893-920.

Yeadon, W. H. and A. W. Yeadon (2001). Handbook of small electric motors. McGraw− Hill Companies.

Zribi, M. and J. Chiasson (1991). Position control of a PM stepper motor by exact linearization. IEEE Transactions on automatic control 36(5), 620-625.