



ASSESSMENT OF DYNAMIC RESPONSE OF OFFSHORE JACKET PLATFORMS SUBJECTS TO RANDOM WAVES

Xiaoshuang Han

Marine Engineering College, Dalian Maritime University, Dalian, P.R.China

Weiliang Qiao

Marine Engineering College, Dalian Maritime University, Dalian, P.R.China.

Bo Zhou

Maritime Production research Pte. Ltd, Singapore., bozhou@dlut.edu.cn

Follow this and additional works at: <https://jmstt.ntou.edu.tw/journal>



Part of the [Hydraulic Engineering Commons](#)

Recommended Citation

Han, Xiaoshuang; Qiao, Weiliang; and Zhou, Bo (2020) "ASSESSMENT OF DYNAMIC RESPONSE OF OFFSHORE JACKET PLATFORMS SUBJECTS TO RANDOM WAVES," *Journal of Marine Science and Technology*. Vol. 28: Iss. 3, Article 2.

DOI: 10.6119/JMST.202006_28(3).0002

Available at: <https://jmstt.ntou.edu.tw/journal/vol28/iss3/2>

This Research Article is brought to you for free and open access by Journal of Marine Science and Technology. It has been accepted for inclusion in Journal of Marine Science and Technology by an authorized editor of Journal of Marine Science and Technology.

ASSESSMENT OF DYNAMIC RESPONSE OF OFFSHORE JACKET PLATFORMS SUBJECTS TO RANDOM WAVES

Acknowledgements

Author Contributions: Conceptualization, Xiaoshuang Han and Bo Zhou; methodology, Bo Zhou and Xiaoshuang Han; validation, Weiliang Qiao; writing—review and editing, Wie Min Gho. Funding: This research is funded by LiaoNing Revitalization Talents Program, grant number XLYC1807190; Liaoning Provincial Natural Science Foundation Guidance Project, grant number 20180550186; Open Project Funding of State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, grant number LP1906; and supported by “China Fundamental Research Funds for the Central Universities” (3132019368).

ASSESSMENT OF DYNAMIC RESPONSE OF OFFSHORE JACKET PLATFORMS SUBJECTS TO RANDOM WAVES

Xiaoshuang Han¹, Weiliang Qiao¹, Bo Zhou^{2*}, and Wie Min Gho³

Key words: pseudo-excitation method; Ritz method; random wave force; frequency domain response; Timoshenko beam theory

ABSTRACT

This paper presents a simplified approach to analyze the dynamic response of offshore jacket platform based on Timoshenko beam theory. In the study, the jacket platform is simulated as a non-uniform cantilever beam structure subjects to random waves. The assessment of dynamic performance is based on the power spectral densities, variances and spectral moments due to wave spectrum. A combination of Pseudo-excitation and Ritz method is proposed and adopted, such as the dynamic characteristics of the jacket platform can be solved more efficiently without a need to examine the modes of vibration. The proposed analytical method also provides a more straightforward solution to determine the dynamic response of offshore jacket platforms that meet the required level of engineering practice for preliminary design.

I. INTRODUCTION

For the offshore oil and gas industry, the template type jacket platform is the most widely used offshore structures in the shallow and intermediate waters. The strength assessment for this type of offshore platforms has been well established for accurate structural analysis. However, one of the most important considerations for engineering application and design is the dynamic response of offshore platforms subject to environmental forces (Moharrami,2014; Wei et al,2017). In the current study, an alternative solution of utilizing a

non-uniform cantilever beam concept to predict the dynamic response of offshore jacket platform under random waves is proposed.

For structural analysis, a finite element (FE) model of a jacket platform is generally created using beam and some plate elements to simulate its behavior in the marine environment, so as to study the complex problems associated with structure, water and soil interactions as well as to provide a useful insight of facilitating solutions for solving technical issues. The recent advancement of computational capacity also enables the dynamic performance of jacket platform being evaluated using a more complex three-dimensional (3D) FE model for better analytical solution. Having said, researchers and engineers still seeking for other simplified but accurate dynamic analysis method to produce results equivalent to the actual response of structure under environmental forces with minimum computational time.

From the literature review of FE simulation, some researchers proposed to simplify the FE model for simulating the behavior of offshore jacket platforms prior to detailed structural analysis. Sunder and Connor (1981), however, examined the sensitivity of offshore jacket platforms due to wave by utilizing the two simplified numerical models with assumed rigid foundation. Sunder and Connor further studied the effect of wave characteristics such as the wave period and height, inertia and drag coefficients, structural mass and hysteretic behavior of structural damping. A simplified model to examine the impact of current velocity, inertia and drag force components, random phase angles and wave cancellation, as well as fluid-structure interactions was conducted by Hahn (1992). Asgarian et al. (2004) deliberated a simplified method based on lumped mass model to determine the dynamic response of offshore jacket structures in response to environmental loading. Zhou et al. (2014) used a simplified method based on Bernoulli-Euler beam theory for modelling the offshore jacket platform.

Despite the Bernoulli-Euler beam theory had been regarded to be the most commonly used approach to simplify the FE model for simulating the offshore jacket platforms, it did not provide an upper bound solution for wave velocity. Further, the natural frequencies of structure could be overestimated.

Paper submitted 09/12/19; revised 12/25/19; accepted 03/17/20. Corresponding Author: Bo Zhou (E-mail address: bozhou@dlut.edu.cn).

¹*Marine Engineering College, Dalian Maritime University, Dalian, P.R.China.*

²*State Key Laboratory of Structural Analysis for Industrial Equipment, School of Naval Architecture Engineering, Dalian University of Technology, Dalian, P.R.China*

³*Maritime Production research Pte. Ltd, Singapore.*

Nevertheless, the simulation of beam structures based on Bernoulli-Euler beam theory provided a better result for beams of slender section than that of short section. However, the beam theory was further enhanced by Timoshenko (1921) to include the influential factors of shear and rotary inertia. As a result, the beam theory proposed by Timoshenko not only provided the structure with an upper bound solution for wave velocities, the natural frequencies and mode shapes could also be determined, which were found to be in good agreement with that of the two-dimensional theory (Timoshenko, 1921; Fung, 1965; Graff, 1973; Labuschagne et al., 2009; Balduzzi et al., 2016; Trahair and Ansurian, 2016; Bertolini et al., 2019). Thus, it could be commented that the Timoshenko beam theory was more appropriate for analyzing the transient response of structures, especially so in situation that involved high frequency vibrations and deep beam sections.

Several researchers carried out study on the vibratory response of structures based on Timoshenko beam theory, and with Mindlin-Goodman method of time-dependent boundary conditions to determine the shifting functions (Herrmann, 1955; Berry and Naghdi, 1956). Based on non-uniform Timoshenko beam and general time-dependent boundary conditions, Lee and Lin (1998) examined the orthogonality cases for the Eigen functions of elastic boundary non-uniform beam for the dynamic analysis of structures. Lin and Lee (2002) further investigated the vibration effect of a pre-twisted non-uniform Timoshenko beam with time-dependent elastic boundary conditions. On the other hand, Kim (2016) proposed a procedure of analytical solution for solving the dynamic response of Timoshenko beam excited by support motions, and with the introduction of Eigen function expansion and Mindlin-Goodman method. Subsequently, Pratiher (2012) studied in detail the use of perturbation method for vibration control of cantilever beam with tip mass under transverse base excitation. A closed-form solution derived by Elishakoff and Livshits (1984a, 1984b) was based on both the Bernoulli-Euler and Bresse-Timoshenko beam under stationary random excitation. However, their study eventually led to the development of random vibration of Bresse-Timoshenko beam under concentrated point load and subjected to space wise white noise (Elishakoff and Lubliner, 1985; Elishakoff and Lubliner, 1988). Despite of several research based on Timoshenko beam theory, the research on the effect of axial load on the dynamic performance of structures, however, drew much attention in view of its wide application in the engineering industries (Shaker, 1975; Bokaiaint, 1990; Lee, 1995; Yesilce and Demirdag, 2008; Murin et al., 2013; Zhang et al., 2018; Zhang et al., 2019). One of the examples was the work of Horr and Safi (2003) that was based on the exact Timoshenko pipe elements to determine the dynamic response of offshore platform in the frequency domain. Tan et al. (2019) established a nonlinear Timoshenko model of the coupled vibration of a pipe conveying fluid to distinguish it from the Eu-

ler-Bernoulli coupled model and the Timoshenko model of the transverse vibration in terms of application scope and accuracy. Nguyen and Oterkus (2019) developed a novel bond based Peridynamic model for three-dimensional complex beam structures with 6 degrees of freedom based on Timoshenko beam theory to predict damage in offshore structures.

Theoretically the more comprehensive and more inclusive are the calculation models, the more accurate the computation results are to be expected. However, in many practical engineering, such as fixed offshore structures, the FE models of jacket structures are usually very complex. It is unpractical to acquire all the detailed information for setting up the finite element information in the preliminary design stage. Therefore, despite the reported revised models and computational methods, the current engineering practice still prefers simplified structural calculation model in which a reasonable degree of precisions could be achieved, especially in the preliminary design stage.

This paper presented a simplified method based on Timoshenko beam theory to assess the dynamic response of offshore jacket platforms under random wave loads. The jacket platform was simplified as an axially loaded non-uniform cantilever beam for structural analysis. In the study, the pseudo-excitation method (PEM) was combined with Ritz method to determine the dynamic behaviour of jacket platform in response to random wave loads. The assessment was based on the power spectral densities, variances of response and higher spectral moments, but without generating the normal mode shapes of vibration. An example of analytical procedure and the accuracy of the proposed approach was then verified against the finite element (FE) method for discussion.

In the paper, the proposed method to determine the Pseudo wave force and the solution of partial differential equation are presented in Sections 2 and 3 respectively. A comparison of analytical results on the dynamic behaviour of offshore jacket platform between the proposed simplified approach and the FE method is discussed in Section 4. Finally, a conclusion of the study based on the analysis results is summarised in Section 5.

II. PSEUDO WAVE FORCE BY PSEUDO-EXCITATION METHOD (PEM)

The Pierson-Moskowitz (PM) wave spectrum were widely referenced for the design and analysis of offshore platforms in open sea environment. In the PM spectrum, the wave force spectrum at z height could be presented as follows (Yu and Liu, 2011).

$$S_p(\omega) = \left\{ \begin{array}{l} \left[\frac{1}{\sqrt{2\pi}} K_D H_{1/3} \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)} \right]^2 \\ + [K_M \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)}]^2 \end{array} \right\} S\eta(\omega) \quad (1)$$

where $K_D = \rho_w C_D D / 2$, and $K_M = \rho_w C_M \pi D^2 / 4$. Here, ρ_w is the density of water, D is the diameter of cylindrical cylinder, C_D is the drag coefficient, C_M is the inertia coefficient, $k = 2\pi / L$, $\omega = 2\pi / T$, H is the wave height, L is the wave length, T is the wave period, and $H_{1/3}$ is the significant height. The pseudo wave load was determined in accordance to pseudo-excitation method (PEM) (Lin and Zhang, 2004).

$$\begin{aligned} \tilde{p}(z,t) &= \sqrt{S_p(\omega)} e^{i\omega t} \\ &= \sqrt{\left(\frac{1}{\sqrt{2\pi}} K_D H_{1/3} \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)}\right)^2 + \left(K_M \omega^2 \frac{\cosh(kz)}{\sinh(kd)}\right)^2} \sqrt{S_\eta(\omega)} e^{i\omega t} \end{aligned} \quad (2)$$

From the above, several derivatives of $\tilde{p}(z,t)$ could be derived as shown.

$$\frac{\partial \tilde{p}}{\partial t} = i\omega \sqrt{\left(\frac{1}{\sqrt{2\pi}} K_D H_{1/3} \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)}\right)^2 + \left(K_M \omega^2 \frac{\cosh(kz)}{\sinh(kd)}\right)^2} \sqrt{S_\eta(\omega)} e^{i\omega t} \quad (3)$$

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = -\omega^2 \sqrt{\left(\frac{1}{\sqrt{2\pi}} K_D H_{1/3} \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)}\right)^2 + \left(K_M \omega^2 \frac{\cosh(kz)}{\sinh(kd)}\right)^2} \sqrt{S_\eta(\omega)} e^{i\omega t} \quad (4)$$

$$\frac{\partial \tilde{p}}{\partial z} = \frac{k\omega^2 \sinh(kz) \left(\frac{1}{\pi} K_D^2 H_{1/3}^2 \frac{\cosh^2(kz)}{\sinh^2(kd)} + K_M^2 \right)}{\sqrt{\frac{1}{2\pi} K_D^2 H_{1/3}^2 \cosh^2(kz) + K_M^2 \sinh^2(kd)}} \sqrt{S_\eta(\omega)} e^{i\omega t} \quad (5)$$

$$\begin{aligned} \frac{\partial^2 \tilde{p}}{\partial z^2} &= \left[\frac{k^2 \omega^2 \cosh(kz) \left(\frac{1}{\pi} K_D^2 H_{1/3}^2 \frac{\cosh^2(kz)}{\sinh^2(kd)} + K_M^2 \right) + \frac{2}{\pi} k^2 \omega^2 K_D^2 H_{1/3}^2 \frac{\cosh(kz) \sinh^2(kz)}{\sinh^2(kd)}}{\sqrt{\frac{1}{2\pi} K_D^2 H_{1/3}^2 \cosh^2(kz) + K_M^2 \sinh^2(kd)}} \right. \\ &\quad \left. - \frac{\frac{1}{2\pi} k^2 \omega^2 K_D^2 H_{1/3}^2 \sinh^2(kz) \left(\frac{1}{\pi} K_D^2 H_{1/3}^2 \frac{\cosh^2(kz)}{\sinh^2(kd)} + K_M^2 \right) \cosh(kz)}{\left(\frac{1}{2\pi} K_D^2 H_{1/3}^2 \cosh^2(kz) + K_M^2 \sinh^2(kd) \right)^{3/2}} \right] \sqrt{S_\eta(\omega)} e^{i\omega t} \end{aligned} \quad (6)$$

III. SOLUTION OF PROPOSED EQUATION FOR ANALYSIS

For an axially loaded cantilever Timoshenko beam, the partial differential equation that governed the beam bending motion $w(z,t)$ could be expressed as follows.

$$\begin{aligned} EI(z) \frac{\partial^4 w}{\partial z^4} + \frac{\partial^2}{\partial z^2} EI(z) \frac{\partial^2 w}{\partial z^2} - \left[p(z,t) - \rho A \frac{\partial^2 w}{\partial t^2} - c \frac{\partial w}{\partial t} - N \frac{\partial^2 w}{\partial z^2} \right] + \frac{EI}{k'GA} \frac{\partial^2}{\partial z^2} \left[p(z,t) - \rho A \frac{\partial^2 w}{\partial t^2} - c \frac{\partial w}{\partial t} - N \frac{\partial^2 w}{\partial z^2} \right] \\ - \frac{\rho I}{k'GA} \frac{\partial^2}{\partial t^2} \left[p(z,t) - \rho A \frac{\partial^2 w}{\partial t^2} - c \frac{\partial w}{\partial t} - N \frac{\partial^2 w}{\partial z^2} \right] - \rho I \frac{\partial^4 w}{\partial z^2 \partial t^2} = 0 \end{aligned} \quad (7)$$

Where $EI(z)$ and $\rho A(z)$ were the flexural rigidity of cantilever beam and its mass per unit height, varied with position z . $p(z,t)$ was the wave force varying with position and time that caused the beam motion that could be described by transverse dis-

placement $w(z,t)$. The boundary condition at each end of the cantilever beam was set as follows.

$$(w)_{z=0} = 0, \quad \left(\frac{dw}{dz}\right)_{z=0} = 0, \quad \left(EI \frac{d^2w}{dz^2}\right)_{z=L} = 0, \quad \frac{d}{dz} \left(EI \frac{d^2w}{dz^2}\right)_{z=L} = 0 \quad (8)$$

By rearranging equation Eq. (7),

$$\begin{aligned} EI \frac{\partial^4 w}{\partial z^4} + \left[\rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + N \frac{\partial^2 w}{\partial z^2} \right] - \frac{EI}{k'GA} \frac{\partial^2}{\partial z^2} \left[\rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + N \frac{\partial^2 w}{\partial z^2} \right] + \frac{\rho I}{k'GA} \frac{\partial^2}{\partial t^2} \left[\rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + N \frac{\partial^2 w}{\partial z^2} \right] \\ - \rho I \frac{\partial^4 w}{\partial z^2 \partial t^2} = p(z,t) - \frac{EI}{k'GA} \frac{\partial^2 p(z,t)}{\partial z^2} + \frac{\rho I}{k'GA} \frac{\partial^2 p(z,t)}{\partial t^2} \end{aligned} \quad (9)$$

The initial condition of the beam under static could be set at,

$$w(z,0) = 0, \quad \dot{w}(z,0) = 0 \quad (10)$$

Substituting equations Eqs. (2), (4) and (6) into Eq. (9),

$$\begin{aligned} EI \frac{\partial^4 w}{\partial z^4} + \left[\rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + N \frac{\partial^2 w}{\partial z^2} \right] - \frac{EI}{k'GA} \frac{\partial^2}{\partial z^2} \left[\rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + N \frac{\partial^2 w}{\partial z^2} \right] + \frac{\rho I}{k'GA} \frac{\partial^2}{\partial t^2} \left[\rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + N \frac{\partial^2 w}{\partial z^2} \right] \\ - \rho I \frac{\partial^4 w}{\partial z^2 \partial t^2} = \tilde{p}(z,t) - \frac{EI}{k'GA} \frac{\partial^2 \tilde{p}(z,t)}{\partial z^2} + \frac{\rho I}{k'GA} \frac{\partial^2 \tilde{p}(z,t)}{\partial t^2} \end{aligned} \quad (11)$$

In accordance to the Ritz method, the displacements of the beam structure could be taken as a linear combination of several shape vectors $\varphi_j(z)$ as follows.

$$\tilde{w}(z,t) = \sum_{j=1}^q \tilde{y}_j(t) \varphi_j(z) = [\varnothing] \{\tilde{y}\} \quad (12)$$

By substituting Eq. (12) into Eq. (11), and multiplying $[\varnothing]^T$ on the left side of the equation,

$$\begin{aligned} EI [\varnothing]^T [\varnothing]^{IV} \{\tilde{y}\} + \rho A [\varnothing]^T [\varnothing] \{\ddot{\tilde{y}}\} + c [\varnothing]^T [\varnothing] \{\dot{\tilde{y}}\} \\ + N [\varnothing]^T [\varnothing]'' \{\tilde{y}\} - \frac{EI}{k'GA} \left(\rho A [\varnothing]^T [\varnothing]'' \{\tilde{y}\} + c [\varnothing]^T [\varnothing]'' \{\dot{\tilde{y}}\} + N [\varnothing]^T [\varnothing]^{IV} \{\tilde{y}\} \right) \\ + \frac{\rho I}{k'GA} \left(\rho A [\varnothing]^T [\varnothing] \{\tilde{y}\}^{IV} + c [\varnothing]^T [\varnothing] \{\tilde{y}\}^{III} + N [\varnothing]^T [\varnothing]'' \{\ddot{\tilde{y}}\} \right) - \rho I [\varnothing]^T [\varnothing]'' \{\ddot{\tilde{y}}\} \\ = [\varnothing]^T p(z,t) - \frac{EI}{k'GA} [\varnothing]^T \frac{\partial^2 p(z,t)}{\partial z^2} + \frac{\rho I}{k'GA} [\varnothing]^T \frac{\partial^2 p(z,t)}{\partial t^2} \end{aligned} \quad (13)$$

Integrating the position z of the beam from zero to L yielded the following formulation,

$$\begin{aligned} \int_0^L \frac{\rho^2 I}{k'G} [\varnothing]^T [\varnothing] \{\tilde{y}\}^{IV} dz + \int_0^L \frac{\rho I c}{k'GA} [\varnothing]^T [\varnothing] \{\tilde{y}\}^{III} dz + \int_0^L \rho A [\varnothing]^T [\varnothing] \{\dot{\tilde{y}}\} dz - \int_0^L \frac{EI \rho}{k'G} [\varnothing]^T [\varnothing]'' \{\ddot{\tilde{y}}\} dz + \int_0^L \frac{\rho I N}{k'GA} [\varnothing]^T [\varnothing]'' \{\ddot{\tilde{y}}\} dz \\ - \int_0^L \rho I [\varnothing]^T [\varnothing]'' \{\ddot{\tilde{y}}\} dz + \int_0^L c [\varnothing]^T [\varnothing] \{\dot{\tilde{y}}\} dz - \int_0^L \frac{EI c}{k'GA} [\varnothing]^T [\varnothing]'' \{\dot{\tilde{y}}\} dz + \int_0^L EI [\varnothing]^T [\varnothing]^{IV} \{\tilde{y}\} dz + \int_0^L N [\varnothing]^T [\varnothing]'' \{\tilde{y}\} dz \\ - \int_0^L \frac{EI N}{k'GA} [\varnothing]^T [\varnothing]^{IV} \{\tilde{y}\} dz = \int_0^L [\varnothing]^T p(z,t) dz - \int_0^L \frac{EI}{k'GA} [\varnothing]^T \frac{\partial^2 p(z,t)}{\partial z^2} dz + \int_0^L \frac{\rho I}{k'GA} [\varnothing]^T \frac{\partial^2 p(z,t)}{\partial t^2} dz \end{aligned} \quad (14)$$

By rearranging the equation Eq. (14),

$$[R]\{\ddot{y}\}^{IV} + [S]\{\ddot{y}\}^{III} + [M_1]\{\ddot{y}\} + [M_2]\{\ddot{y}\} + [M_3]\{\ddot{y}\} + [M_4]\{\ddot{y}\} + [C_1]\{\dot{y}\} + [C_2]\{\dot{y}\} + [K_1]\{y\} + [K_2]\{y\} + [K_3]\{y\} = ([P_1] + [P_2] + [P_3])e^{i\omega t} \tag{15}$$

where:

$$[R] = \int_0^L \frac{\rho^2 I}{k'G} [\emptyset]^T [\emptyset] dz \tag{16}$$

$$[C_1] = \int_0^L c [\emptyset]^T [\emptyset] dz \tag{22}$$

$$[S] = \int_0^L \frac{\rho I c}{k'GA} [\emptyset]^T [\emptyset] dz \tag{17}$$

$$[C_2] = - \int_0^L \frac{EIc}{k'GA} [\emptyset]^T [\emptyset]'' dz \tag{23}$$

$$[M_1] = \int_0^L \rho A [\emptyset]^T [\emptyset] dz \tag{18}$$

$$[K_1] = \int_0^L EI [\emptyset]^T [\emptyset]^{IV} dz \tag{24}$$

$$[M_2] = - \int_0^L \frac{EI\rho}{k'G} [\emptyset]^T [\emptyset]'' dz \tag{19}$$

$$[K_2] = \int_0^L N [\emptyset]^T [\emptyset]'' dz \tag{25}$$

$$[M_3] = \int_0^L \frac{\rho IN}{k'GA} [\emptyset]^T [\emptyset]'' dz \tag{20}$$

$$[K_3] = - \int_0^L \frac{EIN}{k'GA} [\emptyset]^T [\emptyset]^{IV} dz \tag{26}$$

$$[M_4] = - \int_0^L \rho I [\emptyset]^T [\emptyset]'' dz \tag{21}$$

$$[P_1] = \int_0^L [\emptyset]^T p(z,t) dz = \sqrt{S_\eta(\omega)} \int_0^L [\emptyset]^T \sqrt{\left(\frac{1}{\sqrt{2\pi}} K_D H_{\frac{1}{3}} \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)}\right)^2 + \left(K_M \omega^2 \frac{\cosh(kz)}{\sinh(kd)}\right)^2} dz \tag{27}$$

$$[P_2] = - \int_0^L \frac{EI}{k'GA} [\emptyset]^T \frac{\partial^2 p(z,t)}{\partial z^2} dz = - \frac{EI}{k'GA} \sqrt{S_\eta(\omega)} \int_0^L [\emptyset]^T \left(\frac{k^2 \omega^2 \cosh(kz) \left(\frac{1}{\pi} K_D^2 H_{\frac{1}{3}}^2 \frac{\cosh^2(kz)}{\sinh^2(kd)} + K_M^2 \right) + \frac{2}{\pi} k^2 \omega^2 K_D^2 H_{\frac{1}{3}}^2 \frac{\cosh(kz) \sinh^2(kz)}{\sinh^2(kd)}}{\sqrt{\frac{1}{2\pi} K_D^2 H_{\frac{1}{3}}^2 \cosh^2(kz) + K_M^2 \sinh^2(kd)}} - \frac{\frac{1}{2\pi} k^2 \omega^2 K_D^2 H_{\frac{1}{3}}^2 \sinh^2(kz) \left(\frac{1}{\pi} K_D^2 H_{\frac{1}{3}}^2 \frac{\cosh^2(kz)}{\sinh^2(kd)} + K_M^2 \right) \cosh(kz)}{\left(\frac{1}{2\pi} K_D^2 H_{\frac{1}{3}}^2 \cosh^2(kz) + K_M^2 \sinh^2(kd) \right)^{3/2}} \right) dz \tag{28}$$

$$[P_3] = \int_0^L \frac{\rho I}{k'GA} [\emptyset]^T \frac{\partial^2 p(z,t)}{\partial t^2} dz = -\omega^2 \frac{\rho I}{k'GA} \sqrt{S_\eta(\omega)} \int_0^L [\emptyset]^T \sqrt{\left(\frac{1}{\sqrt{2\pi}} K_D H_{\frac{1}{3}} \omega^2 \frac{\cosh^2(kz)}{\sinh^2(kd)}\right)^2 + \left(K_M \omega^2 \frac{\cosh(kz)}{\sinh(kd)}\right)^2} dz \tag{29}$$

Table 1. Expressions of [C], [E] and [D] under three types of damping

Types of Damping	[C]	[E]	[D]
Rayleigh damping	$\alpha [M] + \beta [K]$	$[K] - \omega^2 [M]$	$-\omega (\alpha [M] + \beta [K])$
hysteretic damping	$i\varepsilon [K]$	$[K] - \omega^2 [M]$	$-\varepsilon [K]$
Non-proportional damping	[C]	$[K] - \omega^2 [M]$	$-\omega [C]$

For Eqs (19), (20), (21), (23), (24), (25) and (26). By integrating the equations by parts twice, and considering the boundary conditions of cantilever beam in equation Eq. (8) produced,

$$[M_2] = -\int_0^L \frac{EI\rho}{k'G} [\varnothing]^T [\varnothing]'' dz = \frac{EI\rho}{k'G} \int_0^L [\varnothing]^T [\varnothing]' dz \quad (30)$$

$$[M_3] = \int_0^L \frac{\rho IN}{k'GA} [\varnothing]^T [\varnothing]'' dz = -\frac{\rho IN}{k'GA} \int_0^L [\varnothing]^T [\varnothing]' dz \quad (31)$$

$$[M_4] = -\int_0^L \rho I [\varnothing]^T [\varnothing]'' dz = \rho I \int_0^L [\varnothing]^T [\varnothing]' dz \quad (32)$$

$$[C_2] = -\int_0^L \frac{EIc}{k'GA} [\varnothing]^T [\varnothing]'' dz = \frac{EIc}{k'GA} \int_0^L [\varnothing]^T [\varnothing]' dz \quad (33)$$

$$[K_1] = \int_0^L EI [\varnothing]^T [\varnothing]^{IV} dz = EI \int_0^L [\varnothing]^T [\varnothing]'' dz \quad (34)$$

$$[K_2] = \int_0^L N [\varnothing]^T [\varnothing]'' dz = -N \int_0^L [\varnothing]^T [\varnothing]' dz \quad (35)$$

$$[K_3] = -\int_0^L \frac{EIN}{k'GA} [\varnothing]^T [\varnothing]^{IV} dz = -\frac{EIN}{k'GA} \int_0^L [\varnothing]^T [\varnothing]'' dz \quad (36)$$

Substituting equations from (30) to (36) into Eq. (15). Considering the following,

$$[M] = [M_1] + [M_2] + [M_3] + [M_4] \quad (37)$$

$$[C] = [C_1] + [C_2] \quad (38)$$

$$[K] = [K_1] + [K_2] + [K_3] \quad (39)$$

$$[P] = [P_1] + [P_2] + [P_3] \quad (40)$$

Then by substituting equations from (37) to (40) into Eq. (15) yielded

$$[R]\{\tilde{y}\}^{IV} + [S]\{\tilde{y}\}^{III} + [M]\{\ddot{\tilde{y}}\} + [C]\{\dot{\tilde{y}}\} + [K]\{\tilde{y}\} = [P]e^{i\omega t} \quad (41)$$

From the above, the right side was a harmonic excitation. The solution could be expressed in the form of,

$$\{\tilde{y}(t)\} = (\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \quad (42)$$

Thus,

$$\{\dot{\tilde{y}}(t)\} = i\omega(\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \quad (43)$$

$$\{\ddot{\tilde{y}}(t)\} = -\omega^2(\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \quad (44)$$

$$\{\tilde{y}\}^{III} = -i\omega^3(\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \quad (45)$$

$$\{\tilde{y}\}^{IV} = \omega^4(\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \quad (46)$$

By substituting equations from (43) to (46) into Eq. (41) produced the following.

$$\begin{aligned} & \omega^4 [R](\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} - i\omega^3 [S](\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \\ & - \omega^2 [M](\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} + i\omega [C](\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} \\ & + [K](\{\tilde{y}_r\} + i\{\tilde{y}_i\})e^{i\omega t} = [P]e^{i\omega t} \end{aligned} \quad (47)$$

Where r and i denoted the real and imaginary part respectively. By eliminating $e^{i\omega t}$ at both sides of the equation, and comparing the real and imaginary part,

$$[E]\{\tilde{y}_r\} + [D]\{\tilde{y}_i\} = \{P\} \quad (48)$$

$$-[D]\{\tilde{y}_r\} + [E]\{\tilde{y}_i\} = \{0\} \quad (49)$$

Where,

$$[E] = \omega^4 [R] - \omega^2 [M] + [K] \quad (50)$$

$$[D] = \omega^3 [S] - \omega [C] \quad (51)$$

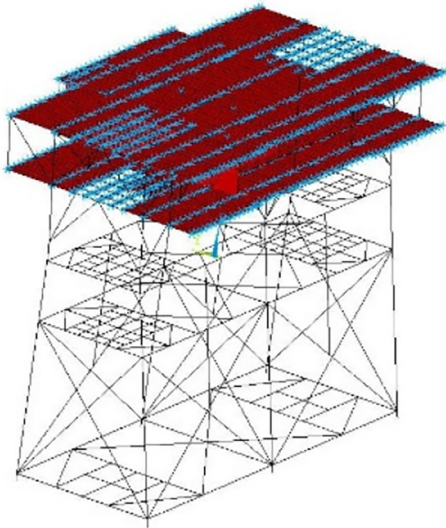


Fig. 1 FE model of offshore jacket platform

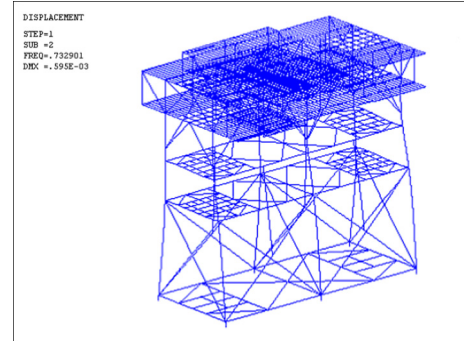


Fig. 2 Fundamental natural frequency of offshore jacket platform

The expressions of [C], [E] and [D] under three (3) types of damping are presented in Table 1.

The following two (2) expressions could be obtained by solving $\{\tilde{y}_r\}$ and $\{\tilde{y}_i\}$.

$$\{\tilde{y}_r\} = ([D]^{-1}[E] + [E]^{-1}[D])^{-1} [D]^{-1} \{P\} \quad (52)$$

$$\{\tilde{y}_i\} = ([D]^{-1}[E] + [E]^{-1}[D])^{-1} [E]^{-1} \{P\} \quad (53)$$

The displacement $\{\tilde{u}\}$ could be determined from the equations Eqs (42) and (12). The PSD of displacement and bending moment could therefore be obtained as,

$$S_{uu}(z, \omega) = |\tilde{u}(z, t)|^2, S_{MM}(z, \omega) = |\tilde{M}(z, t)|^2 \quad (54)$$

Considering,

$$\{\tilde{y}_r\}_{n \times 1} = \begin{bmatrix} y_{r1} \\ y_{r2} \\ M \\ y_m \end{bmatrix}_{n \times 1}, \{\tilde{y}_i\}_{n \times 1} = \begin{bmatrix} y_{i1} \\ y_{i2} \\ M \\ y_{in} \end{bmatrix}_{n \times 1}, [\Phi]_{1 \times n} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ M \\ \Phi_n \end{bmatrix}_{1 \times n}^T$$

Then,

$$\begin{aligned} \{\tilde{u}\} &= [\Phi] \{\tilde{y}\} = [\Phi_1 \ \Phi_2 \ L \ \Phi_n]_{1 \times n} \times \begin{bmatrix} y_{r1} + iy_{i1} \\ y_{r2} + iy_{i2} \\ M \\ y_m + iy_{in} \end{bmatrix}_{n \times 1} \\ &= \Phi_1 (y_{r1} + iy_{i1}) + \Phi_2 (y_{r2} + iy_{i2}) + L + \Phi_n (y_m + iy_{in}) \\ &= (\Phi_1 y_{r1} + \Phi_2 y_{r2} + L + \Phi_n y_m) + i(\Phi_1 y_{i1} + \Phi_2 y_{i2} + L + \Phi_n y_{in}) \end{aligned} \quad (55)$$

As,

$$M = EI \frac{d^2 u}{dz^2}$$

$$\begin{aligned} S_{MM}(z, \omega) &= |M(z, t)|^2 \\ &= E^2 I^2 (z) \left[(\Phi_1'' y_{r1} + \Phi_2'' y_{r2} + L + \Phi_n'' y_m)^2 \right. \\ &\quad \left. + (\Phi_1'' y_{i1} + \Phi_2'' y_{i2} + L + \Phi_n'' y_{in})^2 \right] \end{aligned} \quad (57)$$

After the PSD of response $S_r(\omega)$ was determined, its spectral moments could be computed directly. The zeroth and the second moments were the most useful terms.

$$\lambda_{0,r} = \sigma_r^2 = 2 \int_0^{+\infty} S_r(\omega) d\omega \quad (58)$$

$$\lambda_{2,r} = 2 \int_0^{+\infty} \omega^2 S_r(\omega) d\omega \quad (59)$$

IV. EXAMPLE OF ANALYTICAL PROCEDURE

In this section, a 3D model of 6-leg offshore jacket platform subjected to a wave spectrum was considered as a case study. The geometrical configuration of the jacket platform can be seen in Fig. 1. The height of the platform was 68m. It was located at 23 m water depth. The overall horizontal dimension of the platform was 48m by 20m at the mud level, and 48m by 14m at the top elevation. The foundation of the jacket substructure was assumed fixed with a total of 6 groups of grouted piles through the jacket legs. The inclination of the jacket legs was 10 to 1. The jacket leg was tubular circular hollow section of diameter $\Phi 1333\text{mm}$ and wall thickness 19mm. The geometrical properties of the 12 tubular members

Table 2. Element type and element number

Element Type	Area Element	Beam Element	Mass Element	Node
Element No.	35152	32806	2308	47420

Table 3 Comparison of natural frequencies

Mode	3-D FEM Model (Hz)	Timoshenko beam model (Hz)
1	0.73	0.75
2	2.45	2.51

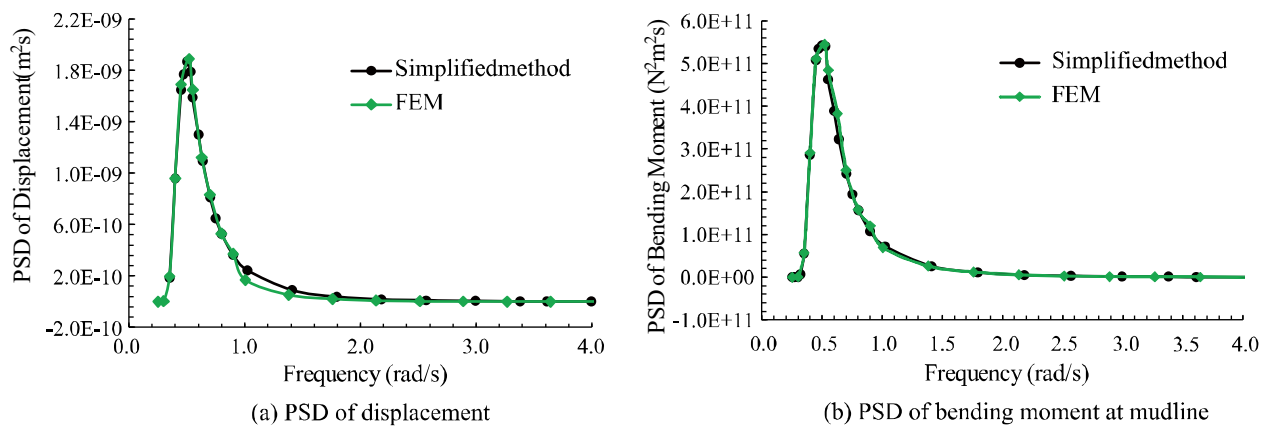


Fig. 3. PSD of offshore jacket platform FE model and simplified beam model.

at the bottom and the top were $\Phi 762 \times 16\text{mm}$ and $\Phi 610 \times 13\text{mm}$, respectively. The mass of the three decks of the superstructure, from the bottom to the top level, were 93 tons, 267 tons and 1,231 tons, respectively.

The offshore jacket platform was modeled in accordance to the dimensional and geometrical properties as stated in the AS-BUILT drawings. The entire platform was modeled as a plate-girder steel composite structure, while the deck and other platform components as shell elements. The space frames of the jacket structure were modelled as beam elements. The pile group effect between the adjacent legs of the jacket foundation was ignored, taking into consideration that the spacing between the jacket legs is 5 to 10 times the leg diameter. However, the effect of additional mass of the joints of beam elements underwater was included. The end of the piles was simulated as fixed at the seabed level. A FE package ANSYS with capability to perform both static and dynamic analyses of offshore jacket platforms under wave loading was utilized for structural analysis. The element type and element number were tabulated in Table 2.

The 3D offshore jacket platform model was simplified as a cantilever beam model based on Timoshenko beam theory. The inertia moment and the steel density were taken as $I(z) = 55 - z$ (see ref. [6]) and $\rho = 7800 \text{ kg/m}^3$, respectively. The sectional mass per unit height, taking account the effect of water, was $\rho A(z) = 15545 - 91z$ (see ref. [6]). A total of 1,591

tons topside load was applied at the top end of the cantilever beam.

An Eigenvalue analysis was performed and the natural frequencies of the simplified model were determined (refer to Fig. 2 for the first order natural frequency). The natural frequencies of the 3D FE model and the simplified model for the first 2 modes is presented in Table 3. A comparison of the natural frequencies between the 2 models showed that the difference of results was small. It could therefore be commented that the simplified model could actually be used in the preliminary design to assess the dynamic response of offshore jacket platforms.

To further justify the use of Timoshenko beam model for offshore jacket platforms, the P-M wave spectrum was applied. The significant wave height and the wave period were taken as 5.63m (after conversion) and $v_{a19.5} = 16.24 \text{ m/s}$, respectively.

A q-dimensional Ritz vector defined as $R_1(q) = [\eta^2, \eta^3, \dots, \eta^{q+1}]$ which $\eta = z / L$, was considered. It was clearly noted that the elements in the Ritz vector were linearly independent and satisfied the geometrical boundary conditions. From the calculation, the parameters were determined as $\epsilon = 0.05$ and $q = 3$.

The PSD of the displacement at the top and that of the bending moment at the mud line of the offshore jacket platform can be seen in Fig. 3. In the figure, the green line was the

Table4. Several spectral moments generated from the 2 models.

Item	jacket-top displacement (m ² s)		bending moment of jacket at mud line (N ² m ² s)	
	Simplified method	FEM	Simplified Method	FEM
σ_r^2	7.20e-10	7.23e-10	2.16e11	2.20e11
$\lambda_{2,r}$	7.53e-10	7.60e-10	1.98e11	2.01e11

results generated from the FE model, while the black line was the results from the proposed simplified method. The corresponding variances σ_r^2 and second spectral moments $\lambda_{2,r}$ of the FE model and the proposed simplified model are presented in Table4.

The PSD results in Fig. 3 showed that the maximum values of displacement at the top elevation of offshore jacket platform generated by the FE model and the proposed simplified model were 1.89×10^{-9} m²s (at 0.52 rad/s) and 1.87×10^{-9} m²s (at 0.51 rad/s), respectively. Similarly, the PSD results showed that the maximum values of bending moment at the mud line of offshore jacket platform generated by the FE model and the proposed simplified model were 5.45×10^{11} N²m²s (at 0.52 rad/s), and 5.43×10^{11} N²m²s (at 0.51 rad/s), respectively.

A comparison of results in Fig. 3 indicated that the maximum PSD and the corresponding frequency of the simplified model was slightly lower than those of the FE model. The curves plotted for the FE model and the simplified model were almost identical, and having the same peak value. It could therefore be concluded that the simplified model could actually provide an assessment on the dynamic response of offshore jacket platforms in the preliminary design stage.

A comparison of spectral moments as shown in Table 4 revealed that the results of the simplified model agreed very well with that of the FE model. This indicated that the prediction of spectral moment by the simplified model was quite similar to that by the FE model. However, considering the complexity of the structural behavior of offshore jacket platform in response to waves, the proposed simplified model presented in this paper could only be utilized as an analytical tool to predict the dynamic performance of offshore jacket platforms.

V. CONCLUDING REMARKS

From the comparison of power spectral densities (PSDs), variances and second spectral moments of an offshore jacket platform between the simplified model and the FE model, it can be concluded that,

(1) The simplified model predicts the dynamic response of jacket platform under random wave is as accurate as the FE model in the frequency domain.

(2) The assessment with the combined pseudo-excitation and classical Ritz method provides accurate and efficient solutions to determine the dynamic response of offshore jacket platform, without a need to generate the normal mode shapes of structure. This approach of analysis would have computa-

tional advantage and could be very useful for analyzing complex structures in the frequency domain.

ACKNOWLEDGEMENT

Author Contributions: Conceptualization, Xiaoshuang Han and Bo Zhou; methodology, Bo Zhou and Xiaoshuang Han; validation, Weiliang Qiao; writing—review and editing, Wie Min Gho.

Funding: This research is funded by Liaoning Revitalization Talents Program, grant number XLYC1807190; Liaoning Provincial Natural Science Foundation Guidance Project, grant number 20180550186; Open Project Funding of State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, grant number LP1906; and supported by “China Fundamental Research Funds for the Central Universities” (3132019368).

REFERENCES

- Asgarian, B., A. Mohebbinejad and R. H. Soltani (2004). Simplified method to assess dynamic response of jacket type offshore platforms subjected to wave loading. In: Proceedings of the twenty-third offshore mechanics and arctic engineering conference. Vancouver: British Columbia, 685-92.
- Balduzzi, G., M. Aminbaghai, E. Sacco, J. Füssl, J. Eberhardsteiner and F. Auricchio (2016). Non-prismatic beams: a simple and effective Timoshenko-like model. *International Journal of Solids and Structures* 90, 236–250.
- Berry, J.G. and P.M. Naghdi (1956). On the vibration of elastic bodies having time-dependent boundary conditions, *Quarterly of Applied Mathematics*, 14, 43–50.
- Bertolini, P., M. Eder, L. Tagliacarne and P. Valvo (2019). Stresses in constant tapered beams with thin-walled rectangular and circular cross sections. *Thin walled structures* 137, 527-540.
- Bokaian, A. (1990). Natural frequencies of beams under tensile axial loads. *Journal of Sound and Vibration*, 142 481–498.
- Elishakoff, I. and D. Livshits (1984a). Some closed-form solutions in random vibration of Bernoulli-Euler beams. *International Journal of Engineering Science* 22 (11), 1291–1301.
- Elishakoff, I. and D. Livshits (1984b). Some closed-form solutions in random vibration of Bresse-Timoshenko beams. *Probabilistic Engineering Mechanics* 4 (1), 49–54.
- Elishakoff, I. and E. Lubliner (1985) *Random Vibration of a Structure via Classical and Nonclassical Theories*, Probabilistic Methods in the Mechanics of Solids and Structures, Springer, Heidelberg, Berlin, 455–467.
- Elishakoff, I. and E. Lubliner (1988). *Random Vibrations of Structures with Complicating Effects*, Stochastic Structural Dynamics Progress: Theory and Applications, the Y.K. Lin Anniversary Volume, Elsevier, London, 47–64.
- Fung, Y. C. (1965). *Foundations of Solid Mechanics*. Prentice-Hall Inc., New Jersey.

- Graff, K. F. (1973). *Wave Motion in Elastic Solids*. Dover Publications, New York.
- Hahn, G. D. (1992) Sensitivity of the dynamic response of simple offshore structural models. *Ocean Engineering*, 19(3), 289–301.
- Herrmann, G. (1955). Forced motions of Timoshenko beams. *Journal of Applied Mechanics*. ASME. 22, 53-56.
- Horr, A. M. and M. Safi (2003). Full dynamic analysis of offshore platform structures using exact Timoshenko pipe element. *Journal of Offshore Mechanics and Arctic Engineering*, 125(3):168–75.
- Kim, Y. W. (2016). Dynamic analysis of Timoshenko beam subjected to support motions. *Journal of Mechanical Science & Technology* 30 (9), 4167–4176.
- Labuschagne, A., N. F. J. Van Rensburg and A. J. Van der Merwe (2009). Comparison of linear beam theory. *Mathematical & Computer Modelling*, 49, 20–30.
- Lee, H.P. (1995) Dynamic response of a rotating Timoshenko shaft subject to axial forces and moving loads. *Journal of Sound and Vibration* 181, 169–177.
- Lee, S. Y. and S. M. Lin (1998). Non-uniform Timoshenko beams with time-dependent elastic boundary conditions. *Journal of Sound and Vibration* 217 (2), 223–238.
- Lin, J. H. and Y. H. Zhang (2004). *Pseudo excitation method of random vibration*. Science Press.
- Lin, S. M. and S. Y. Lee (2002). The forced vibration and boundary control of pre-twisted Timoshenko beams with general time dependent elastic boundary conditions, *Journal of Sound and Vibration* 254 (1), 69–90.
- Moharrami, M. and M. Tootkaboni (2014). Reducing response of offshore platforms to wave loads using hydrodynamic buoyant mass dampers. *Engineering Structures*, 81, 162-174.
- Murin, J., M. Aminbaghai, V. Kutis and J. Hrabovsky (2013) Modal analysis of the FGM beams with effect of axial force under longitudinal variable elastic Winkler foundation. *Engineering Structures* 49, 234–247.
- Nguyen, C. T. and S. Oterkus (2019). Peridynamics formulation for beam structures to predict damage in offshore structures. *Ocean Engineering* 173, 244–267.
- Pratiher, B. (2012). Vibration control of a transversely excited cantilever beam with tip mass. *Archive of Applied Mechanics* 82, 31–42.
- Shaker, F. J. (1975). Effect of Axial Load on Mode Shapes and Frequencies of Beams, NASA Technical Note (NASA-TN-8109).
- Sunder, S. S. and J. J. Connor (1981). Sensitivity analyses for steel jacket offshore platforms. *Applied Ocean Research* 3(1), 13–26.
- Tan, X., H. Ding and L. Q. Chen (2019). Nonlinear frequencies and forced responses of pipes conveying fluid via a couples Timoshenko model. *Journal of Sound and Vibration* 455, 241–255.
- Timoshenko, S. P. (1921). On the correction for shear of the differential equation for transverse vibrations of bars of uniform cross-section. *Philosophical Magazine* 41, 744–746.
- Timoshenko, S. P. (1922). On the transverse vibrations of bars of uniform cross-section. *Philosophical Magazine* 43, 125–131.
- Trahair, N. and P. Ansourian (2016). In-plane behavior of web-tapered beams. *Engineering Structures* 108, 47–52.
- Wei, K., A. T. Myers and S. R. Arwade (2017). Dynamic effects in the response of offshore wind turbines supported by jackets under wave loading, *Engineering Structures*, 142, 36-45.
- Yesilce, Y. and O. Demirdag (2008). Effect of axial force on free vibration of Timoshenko multi-span beam carrying multiple spring-mass systems, *International Journal of Mechanical Sciences* 50, 995–1003.
- Yu, Y. X. and S. X. Liu (2011). *Random Wave and Its Applications to Engineering* (4th Edition). Dalian University of Technology Press.
- Zhang, G.Y., S. Q. Wang, H. Lu, Z. Q. Zhang and Z. Zong (2018). Coupling immersed method with node-based partly smoothed point interpolation method (NPS-PIM) for large-displacement fluid-structure interaction problems, *Ocean Engineering*, 157, 180–201.
- Zhang, G. Y., S. Q. Wang, Z. X. Sui, L. Sun, Z. Q. Zhang and Z. Zong (2019). Coupling of SPH with smoothed point interpolation method for violent fluid structure interaction problems. *Engineering Analysis with Boundary Elements* 103, 1–10.
- Zhou, B., X. Han and S. Tan (2014). A simplified computational method for random seismic responses of a Jacket Platform. *Ocean Engineering* 82, 85-90.