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EVALUATION OF DISTRIBUTION CENTER LOCATION BY FMCGDM BASED ON EXTENDED FUZZY PREFERENCE RELATION

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Key words: distribution center; extended fuzzy preference relation; FMCGDM; location evaluation.

ABSTRACT

Evaluation of distribution center location, under uncertain environment belonging to fuzzy multi-criteria group decision-making (FMCGDM) problems, is an essential element of global supply chain management. In order to take into imprecision and vagueness of decision-making, this study uses Lee's (2005a; 2005b) extended fuzzy preference relation to evaluate the problem of distribution center location in global supply chain management. In the location evaluation problem, all evaluation ratings and criteria weights, with the exception of investment cost evaluation ratings, are given linguistic values, and then transformed into trapezoidal fuzzy numbers. By Lee's (2005a; 2005b) extended fuzzy preference relation, preference degrees between the evaluation ratings of candidate locations and the ideal (or anti-ideal) solution for all criteria are first derived, and then the preference degrees are multiplied by their corresponding criteria weights into weighted preference degrees for all criteria for separate locations. The weighted preference degrees and their related locations are aggregated, and then converted into corresponding location evaluation indices. Finally, these locations are ranked according to their corresponding evaluation indices, and the best distribution center location is determined.

I. INTRODUCTION

In global supply chains (Bowersox et al., 2019), distribution centers play a crucial role in connecting suppliers and demanders. Practically, suppliers include manufacturers and distributors, whereas demanders are retailers and end-customers. Therefore, distribution center location is a critical issue encountered by decision-makers because it determines the convenience with which suppliers and demanders are connected. Generally, a good distribution center increases revenue and decreases transportation costs between suppliers and demanders. The problem of identifying an optimal location of distribution center is a location problem, and such location problems are regarded as layout problems. In the distribution center location layout problem (Bowersox et al., 2019), decision-makers must consider some important factors (Wang and Lee, 2007, Wang, 2015), including climate conditions, demand quantity, expansion possibility, investment costs, labor force quality, transportation availability, etc., according to global supply chain management. These factors are used as evaluation criteria for evaluating the location of a distribution center, and thus the problem is also a multi-criteria decision-making (MCDM) problem (Churchman et al., 1957; Hwang and Yoon, 1981, Fu et al., 2019). An MCDM problem is formatted as

$$G = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \text{ and } W = \left[W_1, W_2, ..., W_n\right],$$

where G_{ij} is the evaluation rating of the *ith* alternative on the *jth* criterion, and W_j is the weight of the *jth* criterion. Some MCDM problems are fuzzy multi-criteria decision-making (FMCDM) problems (Igoulalene and Benyoucef, 2014), in which evaluation ratings and criteria weights include imprecision, subjectivity or vagueness. These ratings and weights are often presented as linguistic terms (Delgado et al., 1992; Herrera et al., 1996), and then transformed into fuzzy numbers (Zadeh, 1965). Further, FMCDM employed by a decision-making group is called fuzzy multi-criteria group decision-making (FMCGDM) (Kacprzyk et al., 1992; Hsu and Chen, 1996).

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Previous studies used classical MCDM methods under uncertain environments to solve FMCGDM problems. The utilizations of classical MCDM are usually classified into two categories, defuzzification and fuzzy extension methods. Defuzzification methods often lose lots of messages, so some (Liang, 1999; Raj and Kumar, 1999; Chen, 2000) extended classical MCDM methods under fuzzy environments. However, their works of fuzzy extension had some drawbacks. Liang (1999) and Raj and Kumar (1999) they applied maximizing and minimizing sets developed by Chen (1985) to rank evaluated values presented by approximate trapezoidal fuzzy numbers. The ranking of approximate trapezoidal fuzzy numbers was complicated and hard. Additionally, distance values from two different alternatives to the ideal (or anti-ideal) solution would be equal when intersections of the two alternatives and the ideal (or anti-ideal) solution on all criteria were null. In Chen's (2000) fuzzy extension, (1,1,1) and (0,0,0) were viewed as the best and worst values, which might be far from away the maximum and minimum values. Therefore, (1,1,1) and (0,0,0) could not be used for the maximum and minimum values of alternatives on criteria. Besides, weighted ratings were presented by triangular fuzzy numbers as evaluation ratings, and criteria weights were also triangular fuzzy numbers in Chen's method. Practically, the product of multiplying two triangular fuzzy numbers will not be a triangular fuzzy number, whereas it should be an approximate triangular fuzzy number. To avoid the above drawbacks, this study utilizes an FMCGDM method based on extended fuzzy preference relation proposed by Lee (2005a; 2005b) to evaluate candidate locations in establishing a distribution center. By FMCGDM-based extended fuzzy preference relation, the evaluation of distribution center location can be easily and quickly achieved.

For the sake of clarity, mathematical theory concerning fuzzy numbers is presented in Section 2. Lee's extended fuzzy preference relation (2005a; 2005b) on fuzzy numbers is also expressed in the section. Traditional MCDM methods applied in uncertain environments are described in Section 3. In Section 4, FMCGDM based on the extended fuzzy preference relation is proposed to evaluate the location of a distribution center. A numerical example evaluating several locations for establishing a distribution center solved by FMCGDM based on the extended fuzzy preference relation is illustrated in Section 5.

II. MATHEMATICAL PRELIMINARIES

This section provides definitions concerning the rationales of fuzzy sets (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991).

Definition 2.1: Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A(x) \to [0,1]$, where $\mu_A(x)$, and $\forall x \in U$ indicates the degree of x in A.

Definition 2.2: The $\alpha - cut$ of fuzzy set A is a crisp set $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$.

Definition 2.3: The support of fuzzy set A is a crisp set $Supp(A) = \{x \mid \mu_A(x) > 0\}$.

Definition 2.4: A fuzzy subset A of U is normal iff $\sup_{x \in U} \mu_A(x) = 1$.

Definition 2.5: A fuzzy subset A of U is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geq (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in U$, $\forall \lambda \in [0,1]$, where $\wedge$ denotes the minimum operator.

Definition 2.6: A fuzzy subset A of U is a fuzzy number iff A is both normal and convex.

Definition 2.7: A trapezoidal fuzzy number A is a fuzzy set with membership function μ_A defined by

$$\mu_A = \begin{cases} \dfrac{x - a_l}{a_h - a_l}, & a_l \leq x \leq a_h \\ 1, & a_h \leq x \leq a_r \\ \dfrac{a_u - x}{a_u - a_r}, & a_r \leq x \leq a_u \\ 0, & otherwise \end{cases},$$

which is denoted as a quartet (a_l, a_h, a_r, a_u) (Zadeh, 1965).

Definition 2.8: Let $\circ$ be an operation on real numbers, such as $+$, $-$, $*$, $\wedge$, $\vee$, etc. Let A and B be two fuzzy numbers. By extension principle, an extended operation $\circ$ on fuzzy numbers is defined by

$$\mu_{A \circ B}(z) = \sup_{x,y:z=x\circ y}\{\mu_A(x) \wedge \mu_B(y)\}. \tag{1}$$

Definition 2.9: Let A be a fuzzy number. Then A_α^L and A_α^U are respectively defined as $A_\alpha^L = \inf_{\mu_A(z) \geq \alpha}(z)$ and $A_\alpha^U = \sup_{\mu_A(z) \geq \alpha}(z)$.

Definition 2.10: A fuzzy preference relation R is a fuzzy subset of $\Re \times \Re$ with a membership function $\mu_R(A, B)$ representing the preference degree of fuzzy numbers A over B (Epp, 1990; Lee, 2005a; Lee, 2005b).

(1) R is reciprocal iff $\mu_R(A, B) = 1 - \mu_R(B, A)$ for all fuzzy numbers A and B.
(2) R is transitive iff $\mu_R(A, B) \geq 0.5$ and $\mu_R(B, C) \geq 0.5 \Rightarrow \mu_R(A, C) \geq 0.5$ for all fuzzy numbers A, B and C.
(3) R is fuzzy total ordering iff R is both reciprocal and transitive.

Based on the above, A is preferred to B iff $\mu_R(A,B) > 0.5$. Additionally, A is equal to B iff $\mu_R(A,B) = 0.5$.

Definition 2.11: An extended fuzzy preference relation R' (Lee, 2005a; Lee, 2005b) is a fuzzy subset of $\Re \times \Re$ with a membership function $-\infty \le \mu_{R'}(A,B) \le \infty$ representing the preference degree of fuzzy numbers A over B.

(1) R' is reciprocal iff $\mu_{R'}(A,B) = -\mu_{R'}(B,A)$ for all fuzzy numbers A and B.
(2) R' is transitive iff $\mu_{R'}(A,B) \ge 0$ and $\mu_{R'}(B,C) \ge 0 \Rightarrow \mu_{R'}(A,C) \ge 0$ for all fuzzy numbers A, B and C.
(3) R' is additive iff $\mu_{R'}(A,C) = \mu_{R'}(A,B) + \mu_{R'}(B,C)$.
(4) R' is a total ordering relation iff R' is reciprocal, transitive and additive.

Therefore, A is not smaller than B iff $\mu_{R'}(A,B) \ge 0$. Further, A is preferred to B iff $\mu_{R'}(A,B) > 0$. Additionally, A and B are equal iff $\mu_{R'}(A,B) = 0$.

Definition 2.12: For two fuzzy numbers A and B, Lee (2005a; 2005b) proposed an extended fuzzy preference relation R^* with membership function $\mu_{R^*}(A,B)$ representing the extended fuzzy preference degree of fuzzy numbers A over B defined as

$$\mu_{R^*}(A,B) = \int_0^1 ((A_\alpha^U - B_\alpha^L) + (A_\alpha^L - B_\alpha^U))d\alpha . \quad (2)$$

Lemma 2.1: R^* is reciprocal due to

$$\mu_{R^*}(A,B) = -\mu_{R^*}(B,A) . \quad (3)$$

Lemma 2.2: R^* is transitive because

$$\mu_{R^*}(A,B) \ge 0 \text{ and } \mu_{R^*}(B,C) \ge 0 \;\Rightarrow\; \mu_{R^*}(A,C) \ge 0 . \quad (4)$$

Lemma 2.3: R^* is additive because

$$\mu_{R^*}(A,B) + \mu_{R^*}(B,C) = \mu_{R^*}(A,C) . \quad (5)$$

According to the three above lemmas, Lee's extended fuzzy preference relation R^* is a total ordering relation.

Definition 2.13: If $A = (a_l, a_h, a_r, a_u)$ and $B = (b_l, b_h, b_r, b_u)$ are two trapezoidal fuzzy numbers, addition $\oplus$ of the two trapezoidal fuzzy numbers by extension principle (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991) is defined as

$$\begin{aligned} A \oplus B &= (a_l, a_h, a_r, a_u) \oplus (b_l, b_h, b_r, b_u) \\ &= (a_l + b_l, a_h + b_h, a_r + b_r, a_u + b_u) \end{aligned} \quad (6)$$

Definition 2.14: In case that $A = (a_l, a_h, a_r, a_u)$, multiplication $\otimes$ of the trapezoidal fuzzy number and a real number t based on extension principle (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991) is defined as

$$A \otimes t = (a_l, a_h, a_r, a_u) \otimes t = (ta_l, ta_h, ta_r, ta_u) . \quad (7)$$

Definition 2.15: Let $A = (a_l, a_h, a_r, a_u)$ and $B = (b_l, b_h, b_r, b_u)$ be two trapezoidal fuzzy numbers. Lee's extended fuzzy preference relation R^* (Lee, 2005a; Lee, 2005b) with the membership function $\mu_{R^*}(A,B)$ representing the extended fuzzy preference degree of fuzzy numbers A over B is defined as

$$\mu_{R^*}(A,B) = \frac{(a_l - b_u) + (a_h - b_r) + (a_r - b_h) + (a_u - b_l)}{2} , \quad (8)$$

and

$$\begin{aligned} &\mu_{R^*}(A,B) \ge 0 \text{ iff} \\ &(a_l - b_u) + (a_h - b_r) + (a_r - b_h) + (a_u - b_l) \ge 0 . \end{aligned} \quad (9)$$

In addition,

$$\mu_{R^*}(A,0) = \frac{a_l + a_h + a_r + a_u}{2} \quad (10)$$

where a crisp value 0 is extended into a trapezoidal fuzzy number $(0,0,0,0)$.

III. TRADITIONAL MCDM METHODS APPLIED IN UNCERTAIN ENVIRONMENTS

Traditional MCDM methods are often applied in uncertain environments to solve decision-making problems (Abdel-Baset et al., 2019). In this study, the MCDM methods applied include simple additive weighting (SAW) (Churchman et al., 1957) and technique for order preference by similarity to ideal solution (TOPSIS) (Hwang and Yoon, 1981). These two well-known methods are useful for developing FMCGDM in this paper. To more clearly describe the proposed FMCGDM, the computation steps of SAW are presented below:

Step 1: Identify a decision matrix in a giving MCDM problem.
The decision matrix G is similar to the one shown in Section 1.

Step 2: Normalize elements of the decision matrix into a normalized decision matrix.

Let G_{ij} be an evaluation rating in the decision matrix, and

$$g_{ij} = \begin{cases} \dfrac{G_{ij}}{\max_i \{G_{ij}\}} & if \quad j \in J \\ \dfrac{\min_i \{G_{ij}\}}{G_{ij}} & if \quad j \in J' \end{cases} \quad (11)$$

be the normalization of G_{ij}, where J is a set composed of benefit criteria, and J' is a set consisting of cost criteria for $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

Step 3: Obtain a weighted decision matrix from the normalized decision matrix.

Let $g_{ij} \times W_j$ be the weighted rating of g_{ij} to construct a weighted decision matrix for $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

Step 4: Derive alternative evaluation indices from the weighted decision matrix.

Let

$$A_i' = \sum_{j=1}^{n} g_{ij} \times W_j \quad (12)$$

be the evaluation index of the *ith* alternative, where $i = 1, 2, ..., m$.

Step 5: Rank alternatives according to related evaluation indices.

Alternatives are ranked according to their related evaluation indices $A_1', A_2', ..., A_m'$. The greater the evaluation index, the better the corresponding alternative.

Additionally, the computation steps of TOPSIS are described below:

Step 1: Identify a decision matrix for a given MCDM problem.

The decision matrix is similar to that described in Section 1.

Step 2: Standardize the decision matrix.

Let

$$g_{ij} = \frac{G_{ij}}{\sum_{i=1}^{m} G_{ij}} \quad (13)$$

be the standardized value of G_{ij} in the decision matrix, where $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

Step 3: Derive a weighted decision matrix from the standardized decision matrix.

Let

$$u_{ij} = g_{ij} \times W_j \quad (14)$$

be the weighted value of g_{ij} in the standardized decision matrix, where $i = 1, 2, ..., m$; $j = 1, 2, ..., n$.

Step 4: Find the ideal solution A^+ and the anti-ideal solution A^-.

Let $A^+ = [u_1^+, u_2^+, ..., u_n^+]$ and $A^- = [u_1^-, u_2^-, ..., u_n^-]$, where

$$u_j^+ = \begin{cases} \max_i \{u_{ij}\} & if \quad j \in J \\ \min_i \{u_{ij}\} & if \quad j \in J' \end{cases}, \quad (15)$$

$$u_j^- = \begin{cases} \min_i \{u_{ij}\} & if \quad j \in J \\ \max_i \{u_{ij}\} & if \quad j \in J' \end{cases}, \quad (16)$$

J is a set consisting of benefit criteria, and J' is a set composed of cost criteria.

Step 5: Obtain distance values between alternatives and the ideal solution/anti-ideal solution.

Let A_i^+ be the distance between the *ith* alternative and the ideal solution, and A_i^- be the distance between the *ith* alternative and the anti-ideal solution,
where

$$A_i^+ = (\sum_{j=1}^{n} (u_{ij} - u_j^+)^2)^{1/2}, \forall i \quad (17)$$

and

$$A_i^- = (\sum_{j=1}^{n} (u_{ij} - u_j^-)^2)^{1/2}, \ \forall i. \quad (18)$$

Step 6: Calculate the relative closeness coefficients of all alternatives.

Let

$$A_i^* = \frac{A_i^-}{A_i^- + A_i^+} \quad (19)$$

be the relative closeness coefficient of A_i, $i = 1, 2, ..., m$.

Step 7: Rank alternatives according to their relative closeness coefficients.

Obviously, $0 \le A_i^* \le 1$. Alternatives are ranked according to their related evaluation indices $A_1^*, A_2^*, ..., A_m^*$. The greater the evaluation index, the better the corresponding alternative. Therefore, the best alternative is the one that has the maximum relative closeness coefficient among feasible alternatives.

Using the definitions given in Section 2 and the two methods above, this study develops the proposed FMCGDM based on extended fuzzy preference relation (Lee, 2005a; Lee, 2005b) for trapezoidal fuzzy numbers.

IV. FMCGDM BASED ON EXTENDED FUZZY PREFERENCE RELATION

To evaluate the problem of distribution center location, the proposed FMCGDM based on Lee's extended fuzzy preference relation (2005a; 2005b) is used in this section. First, *m* locations $A_1, A_2, ..., A_m$ are evaluated by a decision-making group of experts $E_1, E_2, ..., E_t$ using criteria $C_1, C_2, ..., C_n$ in the evaluation problem. The FMCGDM method is described below. Let $G_{ijk} = (g_{ijkl}, g_{ijkh}, g_{ijkr}, g_{ijku})$ be the evaluation rating given by expert E_k to distribution center A_i for criterion C_j, where $i = 1,2,...,m$; $j = 1,2,...,n$; $k = 1,2,...,t$. Then

$$G_{ij} = \frac{1}{t} \otimes (G_{ij1} \oplus G_{ij2} \oplus ... \oplus G_{ijt}) = (g_{ijl}, g_{ijh}, g_{ijr}, g_{iju}) \quad (20)$$

indicates the evaluation rating of A_i for C_j, where $g_{ijl} = \frac{1}{t}\sum_{k=1}^{t} g_{ijkl}$, $g_{ijh} = \frac{1}{t}\sum_{k=1}^{t} g_{ijkh}$, $g_{ijr} = \frac{1}{t}\sum_{k=1}^{t} g_{ijkr}$, and $g_{iju} = \frac{1}{t}\sum_{k=1}^{t} g_{ijku}$ for $i = 1,2,...,m$; $j = 1,2,...,n$. Then $\tilde{G}_{ij} = (\tilde{g}_{ijl}, \tilde{g}_{ijh}, \tilde{g}_{ijr}, \tilde{g}_{iju})$ is assumed to be the normalized value of G_{ij}, and classified into three situations presented in the following.

(i) As G_{ij} is evaluated by linguistic, ordinal or qualitative terms (Delgado et al., 1992; Herrera et al. 1996), and then transferred into a trapezoidal fuzzy number in the interval [0,1], $\tilde{G}_{ij} = G_{ij}$.

(ii) As G_{ij} belongs to cost criteria,

$$\tilde{G}_{ij} = (\frac{g_{jl}^-}{g_{iju}}, \frac{g_{jl}^-}{g_{ijr}}, \frac{g_{jl}^-}{g_{ijh}}, \frac{g_{jl}^-}{g_{ijl}}), \quad (21)$$

where $g_{jl}^- = \min_i \{g_{ijl}\}$, $\forall j$.

(iii) As G_{ij} belongs to benefit criteria,

$$\tilde{G}_{ij} = (\frac{g_{ijl}}{g_{jr}^+}, \frac{g_{ijh}}{g_{jr}^+}, \frac{g_{ijr}}{g_{jr}^+}, \frac{g_{ijr}}{g_{ju}^+}), \quad (22)$$

where $g_{ju}^+ = \max_i \{g_{iju}\}$, $\forall j$.

Additionally, $W_{jk} = (w_{jkl}, w_{jkh}, w_{jkr}, w_{jku})$ denotes the weight of criterion C_j evaluated by expert E_k, where $j = 1,2,...,n$; $k = 1,2,...,t$. Let

$$W_j = \frac{1}{t} \otimes (W_{j1} \oplus W_{j2} \oplus ... \oplus W_{jt}) = (w_{jl}, w_{jh}, w_{jr}, w_{ju}) \quad (23)$$

be the weight of C_j, where $w_{jl} = \frac{1}{t}\sum_{k=1}^{t} w_{jkl}$, $w_{jh} = \frac{1}{t}\sum_{k=1}^{t} w_{jkh}$, $w_{jr} = \frac{1}{t}\sum_{k=1}^{t} w_{jkr}$ and $w_{ju} = \frac{1}{t}\sum_{k=1}^{t} w_{jku}$, for $j = 1,2,...,n$.

Based on the above evaluation ratings and criteria, normalized ratings of the *ith* distribution center for n criteria are used to derive a vector $A_i = [\tilde{G}_{i1}, \tilde{G}_{i2}, ..., \tilde{G}_{in}]$ for $i = 1,2,...,m$. Then the ideal solution and the anti-ideal solution (Wang, 2014) are found as the computation of TOPSIS (Hwang et al. 1981). Thus, the ideal solution and the anti-ideal solution (Wang, 2014) obtained from the *m* normalized vectors of locations for *n* criteria are

$A^+ = [\tilde{G}_1^+, \tilde{G}_2^+, ..., \tilde{G}_n^+]$ and $A^- = [\tilde{G}_1^-, \tilde{G}_2^-, ..., \tilde{G}_n^-]$ respectively. Herein,

$$\tilde{G}_j^+ = (\max_i \{\tilde{g}_{ijl}\}, \max_i \{\tilde{g}_{ijh}\}, \max_i \{\tilde{g}_{ijr}\}, \max_i \{\tilde{g}_{iju}\}) \quad (24)$$

are the maximum rating for the *jth* criterion for all normalized vectors of locations, and

$$\tilde{G}_j^- = (\min_i \{\tilde{g}_{ijl}\}, \min_i \{\tilde{g}_{ijh}\}, \min_i \{\tilde{g}_{ijr}\}, \min_i \{\tilde{g}_{iju}\}) \quad (25)$$

are the minimum rating for the *jth* criterion for all normalized vectors of locations, where $j = 1,2,...,n$. Then $\mu_{R*}(\tilde{G}_j^+, \tilde{G}_{ij})$ denotes the extended fuzzy preference degree of A^+ over A_i for the *jth* criterion, and $\mu_{R*}(\tilde{G}_{ij}, \tilde{G}_j^-)$ indicates the extended fuzzy preference degree of A_i over A^- for the criterion, where $i = 1,2,...,m$; $j = 1,2,...,n$.

By Definition 2.15,

$$\mu_{R*}(\tilde{G}_j^+,\tilde{G}_{ij})=\frac{(\max_i\{\tilde{g}_{ijl}\}-\tilde{g}_{iju})+(\max_i\{\tilde{g}_{ijh}\}-\tilde{g}_{ijr})+(\max_i\{\tilde{g}_{ijr}\}-\tilde{g}_{ijh})+(\max_i\{\tilde{g}_{iju}\}-\tilde{g}_{ijl})}{2} \tag{26}$$

and

$$\mu_{R*}(\tilde{G}_{ij},\tilde{G}_j^-)=\frac{(\tilde{g}_{ijl}-\min_i\{\tilde{g}_{iju}\})+(\tilde{g}_{ijh}-\min_i\{\tilde{g}_{ijr}\})+(\tilde{g}_{ijr}-\min_i\{\tilde{g}_{ijh}\})+(\tilde{g}_{iju}-\min_i\{\tilde{g}_{ijl}\})}{2}, \tag{27}$$

where $i=1,2,...,m$; $j=1,2,...,n$.

Using $\mu_{R*}(\tilde{G}_j^+,\tilde{G}_{ij})$, $\mu_{R*}(\tilde{G}_{ij},\tilde{G}_j^-)$ and W_j, D_i^+ is obtained to denote the weighted preference degree of A^+ over A_i, and D_i^- is derived to indicate the weighted preference degree of A_i over A^-, for $i=1,2,...,m$. Define

$$D_i^+=(W_1\otimes\mu_{R*}(\tilde{G}_1^+,\tilde{G}_{i1}))\oplus(W_2\otimes\mu_{R*}(\tilde{G}_2^+,\tilde{G}_{i2}))\oplus...\oplus(W_n\otimes\mu_{R*}(\tilde{G}_n^+,\tilde{G}_{in})) \tag{28}$$

and

$$D_i^-=(W_1\otimes\mu_{R*}(\tilde{G}_{i1},\tilde{G}_1^-))\oplus(W_2\otimes\mu_{R*}(\tilde{G}_{i2},\tilde{G}_2^-))\oplus...\oplus(W_n\otimes\mu_{R*}(\tilde{G}_{in},\tilde{G}_n^-)), \tag{29}$$

where $i=1,2,...,m$. Obviously, D_i^+ and D_i^- are trapezoidal fuzzy numbers for $i=1,2,...,m$. Let $D_i^+=(d_{il}^+,d_{ih}^+,d_{ir}^+,d_{iu}^+)$ and $D_i^-=(d_{il}^-,d_{ih}^-,d_{ir}^-,d_{iu}^-)$, where $i=1,2,...,m$.

Finally, the evaluation index D_i of A_i is defined as

$$D_i=\frac{\mu_{R*}(D_i^-,0)}{\mu_{R*}(D_i^+,0)}=\frac{d_{il}^-+d_{ih}^-+d_{ir}^-+d_{iu}^-}{d_{il}^++d_{ih}^++d_{ir}^++d_{iu}^+}, \tag{30}$$

where $\mu_{R*}(D_i^+,0)=\frac{d_{il}^++d_{ih}^++d_{ir}^++d_{iu}^+}{2}$ and $\mu_{R*}(D_i^-,0)=\frac{d_{il}^-+d_{ih}^-+d_{ir}^-+d_{iu}^-}{2}$ for $i=1,2,...,m$. Obviously, the larger the D_i value, the closer the ideal solution is. In contrast, the smaller the D_i value, the closer the anti-ideal solution is. Thus m distribution centers can be ranked according to their corresponding evaluation indices. In other words, once the evaluation indices of distribution center location are derived, the best distribution center location can be found.

V. A NUMERICAL EXAMPLE OF EVALUATING DISTRIBUTION CENTER LOCATION

As in the example described by Wang (2014), some decision-makers in a supply chain want to select the best location for a new distribution center in order to most efficiently connect suppliers with demanders. Four experts E_1, E_2, E_3 and E_4 are employed in a decision-making group to evaluate the distribution center location problem. The four experts evaluate possible locations based on five criteria, including climate and transportation availability (C_1), demand quantity (C_2), expansion possibility (C_3), labor force (C_4), and investment cost (C_5). Of the five criteria, investment cost (unit: millions of dollars) is a cost criterion, and the remainder are benefit perspectives. Through initial evaluation, the experts find that A_1, A_2 and A_3 are candidate locations.

Additionally, the following linguistic terms are employed to present evaluation ratings and criteria weights. These linguistic terms of evaluation ratings and criteria weights represented by trapezoidal fuzzy numbers are respectively shown in Table 1 and Table 2.

The linguistic terms in Table 1 used by the four experts present evaluation ratings, denoted as the entries in Table 3, for three candidate locations based on five criteria. Moreover, the linguistic terms in Table 2 were utilized to express criteria weights for five criteria, also shown in Table 3.

Through the entries of Table 1 and Table 3, normalized group fuzzy ratings of three candidate locations based on five criteria are derived and shown in Table 4. Similarly, criteria weights of five criteria based on the entries of Table 2 and Table 3 are obtained and displayed in Table 5.

According to the entries of Table 4, the ideal solution A^+ and the anti-ideal solution A^- of three candidate locations based on five criteria are obtained and presented in Table 6. Based on the entries of Table 4 and Table 6, the extended fuzzy preference degrees of the ideal solution over three candidate locations for five criteria are given in Table 7, and the extended fuzzy preference degrees of the locations over the anti-ideal solution for the five criteria are presented in Table 8.

Using the entries of Table 5 and Table 7, weighted preference degrees of the ideal solution over three candidate locations based on five criteria are derived and expressed in Table 9. Then the weighted preference degrees of Table 9 within the first, second and third locations are respectively aggregated into their weighted preference degrees of the ideal solution over locations, expressed in Table 10.

Table 1. The linguistic terms of evaluation ratings and corresponding fuzzy numbers

Linguistic terms	Corresponding fuzzy numbers
Very poor (VP)	(0, 0, 0.05, 0.15)
Poor (P)	(0.05, 0.15, 0.25, 0.35)
Medium poor (MP)	(0.2, 0.3, 0.4, 0.5)
Fair (F)	(0.35, 0.45, 0.55, 0.65)
Medium good (MG)	(0.5, 0.6, 0.7, 0.8)
Good (G)	(0.65, 0.75, 0.85, 0.95)
Very good (VG)	(0.85, 0.95, 1, 1)

Table 2. The linguistic terms of criteria weights and corresponding fuzzy numbers

Linguistic terms	Corresponding fuzzy numbers
Very low (VL)	(0, 0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.3, 0.4)
Medium (M)	(0.3, 0.4, 0.6, 0.7)
High (H)	(0.6, 0.7, 0.8, 0.9)
Very high (VH)	(0.8, 0.9, 1, 1)

Table 3. The linguistic ratings of four experts for three candidate locations based on five criteria and the linguistic weights of four experts for five criteria

Locations and weights	Criteria	E_1	E_2	E_3	E_4
Location A_1	C_1	VG	G	G	VG
	C_2	VG	MG	G	MG
	C_3	MG	F	MG	MG
	C_4	G	VG	G	G
	C_5	8	10	8	10
Location A_2	C_1	G	MG	VG	MG
	C_2	F	MG	F	F
	C_3	VG	VG	MG	VG
	C_4	G	G	MG	MG
	C_5	4	6	3	5
Location A_3	C_1	F	G	MG	F
	C_2	G	VG	G	VG
	C_3	MG	G	MG	G
	C_4	F	G	G	F
	C_5	8	9	9	10
Weights	C_1	H	M	VH	H
	C_2	M	H	H	H
	C_3	H	M	VH	H
	C_4	VH	H	H	VH
	C_5	H	M	H	M

Table 4. Normalized group fuzzy ratings of three candidate locations based on five criteria

	A_1	A_2	A_3
C_1	(0.75, 0.85, 0.925, 0.975)	(0.625, 0.725, 0.8125, 0.8875)	(0.4625, 0.5625, 0.6625, 0.7625)
C_2	(0.625, 0.725, 0.8125, 0.8875)	(0.3875, 0.4875, 0.5875, 0.6875)	(0.75, 0.85, 0.925, 0.975)
C_3	(0.4625, 0.5625, 0.6625, 0.7625)	(0.7625, 0.8625, 0.925, 0.95)	(0.575, 0.675, 0.775, 0.875)
C_4	(0.7, 0.8, 0.8875, 0.9625)	(0.575, 0.675, 0.775, 0.875)	(0.5, 0.6, 0.7, 0.8)
C_5	(0.5, 0.5, 0.5, 0.5)	(1, 1, 1, 1)	(0.5, 0.5, 0.5, 0.5)

Table 5. Criteria weights of five criteria

	Criteria weights
W_1	(0.575, 0.675, 0.8, 0.875)
W_2	(0.525, 0.625, 0.75, 0.85)
W_3	(0.575, 0.675, 0.8, 0.875)
W_4	(0.7, 0.8, 0.9, 0.95)
W_5	(0.45, 0.55, 0.7, 0.8)

Table 6. The ideal solution and the anti-ideal solution of three candidate locations based on five criteria

	A^+	A^-
C_1	(0.75, 0.85, 0.925, 0.975)	(0.4625, 0.5625, 0.6625, 0.7625)
C_2	(0.75, 0.85, 0.925, 0.975)	(0.3875, 0.4875, 0.5875, 0.6875)
C_3	(0.7625, 0.8625, 0.925, 0.95)	(0.4625, 0.5625, 0.6625, 0.7625)
C_4	(0.7, 0.8, 0.8875, 0.9625)	(0.5, 0.6, 0.7, 0.8)
C_5	(1, 1, 1, 1)	(0.5, 0.5, 0.5, 0.5)

Table 7. The extended fuzzy preference degrees of the ideal solution over three candidate locations based on five criteria

$\mu_{R*}(\tilde{G}_j^+, \tilde{G}_{ij})$	A_1	A_2	A_3
C_1	0	0.225	0.525
C_2	0.225	0.675	0
C_3	0.525	0	0.3
C_4	0	0.225	0.375
C_5	1	0	1

Table 8. The extended fuzzy preference degrees of three candidate locations over the anti-ideal solution based on five criteria

$\mu_{R*}(\tilde{G}_{ij}, \tilde{G}_j^-)$	A_1	A_2	A_3
C_1	0.525	0.3	0
C_2	0.45	0	0.675
C_3	0	0.525	0.225
C_4	0.375	0.15	0
C_5	0	1	0

Table 9. Weighted preference degrees of the ideal solution over three candidate locations based on five criteria

$W_j \otimes \mu_{P*}(\tilde{G}_j^+, \tilde{G}_{ij})$	A_1	A_2	A_3
C_1	(0, 0, 0, 0)	(0.12938, 0.15188, 0.18, 0.19688)	(0.30188, 0.35438, 0.42, 0.45938)
C_2	(0.11813, 0.14063, 0.16875, 0.19125)	(0.35438, 0.42188, 0.50625, 0.57375)	(0, 0, 0, 0)
C_3	(0.30188, 0.35438, 0.42, 0.45938)	(0, 0, 0, 0)	(0.1725, 0.2025, 0.24, 0.2625)
C_4	(0, 0, 0, 0)	(0.1575, 0.18, 0.2025, 0.21375)	(0.2625, 0.3, 0.3375, 0.35625)
C_5	(0.45, 0.55, 0.7, 0.8)	(0, 0, 0, 0)	(0.45, 0.55, 0.7, 0.8)

Table 10. Weighted preference degrees of the ideal solution over three candidate locations

	Weighted preference degrees
D_1^+	(0.87, 1.045, 1.28875, 1.45063)
D_2^+	(0.64125, 0.75375, 0.88875, 0.98438)
D_3^+	(1.18688, 1.40688, 1.6975, 1.87813)

Table 11. Weighted preference degrees of three candidate locations over the anti-ideal solution based on five criteria

$W_j \otimes \mu_{P*}(\tilde{G}_{ij}, \tilde{G}_j^-)$	A_1	A_2	A_3
C_1	(0.301875, 0.35438, 0.42, 0.45938)	(0.1725, 0.2025, 0.24, 0.2625)	(0, 0, 0, 0)
C_2	(0.23625, 0.28125, 0.3375, 0.3825)	(0, 0, 0, 0)	(0.354375, 0.42188, 0.50625, 0.57375)
C_3	(0, 0, 0, 0)	(0.301875, 0.35438, 0.42, 0.45938)	(0.129375, 0.15188, 0.18, 0.19688)
C_4	(0.2625, 0.3, 0.3375, 0.35625)	(0.105, 0.12, 0.135, 0.1425)	(0, 0, 0, 0)
C_5	(0, 0, 0, 0)	(0.45, 0.55, 0.7, 0.8)	(0, 0, 0, 0)

Table 12. Weighted preference degrees of three candidate locations over the anti-ideal solution

	Weighted preference degrees
D_1^-	(0.800625, 0.93563, 1.095, 1.19813)
D_2^-	(1.029375, 1.22688, 1.495, 1.66438)
D_3^-	(0.48375, 0.57375, 0.68625, 0.77063)

Table 13. The evaluation indices of three candidate locations

	Evaluation indices
D_1	0.86572
D_2	1.65710
D_3	0.40756

Through the entries of Table 5 and Table 8, weighted preference degrees of the three candidate locations over the anti-ideal solution based on five criteria are obtained and shown in Table 11. The weighted preference degrees of Table 11 within the first, second and third locations are then respectively summarized into their weighted preference degrees of three candidate locations over the anti-ideal solution, presented in Table 12.

Using the entries of Tables 10 and 12, the evaluation indices of three candidate locations are derived, and shown in Table 13.

The evaluation indices of the three candidate locations are sorted as $D_2 > D_1 > D_3$, and thus the rank order determined by the evaluation indices is $A_2 > A_1 > A_3$. Obviously, A_2 is the best location for establishing a distribution center.

VI. CONCLUSIONS

This study used FMCGDM based on Lee's (2005a; 2005b) extended fuzzy preference relation to evaluate the problem of distribution center location. From previous computations, FMCGDM can be regarded as the fuzzy extension of SAW and TOPSIS. By FMCGDM, weighted preference degrees of the ideal solution over three candidate locations, and the locations over the anti-ideal solution for all criteria were obtained. Then the evaluation indices of three candidate locations based on the extended fuzzy preference relation were derived after aggregating the weighted preference degrees along separate locations for all criteria. Thus, the three candidate locations were simply ranked and the location evaluation problem was easily solved. Additionally, the FMCGDM provides preference degrees with weights based on criteria besides location evaluation indices, so decision-makers can evaluate candidate locations from different perspectives.

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